DSC190: Network Science and Graph Theory **Winter 2025**

Homework Assignment 2

Topics: Centrality

Instructor: **Prof. Gal Mishne**
Deadline: **Friday Jan. 24 (1/24/2025)**
Submission: **Gradescope**
Write full answers to the questions below. Show your calculations.

Note: GenAI tools are permitted solely for problems: 3 and 4, for the coding components.

1. *How central are you?*

   For the following undirected graph, calculate by hand the following centralities for vertices B, C and D:

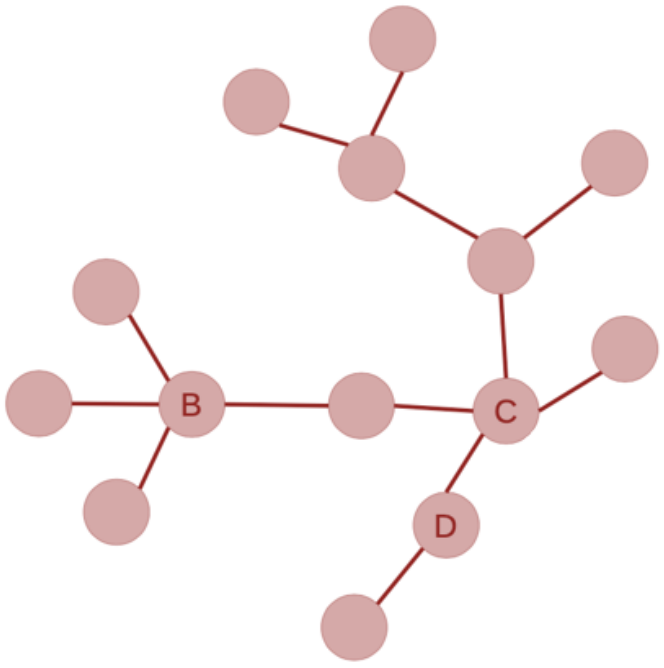
   (a) Degree centrality

   (b) Closeness centrality

   (c) Betweenness centrality

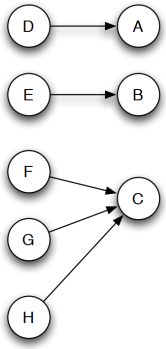
   (d) Eigenvector centrality

   Provide a short interpretation of which of the three vertices is most "important" under each measure and discuss the differences. Why might there be discrepancies between these centrality measures?

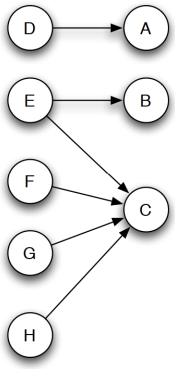


2. *Hubs and Authorities*

   (a) Show the values that you get if you run two rounds of computing hub and authority values on the following network by hand. Show the values both before and after the final normalization step, in which we divide each authority score by the sum of all authority scores, and divide each hub score by the sum of all hub scores. (We will call the scores obtained after this dividing-down step the normalized scores.



   (b) Due to the symmetry of nodes A and B in part (a), you should have seen that they get the same authority scores. Now let's look at what happens to the scores when node E, which links to B, decides to link to C as well. This produces the new network in the following figure. Show the normalized hub and authority values that each node gets when you run the 2-step hub-authority computation on the new network.



   (c) In (b), which of nodes A or B now has the higher authority score? Give a brief explanation in which you provide some intuition for why the difference in authority scores between A and B in (b) turned out the way it did.

3. *Centrality Measures*

   Using *networkx*, calculate the 1) betweeness, 2) closeness, 3) eigenvector, 4) degree centrality 5) PageRank (use default damping factor) for the following graphs:

   - barbell graph with $m1 = 45, m2 = 10$ (*networkx* graph model)
   - Les Miserables (*networkx* graph model)

- Minnesota road network attached in *minessota.mat* file. The adjancency matrix is given in A and the positions of the nodes in the graph for plotting are given in xy.

For each graph,

(a) Plot the graph (including edges) and **color** the nodes by the centrality measures (5 plots per graph). Make the node size small so that the nodes and edges are visible. Explain which nodes rank highest in each graph and hypothesize why they do so based on the graph's structure.

(b) For each graph calculate the correlation for each pair of centrality measures. Which centrality measures are the most correlated and which are the least? Does the structure of the graph affect this?

4. *PageRank*

Import the adjacency matrix of the directed graph `directed_graph_adj_matrix.npy`. Examine how PageRank changes if:

(a) One node drastically changes its out-degree (e.g., it gains or loses many outbound links).

(b) One node is removed completely (and all edges to/from it).

(c) The damping factor α changes from 0.85 to 0.95.