Topological Classification of Neural Oscillations using Simulated Autoregressive Processes

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0.1 Introduction

The detection of periodicity in timeseries has broad applications. An example is detecting brain oscillations: rhythmic patterns that arise from synchronized activity across large ensembles of neurons, and measurable using electroencephalography. In recordings of electrical activity, neural oscillations often occur spontaneously. Detecting when oscillations occur is an important part of neural signal analysis. Traditional methods often rely on analyzing signals in the frequency domain, using Fourier series to decompose the timeseries into sums of sin waves. However, neural signals are not necessarily sinusoidal, so traditional methods may not fully capture their complex topological features. This project proposes a novel approach for classifying neural signals as periodic (oscillatory) or aperiodic (non-oscillatory) using topological data analysis (TDA) to extract features for classification.

The key idea is to leverage the topological properties of timeseries embeddings to distinguish between periodic and aperiodic signals. A concrete starting point are simple linear time processes, specifically autoregressive order two (ar-2) processes. In an ar-2 process, each value is a linear combination of its two previous values, and the underlying parameters determine whether it exhibits oscillatory behavior. In this project I will simulate ar-2 processes, because the choice of theoretical parameters leads to well-defined oscillatory dynamics, as a training dataset for classification. Distinct topological features will be computed from the tools of persistent homology. Finally, the generalizability of this classifier will be tested on real neural data.

0.1.1 Classification Pipeline

The proposed classification pipeline consists of the following steps:

1. Simulate ar-2 processes: generate a dataset of ar-2 timeseries with varying parameters, ensuring a balanced representation of periodic and aperiodic signals.

2. Takens' embedding: construct point clouds from timeseries using Takens' embedding, a method that preserves topological properties of dynamic systems [Tak, SBDH].

3. Vietoris-Rips persistence: compute persistent homology on the point cloud using Vietoris-Rips complexes, to capture topological features at different scales [ELZ].

4. Topological feature extraction: extract relevant topological features from the persistence diagrams, such as entropy, to quantify the presence and persistence of structures like loops (oscillations) and voids [AGDR].

5. Classification algorithm: train a machine learning classifier (e.g. support vector machine, random forest) on the extracted features, using the theoretical periodicity labels of the simulated ar-2 process.

6. Neural data application: apply the trained classifier to neural timeseries to detect and classify oscillatory bursts.

0.2 Experiments

0.2.1 Notation

Let $\{X_t, t \in \mathbb{Z}\}$ be a stationary stochastic process, and $x_t = \{x_1, x_2, ..., x_T\}$ be a finite sample of X_t. For simplicity, assume X_t and x_t have zero mean. The Takens' embedding is a two-parameter model that maps the time domain to d-dimensional vectors in Euclidean space:

$$\mathrm{emb}(x_t) \mapsto \{x_t, x_{t-\tau}, x_{t-2\tau}, ..., x_{t-(d-1)\tau}\}. \tag{1}$$

where τ is the time delay and d is the dimension in $\mathbb{R}^d$.

Further, stationarity implies X_t has a constant mean and variance, and autocorrelations that only depend on time lag k:

$$\rho_k = \mathrm{corr}(X_t, X_{t-k}). \tag{2}$$

Further, autocorrelation functions (acf) can be estimates from finite samples:

$$\hat{\rho}_k = (\sum_{t=1}^{T} x_t^2)^{-1}(\sum_{t=k+1}^{T} x_t x_{t-k}), \tag{3}$$

where the notation assumes a centered series. In signals with complex or noisy components autocorrelations tend to average out noise, making more prominent periodic components. Therefore, the Takens' embedding for acfs can also be defined as:

$$\mathrm{emb}(\hat{\rho}_k) \mapsto \{\hat{\rho}_k, \hat{\rho}_{k-\tau}, \hat{\rho}_{k-2\tau}, ..., \hat{\rho}_{k-(d-1)\tau}\}. \tag{4}$$

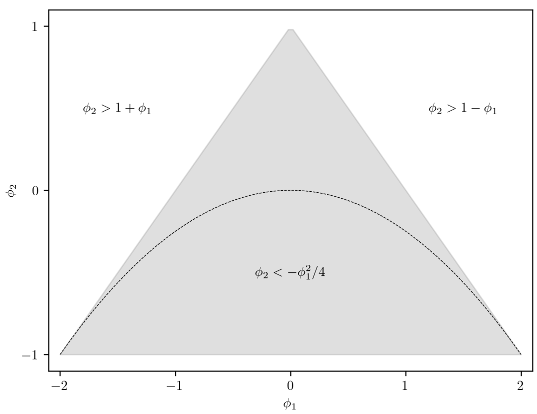
The embeddings for $x_t(1)$ and $\hat{\rho}_k(4)$ provide information on the periodicity of underlying ar-2 process, and are the basis for the proposed topological feature generation pipeline.

0.2.2 Simulations

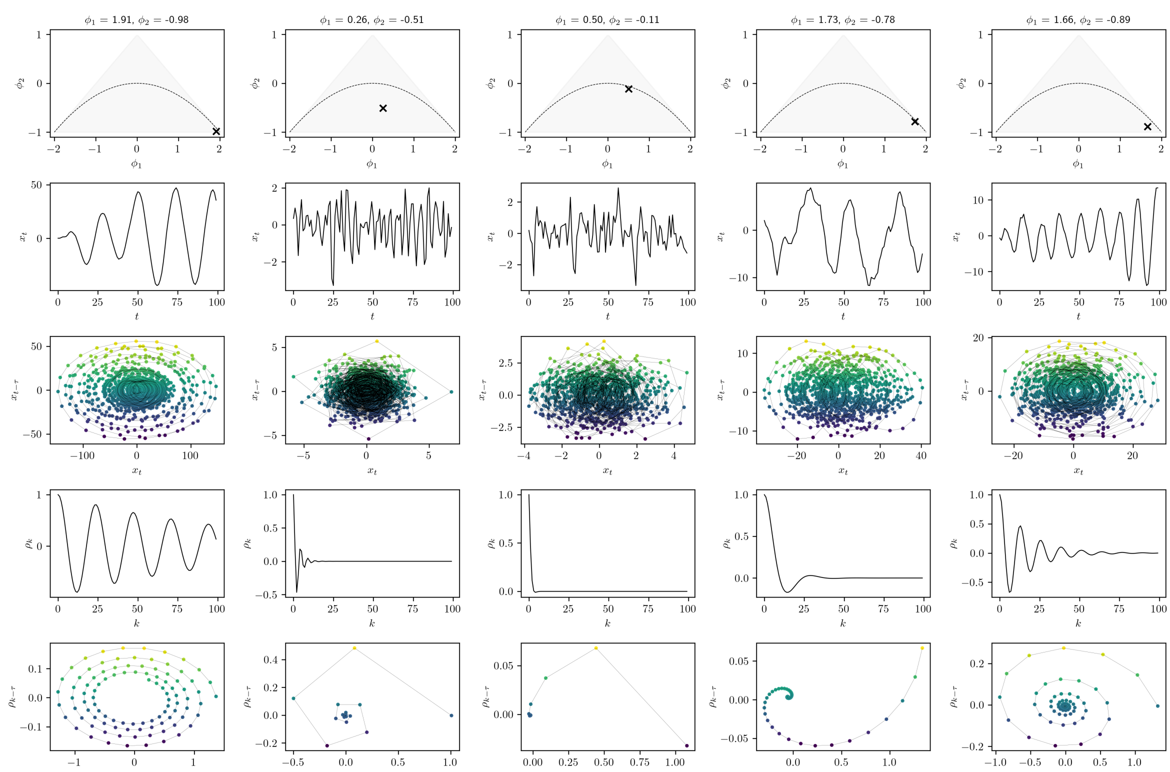
As an example, here I simulate samples of stationary timeseries $x_t = \{x_1, x_2, ..., x_T\}$ according to an ar-2 process:

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + e_t, e_t \overset{\text{iid}}{\sim} \mathcal{N}(0,1). \tag{5}$$

The stationarity triangle in the (ϕ_1, ϕ_2) plane defines the region where the ar-2 process is stationary. Specifically, the theoretical boundary between periodic and aperiodic signals exists at $\phi_2 = -\phi_1^2/4$:



For ar-2 processes generated below this boundary, we can see emergent properties like spirals in the time delay embedding space of autocorrelations:



0.2.3 Expected Outcomes

This project is expected to develop a method for classifying neural oscillations as periodic or not using the tools learned in DSC214. This approach will focus on interoperability of embedding spaces and persistence diagrams, to gain insights into the geometric and topological features that may not be evident in the time or frequency domains.

References

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