DSC190/291: Network Science and Graph Theory **Winter 2025**

Homework Assignment 3

Topics: Random Networks

Instructor: **Prof. Gal Mishne**
Deadline: **Friday Jan. 31 (1/31/2025)**
Submission: **Gradescope**
Write full answers to the questions below. Show your calculations.

Note: GenAI tools are permitted solely for problems: 3, 4.

1. *Understanding Erdős-Rényi Networks*

   Consider an Erdős-Rényi network with $N = 3,000$ nodes, connected to each other with probability $p = 10^{-2}$.

   (a) What is the expected number of links, $\langle L \rangle$?

   (b) In which regime is the network?

   (c) Calculate the probability p_C so that the network is at the critical point.

   (d) Given the new linking probability $p = 10^{-3}$, calculate the number of nodes $N^{C.r}$ so that the network has only one component.

   (e) For the network in (d), calculate the average degree $\langle k^{Cr} \rangle$ and the average distance between two randomly chosen nodes $\langle d \rangle$.

   (f) Calculate the degree distribution p_k of this network (approximate with a Poisson degree distribution).

2. *Snobbish Networks*

   Consider a network of N red and N blue nodes. The probability that there is a link between nodes of identical color is p and the probability that there is a link between nodes of different color is q. A network is snobbish if $p > q$, capturing a tendency to connect to nodes of the same color. For $q = 0$ the network has at least two components, containing nodes with the same color.

   (a) Calculate the average degree of the "blue" sub-network made of only blue nodes, and the average degree in the full network.

   (b) Determine the minimal p and q required to have, with high probability, just one component.

   (c) Show that for large N even very snobbish networks ($p >> q$) display the small-world property.

3. *Generating Erdős-Rényi Networks (coding question)*

   Relying on the $G(N, p)$ model, generate networks with $N = 100$ nodes for each of the following average degrees: $p \in \{0.01, 0.05, 0.1, 0.2\}$. For each network:

   - Plot the average degree vs p,
   - Plot the clustering coefficient vs p,
   - Plot the average shortest path length vs p,
   - Visualize the networks

   Discuss your result.

4. *Generating Small world Networks (coding question)*

   Relying on the Watts-Strogatz model, generate a network with $N = 100$ with $k = 3$ and rewiring probability $p \in \{0.01, 0.1, 0.2, 0.5\}$. For each network:

   - Plot the average degree vs p,
   - Plot the clustering coefficient vs p,
   - Plot the average shortest path length vs p,
   - Visualize the networks

   Compare these networks to a random network and discuss your result.