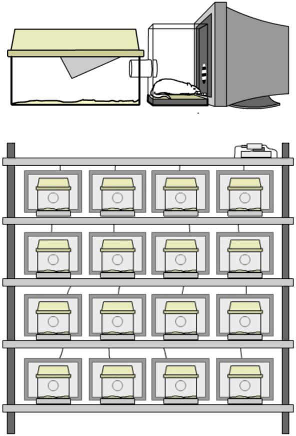
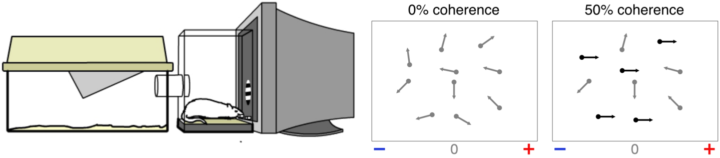
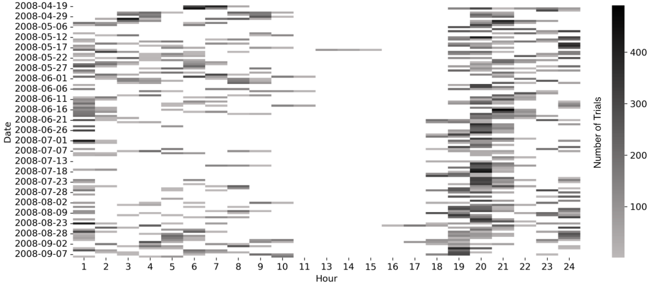
Drift Diffusion Model of Decision Making



Gabriel Riegner (work with Pamela Reinagel, Armin Schwartzman)
UC San Diego, March 2025

Experimental Design

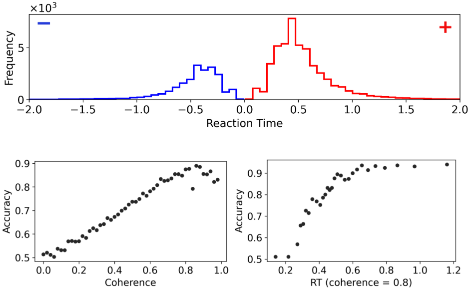




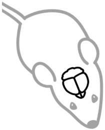
[dot motion example]

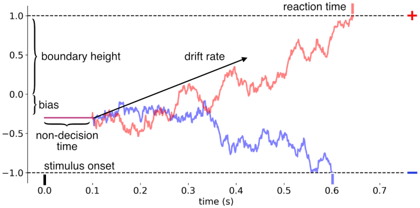
Brain and Behavior

Observed Behaviors:
Speed + Accuracy

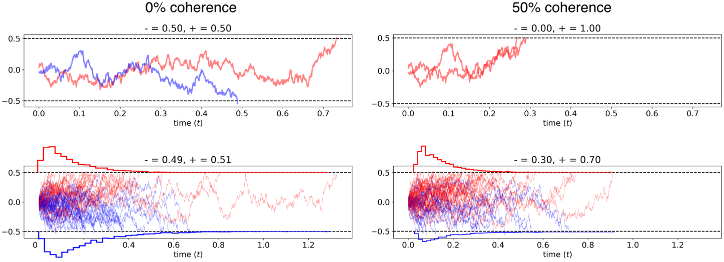


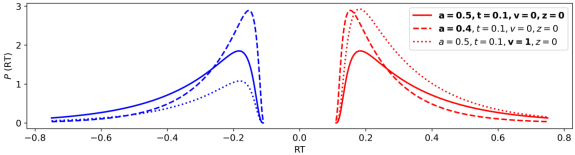
Unobserved Brain Mechanisms:
Drift + Diffusion





Drift Diffusion Model





Drift Diffusion Model

$$Z_\tau = Z_{\tau-1} + e_\tau, \quad e_\tau \sim \mathcal{N}(v\Delta\tau, \sigma^2\Delta\tau) \tag{1}$$

$v \in \mathbb{R}$ is the drift rate
$\Delta\tau \to 0$ is a continuous *drift diffusion* process

RT + Response for $\{Z_\tau : \tau = 0, .., RT\}$

$$RT = \begin{cases} +\min\{\tau > 0 : Z_\tau \geq +a\} \\ -\min\{\tau > 0 : Z_\tau \leq -a\} \end{cases} \tag{2}$$

$a > 0$ is the decision boundary
$|RT| > 0$ is the reaction time
$\text{sign}(RT) \in \{-1, +1\}$ is the response

RTs + Responses for $\{Z_\tau : \tau = 0, .., RT\}_t^T$

$$RT_t = \{RT_1, ..., RT_T\} \sim \mathcal{D}(a, v) \tag{3}$$

$t > 0$ is the trial time index
Δt is nonconstant (unequally sampling)
$\mathcal{D}$ is probability distribution determined by parameters a and v

Probability Density Function

$$f(RT \mid a, v) = \frac{\pi}{a^2} \exp\left(-\frac{va}{2} - \frac{v^2 t}{2}\right) \times \sum_{k=1}^{\infty} k \exp\left(-\frac{k^2 \pi^2 RT}{2a^2}\right) \sin(\frac{k\pi}{2}) \quad (4)$$

from Feller [1], Navarro and Fuss [2]

Likelihood Function

$$L_T(a, v \mid RT_t) = f(RT_t, ..., RT_T \mid a, v) = \prod_{t=1}^{T} f(RT_t \mid a, v) \quad (5)$$

Log-Likelihood Function

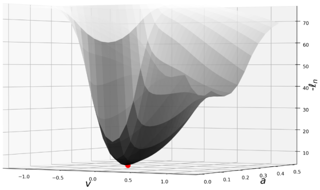
$$\ell_T(a, v \mid RT_t) = \log L_T(a, v \mid RT_t) = \sum_{t=1}^{T} \log f(RT_t \mid a, v) \quad (6)$$

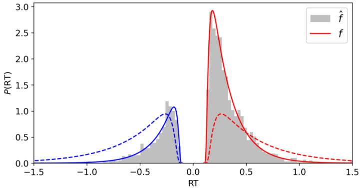
MLE $\hat{\theta}$ of $\theta = (a, v)$

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} - \ell_T(\theta | RT_t) \quad (7)$$

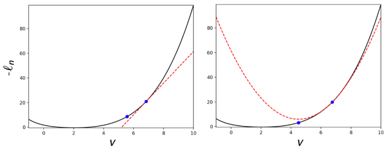
Maximum Likelihood Estimator

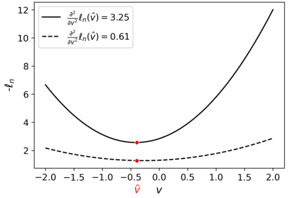
Loss Log-Likelihood





Loss Log-Likelihood Hessian





Asymptotic Covariance Estimators

$$\widehat{\mathcal{H}}_\theta = \frac{1}{T}\sum_{t=1}^{T} -\frac{\partial^2}{\partial\theta\partial\theta'}\log f(RT_t \mid \hat{\theta}) = -\frac{1}{T}\frac{\partial^2}{\partial\theta\partial\theta'}\ell_T(\hat{\theta}) \tag{8}$$

$$\widehat{\mathscr{I}}_\theta = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{\partial}{\partial\theta}\log f(RT_t \mid \hat{\theta})\right)\left(\frac{\partial}{\partial\theta}\log f(RT_t \mid \hat{\theta})\right)' = \frac{1}{T}\sum_{t=1}^{T}\hat{S}_t\hat{S}_t{}' \tag{9}$$

(1) Sample Hessian

$$\widehat{V}_1 = \widehat{\mathcal{H}}_\theta^{-1} \tag{10}$$

(2) Outer Product

$$\widehat{V}_2 = \widehat{\mathscr{I}}_\theta^{-1} \tag{11}$$

(3) Misspecification Robust $\mathscr{I}_\theta \neq \mathcal{H}_\theta$

$$\widehat{V}_3 = \widehat{\mathcal{H}}_\theta^{-1}\widehat{\mathscr{I}}_\theta\widehat{\mathcal{H}}_\theta^{-1} \tag{12}$$

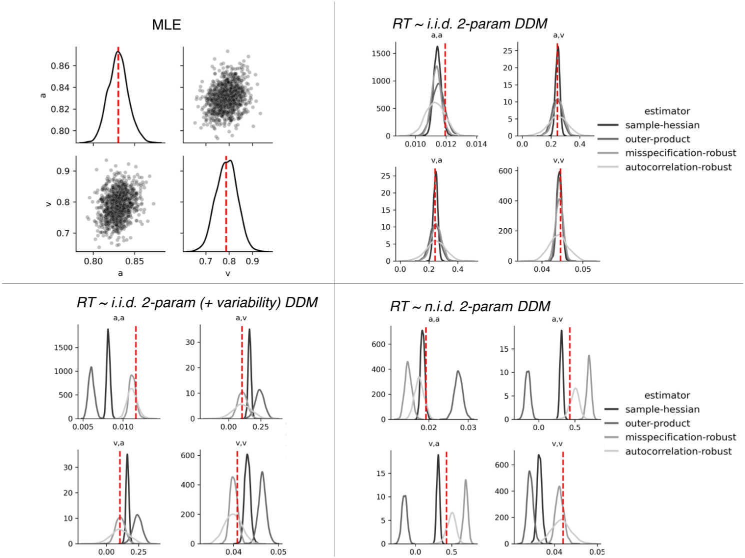
(4) Autocorrelation Robust

$$\widehat{V}_4 = \widehat{\mathcal{H}}_\theta^{-1}\left(\frac{1}{T}\sum_{i,j=1}^{T} w_{|i-j|}\hat{S}_i\hat{S}_j{}'\right)\widehat{\mathcal{H}}_\theta^{-1} \tag{13}$$

from Hansen [3, Chapter 10]

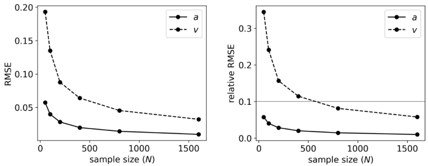
Simulations

Setting: $a = 0.83$, $v = 0.79$, $N = 1000$ repeats, $T = 900$ trials

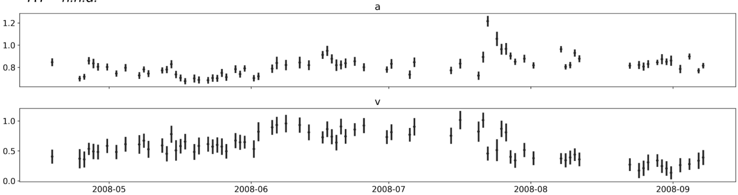


Addressing Non-Stationarity

Dataset: $N = 1$ rat, $T = 120k$ trials over 128 days



$RT \sim n.n.d.$



Shevinsky and Reinagel [4], Nguyen and Reinagel [5]

Conclusions and Future Directions

- Drift diffusion model describes how brains process noisy information during two-choice decision tasks
- MLE provides consistent point and interval estimators for model parameters and their standard errors from speed/accuracy behavioral data
- Generalized estimation framework robust to model misspecification and autocorrelation in reaction times
- Non-stationarity over time addressed by time-varying parameter estimation of freely behaving rats
- Future work: Extending models to incorporate covariates explaining parameter changes over time

References

[1] William Feller. *An introduction to probability theory and its applications*. Wiley series in probability and mathematical statistics. J. Wiley, New York Chichester Brisbane [etc.], third ed. rev edition, 1968. ISBN 978-0-471-25708-0.

[2] Daniel J. Navarro and Ian G. Fuss. Fast and accurate calculations for first-passage times in Wiener diffusion models. *Journal of Mathematical Psychology*, 53(4): 222–230, August 2009. ISSN 00222496. doi: 10.1016/j.jmp.2009.02.003. URL `https://linkinghub.elsevier.com/retrieve/pii/S0022249609000200`.

[3] Bruce E. Hansen. *Probability and statistics for economists*. Princeton University Press, Princeton ; Oxford, 2022. ISBN 978-0-691-23594-3.

[4] Carly A. Shevinsky and Pamela Reinagel. The Interaction Between Elapsed Time and Decision Accuracy Differs Between Humans and Rats. *Frontiers in Neuroscience*, 13: 1211, November 2019. ISSN 1662-453X. doi: 10.3389/fnins.2019.01211. URL `https://www.frontiersin.org/article/10.3389/fnins.2019.01211/full`.

[5] Quynh Nhu Nguyen and Pamela Reinagel. Different Forms of Variability Could Explain a Difference Between Human and Rat Decision Making. *Frontiers in Neuroscience*, 16:794681, February 2022. ISSN 1662-453X. doi: 10.3389/fnins.2022.794681. URL `https://www.frontiersin.org/articles/10.3389/fnins.2022.794681/full`.