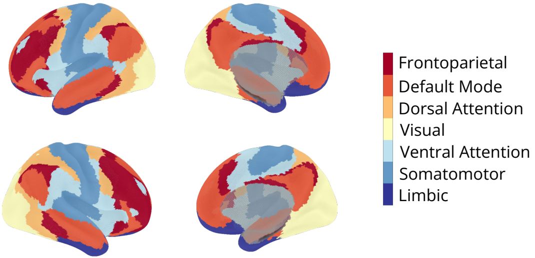
Estimating fMRI Timescale Maps

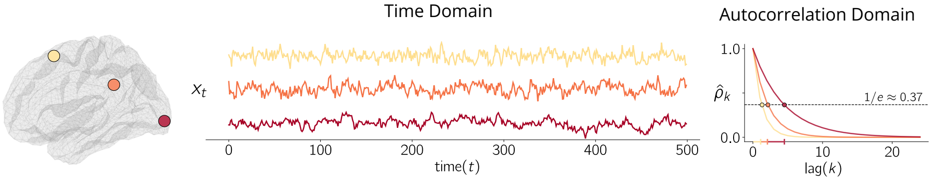
UC San Diego
HALICIOĞLU DATA SCIENCE INSTITUTE

Gabriel Riegner (co-authors Samuel Davenport, Bradley Voytek, Armin Schwartzman)

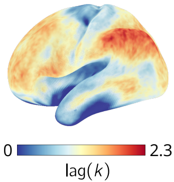
Western North American Region of the International Biometric Society 2025



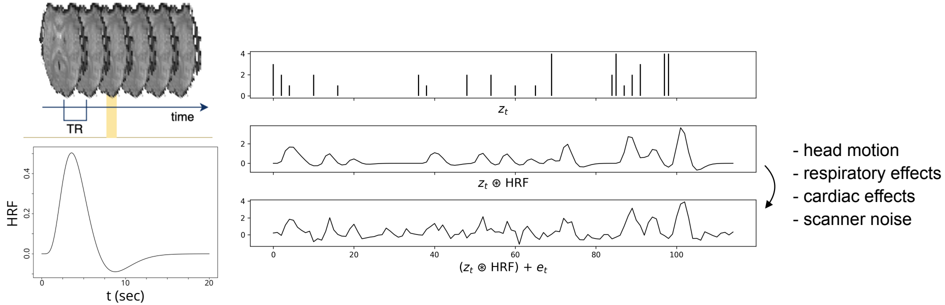
Estimating fMRI **Timescale Maps**



- Neural time series exhibit time-lagged correlations that decay, reflecting how brain regions integrate information over time.
- The timescale parameter quantifies the rate of decay.
- Timescale maps follow a hierarchical organization across the cortex: fast autocorrelation decay in sensory areas, slow in associative areas.

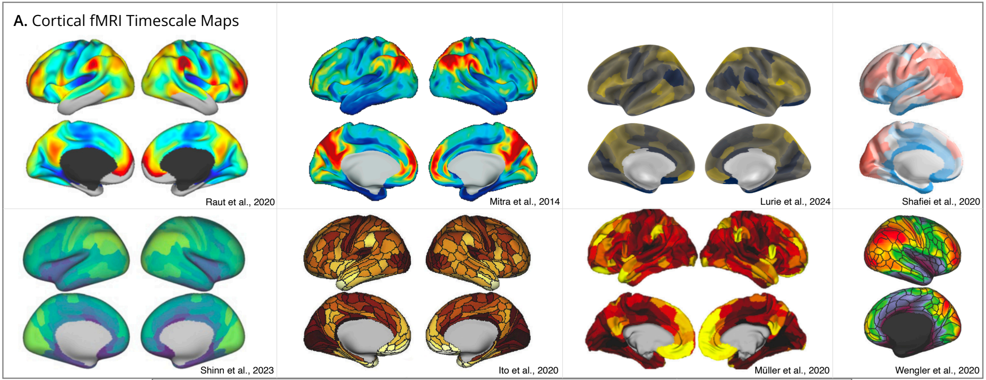


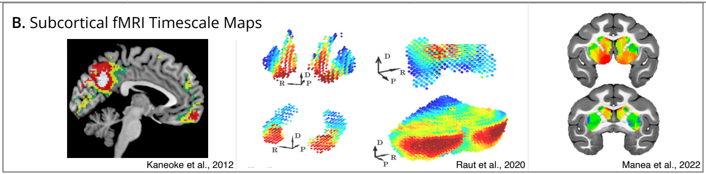
Estimating **fMRI** Timescale Maps



- Resting-state functional magnetic resonance imaging (fMRI) provides whole-brain noninvasive recording of spontaneous brain activity.
- An fMRI time series reflects hemodynamic changes of the underlying electrical activity of neurons.

Estimating **fMRI Timescale Maps**





Estimating fMRI Timescale Maps

Existing Methods

- Time-Domain Linear Model [6/25 papers]
- Autocorrelation-Domain Nonlinear Model [18/25 papers]
- (Frequency-Domain Nonlinear Model [3/25 papers])

Limitations of Existing Methods

- Assume exponentially decaying autocorrelation, which may not hold.
- Provide only point estimates without standard errors, limiting statistical inference.

Contributions

- Generalize assumptions to include all stationary and mixing processes.
- Introduce standard errors that account for misspecified autocorrelation decay.
- Establish statistical properties: consistency and asymptotic normality.
- Demonstrate parameter recovery in realistic simulations.
- Apply methods to fMRI data from the Human Connectome Project (HCP).

Assumptions

Let $\{X_t, t \in \mathbb{Z}\}$ be a discrete-time stochastic process that is **stationary** and **strong mixing**, and $x_t = \{x_1, x_2, ..., x_T\}$ be an observed sample of X_t. Assume X_t and x_t are demeaned.

Stationarity Implies

$$\gamma_k = \text{cov}[X_t, X_{t-k}] = \mathbb{E}[X_t X_{t-k}], \tag{1}$$

$$\rho_k = \text{corr}[X_t, X_{t-k}] = \gamma_0^{-1} \gamma_k. \tag{2}$$

Mixing Implies

- Ergodicity, ensuring consistent estimation by the ergodic theorem.
- Autocorrelations (2) decay sufficiently fast to apply a central limit theorem for correlated observations.

Hansen [2022], White and Domowitz [1984], Newey and West [1987]

Time-Domain Linear Model

Autoregressive Model (AR1) Projection

$$X_t = \phi X_{t-1} + e_t \tag{3}$$

Squared Error Minimization

$$S(\phi) = \mathbb{E}[(X_t - \phi X_{t-1})^2], \quad \phi^* = \underset{\phi}{\operatorname{argmin}}\, S(\phi) \tag{4}$$

Timescale Definition

$$\phi^* = (\mathbb{E}[X_{t-1}^2])^{-1}(\mathbb{E}[X_t X_{t-1}]) \tag{5}$$

$$\tau^* = g(\phi^*) = -\frac{1}{\log(|\phi^*|)} \tag{6}$$

Linear Least Squares (LLS) Estimation

$$\hat{\phi}^*_{\text{LLS}} = \left(\sum_{t=2}^{T} x_{t-1}^2\right)^{-1} \left(\sum_{t=2}^{T} x_t x_{t-1}\right) \tag{7}$$

$$\hat{\tau}^*_{\text{LLS}} = g(\hat{\phi}_{\text{LLS}}) = -\frac{1}{\log(|\hat{\phi}_{\text{LLS}}|)} \tag{8}$$

Time-Domain Linear Model

Autoregressive Model (AR1) Projection

$$X_t = \phi X_{t-1} + e_t \tag{3}$$

Squared Error Minimization

$$S(\phi) = \mathbb{E}[(X_t - \phi X_{t-1})^2], \quad \phi^* = \underset{\phi}{\text{argmin}}\, S(\phi) \tag{4}$$

Timescale Definition

$$\phi^* = (\mathbb{E}[X_{t-1}^2])^{-1}(\mathbb{E}[X_t X_{t-1}]) \tag{5}$$

$$\tau^* = g(\phi^*) = -\frac{1}{\log(|\phi^*|)} \tag{6}$$

Linear Least Squares (LLS) Estimation

$$\hat{\phi}^*_{\text{LLS}} = \left(\sum_{t=2}^{T} x_{t-1}^2\right)^{-1}\left(\sum_{t=2}^{T} x_t x_{t-1}\right) \tag{7}$$

$$\hat{\tau}^*_{\text{LLS}} = g(\hat{\phi}_{\text{LLS}}) = -\frac{1}{\log(|\hat{\phi}_{\text{LLS}}|)} \tag{8}$$

Time-Domain Linear Model

Autoregressive Model (AR1) Projection

$$X_t = \phi X_{t-1} + e_t \tag{3}$$

Squared Error Minimization

$$S(\phi) = \mathbb{E}[(X_t - \phi X_{t-1})^2], \quad \phi^* = \underset{\phi}{\operatorname{argmin}}\, S(\phi) \tag{4}$$

Timescale Definition

$$\phi^* = (\mathbb{E}[X_{t-1}^2])^{-1}(\mathbb{E}[X_t X_{t-1}]) \tag{5}$$

$$\tau^* = g(\phi^*) = -\frac{1}{\log(|\phi^*|)} \tag{6}$$

Linear Least Squares (LLS) Estimation

$$\hat{\phi}^*_{\text{LLS}} = \left(\sum_{t=2}^{T} x_{t-1}^2\right)^{-1} \left(\sum_{t=2}^{T} x_t x_{t-1}\right) \tag{7}$$

$$\hat{\tau}^*_{\text{LLS}} = g(\hat{\phi}_{\text{LLS}}) = -\frac{1}{\log(|\hat{\phi}_{\text{LLS}}|)} \tag{8}$$

Time-Domain Linear Model

Autoregressive Model (AR1) Projection

$$X_t = \phi X_{t-1} + e_t \tag{3}$$

Squared Error Minimization

$$S(\phi) = \mathbb{E}[(X_t - \phi X_{t-1})^2], \quad \phi^* = \underset{\phi}{\operatorname{argmin}} \, S(\phi) \tag{4}$$

Timescale Definition

$$\phi^* = (\mathbb{E}[X_{t-1}^2])^{-1}(\mathbb{E}[X_t X_{t-1}]) \tag{5}$$

$$\tau^* = g(\phi^*) = -\frac{1}{\log(|\phi^*|)} \tag{6}$$

Linear Least Squares (LLS) Estimation

$$\hat{\phi}^*_{\text{LLS}} = \left(\sum_{t=2}^{T} x_{t-1}^2\right)^{-1} \left(\sum_{t=2}^{T} x_t x_{t-1}\right) \tag{7}$$

$$\hat{\tau}^*_{\text{LLS}} = g(\hat{\phi}_{\text{LLS}}) = -\frac{1}{\log(|\hat{\phi}_{\text{LLS}}|)} \tag{8}$$

Autocorrelation-Domain Nonlinear Model

Exponential Decay Model Projection

$$\rho_k = \phi^k + e_k, \text{ for } k \in \{0, 1, \ldots, K\} \tag{9}$$

Squared Error Minimization

$$S(\phi) = \mathbb{E}[(\rho_k - \phi^k)^2], \quad \phi^* = \underset{\phi}{\text{argmin}}\, S(\phi) \tag{10}$$

Timescale Definition

$$\tau^* = g(\phi^*) = -\frac{1}{\log(|\phi^*|)} \tag{11}$$

Nonlinear Least Squares (NLS) Estimation

$$\hat{\phi}^*_{\text{NLS}} = \underset{\phi}{\text{argmin}}\, \widehat{S}(\phi) \tag{12}$$

$$\hat{\tau}^*_{\text{NLS}} = g(\hat{\phi}^*_{\text{NLS}}) = -\frac{1}{\log(|\hat{\phi}_{\text{NLS}}|)} \tag{13}$$

Autocorrelation-Domain Nonlinear Model

Exponential Decay Model Projection

$$\rho_k = \phi^k + e_k, \text{ for } k \in \{0, 1, \ldots, K\} \tag{9}$$

Squared Error Minimization

$$S(\phi) = \mathbb{E}[(\rho_k - \phi^k)^2], \quad \phi^* = \underset{\phi}{\operatorname{argmin}}\, S(\phi) \tag{10}$$

Timescale Definition

$$\tau^* = g(\phi^*) = -\frac{1}{\log(|\phi^*|)} \tag{11}$$

Nonlinear Least Squares (NLS) Estimation

$$\hat{\phi}^*_{\text{NLS}} = \underset{\phi}{\operatorname{argmin}}\, \widehat{S}(\phi) \tag{12}$$

$$\hat{\tau}^*_{\text{NLS}} = g(\hat{\phi}^*_{\text{NLS}}) = -\frac{1}{\log(|\hat{\phi}_{\text{NLS}}|)} \tag{13}$$

Autocorrelation-Domain Nonlinear Model

Exponential Decay Model Projection

$$\rho_k = \phi^k + e_k, \text{ for } k \in \{0, 1, \ldots, K\} \tag{9}$$

Squared Error Minimization

$$S(\phi) = \mathbb{E}[(\rho_k - \phi^k)^2], \quad \phi^* = \underset{\phi}{\operatorname{argmin}}\, S(\phi) \tag{10}$$

Timescale Definition

$$\tau^* = g(\phi^*) = -\frac{1}{\log(|\phi^*|)} \tag{11}$$

Nonlinear Least Squares (NLS) Estimation

$$\hat{\phi}^*_{\text{NLS}} = \underset{\phi}{\operatorname{argmin}}\, \widehat{S}(\phi) \tag{12}$$

$$\hat{\tau}^*_{\text{NLS}} = g(\hat{\phi}^*_{\text{NLS}}) = -\frac{1}{\log(|\hat{\phi}_{\text{NLS}}|)} \tag{13}$$

Autocorrelation-Domain Nonlinear Model

Exponential Decay Model Projection

$$\rho_k = \phi^k + e_k, \text{ for } k \in \{0, 1, \ldots, K\} \tag{9}$$

Squared Error Minimization

$$S(\phi) = \mathbb{E}[(\rho_k - \phi^k)^2], \quad \phi^* = \underset{\phi}{\operatorname{argmin}}\, S(\phi) \tag{10}$$

Timescale Definition

$$\tau^* = g(\phi^*) = -\frac{1}{\log(|\phi^*|)} \tag{11}$$

Nonlinear Least Squares (NLS) Estimation

$$\hat{\phi}^*_{\text{NLS}} = \underset{\phi}{\operatorname{argmin}}\, \widehat{S}(\phi) \tag{12}$$

$$\hat{\tau}^*_{\text{NLS}} = g(\hat{\phi}^*_{\text{NLS}}) = -\frac{1}{\log(|\hat{\phi}_{\text{NLS}}|)} \tag{13}$$

Standard Error Estimation

1. Time-Domain Method

$$\widehat{\text{se}}_{\text{NW}}(\hat{\phi}^*_{\text{LLS}}) = \sqrt{\hat{q}^{-1}\ \hat{\omega}\ \hat{q}^{-1}}, \quad \widehat{\text{se}}_{\text{NW}}(\hat{\tau}^*_{\text{LLS}}) \approx \widehat{\text{se}}_{\text{NW}}(\hat{\phi}^*_{\text{LLS}}) \cdot \frac{d}{d\phi} g(\hat{\phi}^*_{\text{LLS}}) \tag{14}$$

$$\hat{q} = \frac{1}{T}\sum_{t=2}^{T} x_{t-1}^2, \quad \hat{\omega} = \sum_{\ell=-M}^{M}\left(1 - \frac{|\ell|}{M+1}\right)\ \frac{1}{T}\sum_{1\le t-\ell \le T}(x_{t-1}\cdot \hat{e}_t)(x_{t-1-\ell}\cdot \hat{e}_{t-\ell}) \tag{15}$$

2. Autocorrelation-Domain Method

$$\widehat{\text{se}}_{\text{NW}}(\hat{\phi}^*_{\text{NLS}}) = \sqrt{\hat{q}^{-1}\ \hat{\omega}\ \hat{q}^{-1}}, \quad \widehat{\text{se}}_{\text{NW}}(\hat{\tau}^*_{\text{NLS}}) \approx \widehat{\text{se}}_{\text{NW}}(\hat{\phi}^*_{\text{NLS}}) \cdot \frac{d}{d\phi} g(\hat{\phi}^*_{\text{NLS}}) \tag{16}$$

$$\hat{m}_{\phi,k} = k\hat{\phi}^{*k-1}_{\text{NLS}}, \quad \hat{q} = \frac{1}{K}\sum_{k=0}^{K}(\hat{m}_{\phi,k})^2, \quad \hat{\omega} = \sum_{\ell=-M}^{M}\left(1 - \frac{|\ell|}{M+1}\right)\ \frac{1}{K}\sum_{1\le k-\ell\le K}(\hat{m}_{\phi,k}\cdot \hat{e}_k)(\hat{m}_{\phi,k-\ell}\cdot \hat{e}_{k-\ell}) \tag{17}$$

3. Autocorrelation/Time-Domain Method, equation (14) except for:

$$\hat{e}_t = x_t - \hat{\phi}^*_{\text{NLS}} \cdot x_{t-1} \tag{18}$$

Standard Error Estimation

1. Time-Domain Method

$$\widehat{\text{se}}_{\text{NW}}(\hat{\phi}^*_{\text{LLS}}) = \sqrt{\hat{q}^{-1}\ \hat{\omega}\ \hat{q}^{-1}}, \quad \widehat{\text{se}}_{\text{NW}}(\hat{\tau}^*_{\text{LLS}}) \approx \widehat{\text{se}}_{\text{NW}}(\hat{\phi}^*_{\text{LLS}}) \cdot \frac{d}{d\phi} g(\hat{\phi}^*_{\text{LLS}}) \tag{14}$$

$$\hat{q} = \frac{1}{T}\sum_{t=2}^{T} x_{t-1}^2, \quad \hat{\omega} = \sum_{\ell=-M}^{M}\left(1 - \frac{|\ell|}{M+1}\right) \frac{1}{T}\sum_{1\le t-\ell\le T}(x_{t-1}\cdot\hat{e}_t)(x_{t-1-\ell}\cdot\hat{e}_{t-\ell}) \tag{15}$$

2. Autocorrelation-Domain Method

$$\widehat{\text{se}}_{\text{NW}}(\hat{\phi}^*_{\text{NLS}}) = \sqrt{\hat{q}^{-1}\ \hat{\omega}\ \hat{q}^{-1}}, \quad \widehat{\text{se}}_{\text{NW}}(\hat{\tau}^*_{\text{NLS}}) \approx \widehat{\text{se}}_{\text{NW}}(\hat{\phi}^*_{\text{NLS}}) \cdot \frac{d}{d\phi} g(\hat{\phi}^*_{\text{NLS}}) \tag{16}$$

$$\hat{m}_{\phi,k} = k\hat{\phi}^{*k-1}_{\text{NLS}}, \quad \hat{q} = \frac{1}{K}\sum_{k=0}^{K}(\hat{m}_{\phi,k})^2, \quad \hat{\omega} = \sum_{\ell=-M}^{M}\left(1 - \frac{|\ell|}{M+1}\right) \frac{1}{K}\sum_{1\le k-\ell\le K}(\hat{m}_{\phi,k}\cdot\hat{e}_k)(\hat{m}_{\phi,k-\ell}\cdot\hat{e}_{k-\ell}) \tag{17}$$

3. Autocorrelation/Time-Domain Method, equation (14) except for:

$$\hat{e}_t = x_t - \hat{\phi}^*_{\text{NLS}} \cdot x_{t-1} \tag{18}$$

Standard Error Estimation

1. Time-Domain Method

$$\widehat{\text{se}}_{\text{NW}}(\hat{\phi}^*_{\text{LLS}}) = \sqrt{\hat{q}^{-1}\ \hat{\omega}\ \hat{q}^{-1}}, \quad \widehat{\text{se}}_{\text{NW}}(\hat{\tau}^*_{\text{LLS}}) \approx \widehat{\text{se}}_{\text{NW}}(\hat{\phi}^*_{\text{LLS}}) \cdot \frac{d}{d\phi} g(\hat{\phi}^*_{\text{LLS}}) \tag{14}$$

$$\hat{q} = \frac{1}{T}\sum_{t=2}^{T} x_{t-1}^2, \quad \hat{\omega} = \sum_{\ell=-M}^{M}\left(1 - \frac{|\ell|}{M+1}\right)\ \frac{1}{T}\sum_{1 \le t-\ell \le T}(x_{t-1}\cdot \hat{e}_t)(x_{t-1-\ell}\cdot \hat{e}_{t-\ell}) \tag{15}$$

2. Autocorrelation-Domain Method

$$\widehat{\text{se}}_{\text{NW}}(\hat{\phi}^*_{\text{NLS}}) = \sqrt{\hat{q}^{-1}\ \hat{\omega}\ \hat{q}^{-1}}, \quad \widehat{\text{se}}_{\text{NW}}(\hat{\tau}^*_{\text{NLS}}) \approx \widehat{\text{se}}_{\text{NW}}(\hat{\phi}^*_{\text{NLS}}) \cdot \frac{d}{d\phi} g(\hat{\phi}^*_{\text{NLS}}) \tag{16}$$

$$\hat{m}_{\phi,k} = k\hat{\phi}^{*k-1}_{\text{NLS}}, \quad \hat{q} = \frac{1}{K}\sum_{k=0}^{K}(\hat{m}_{\phi,k})^2, \quad \hat{\omega} = \sum_{\ell=-M}^{M}\left(1 - \frac{|\ell|}{M+1}\right)\ \frac{1}{K}\sum_{1 \le k-\ell \le K}(\hat{m}_{\phi,k}\cdot \hat{e}_k)(\hat{m}_{\phi,k-\ell}\cdot \hat{e}_{k-\ell}) \tag{17}$$

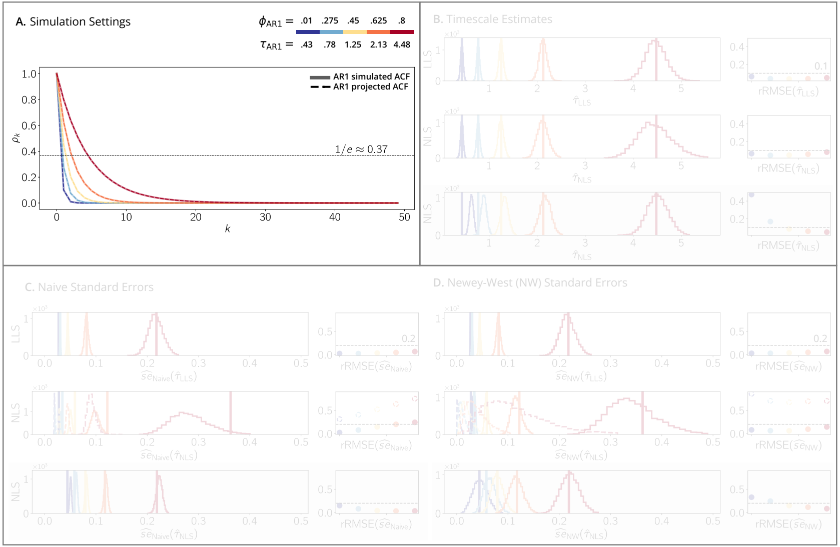
3. Autocorrelation/Time-Domain Method, equation (14) except for:

$$\hat{e}_t = x_t - \hat{\phi}^*_{\text{NLS}} \cdot x_{t-1} \tag{18}$$

AR1 Simulations

$N = 10,000$ repeats, $T = 4,800$ timepoints

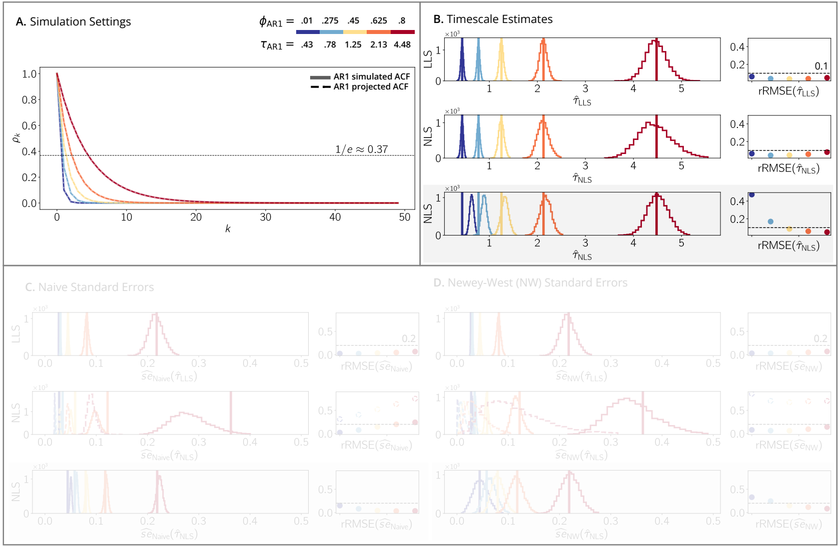
$x_t = \phi x_{t-1} + e_t, \; e_t \sim \mathcal{N}(0,1) \quad \rho_k = \phi^k + e_k, \; e_k \sim \mathcal{N}(0,1)$



AR1 Simulations

$N = 10,000$ repeats, $T = 4,800$ timepoints

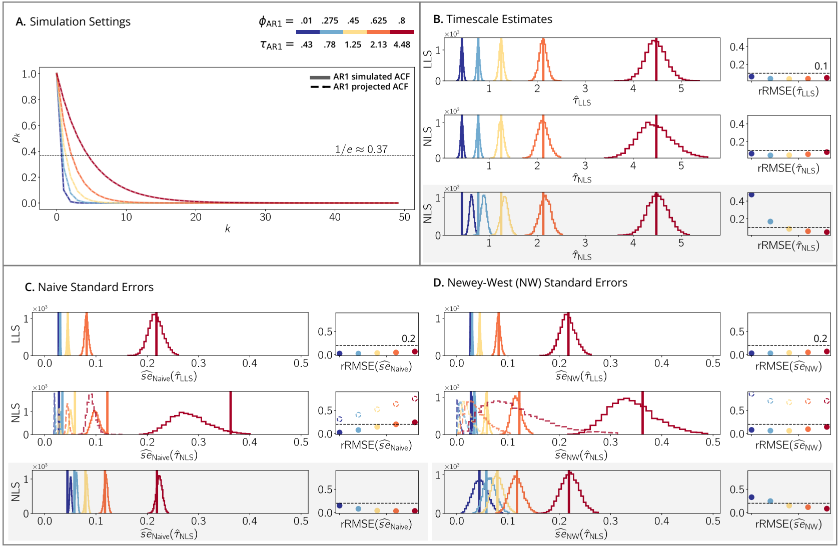
$x_t = \phi x_{t-1} + e_t,\ e_t \sim \mathcal{N}(0,1) \qquad \rho_k = \phi^k + e_k,\ e_k \sim \mathcal{N}(0,1)$



AR1 Simulations

$N = 10,000$ repeats, $T = 4,800$ timepoints

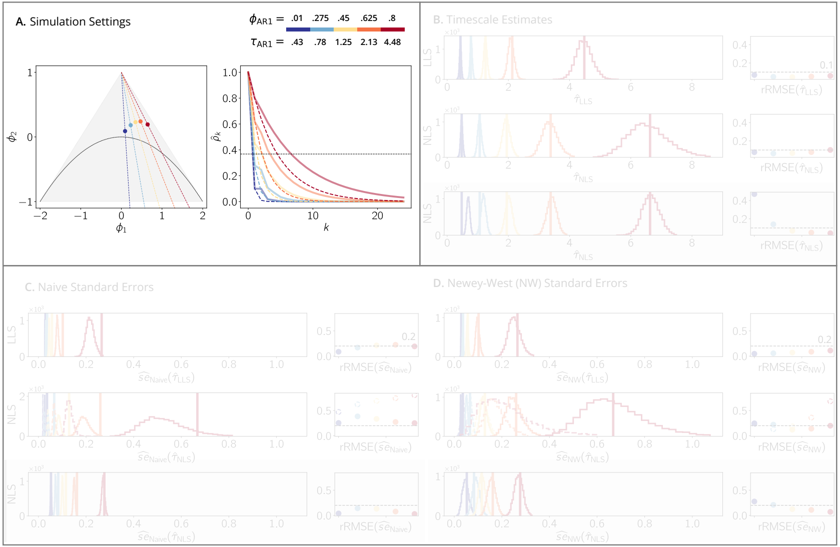
$x_t = \phi x_{t-1} + e_t,\ e_t \sim \mathcal{N}(0,1)$ $\quad \rho_k = \phi^k + e_k,\ e_k \sim \mathcal{N}(0,1)$



AR2 Simulations

$N = 10,000$ repeats, $T = 4,800$ timepoints

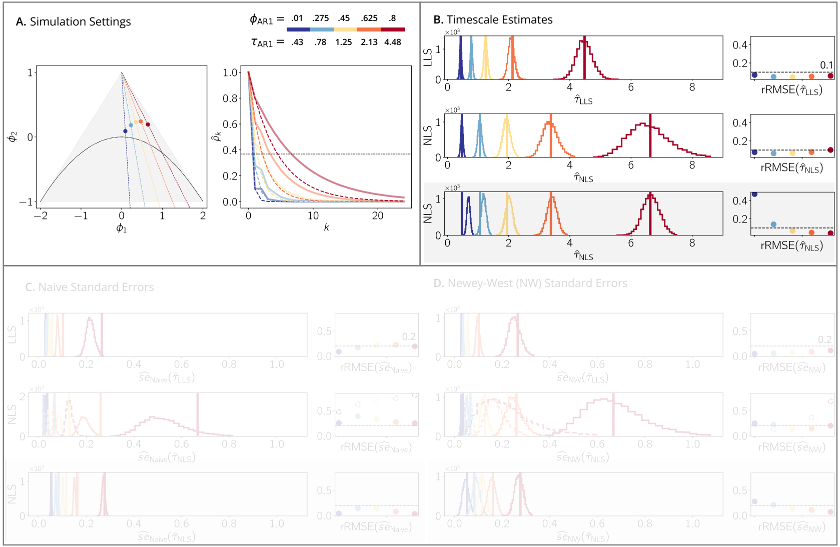
$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + e_t,\ e_t \sim \mathcal{N}(0,1)$ $\quad \rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + e_k,\ e_k \sim \mathcal{N}(0,1)$



AR2 Simulations

$N = 10,000$ repeats, $T = 4,800$ timepoints

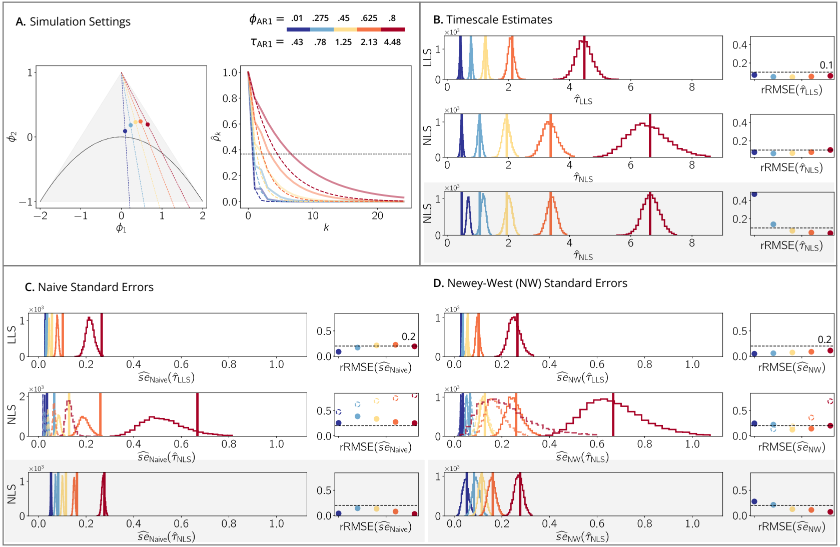
$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + e_t,\ e_t \sim \mathcal{N}(0,1)$ $\quad \rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + e_k,\ e_k \sim \mathcal{N}(0,1)$



AR2 Simulations

$N = 10,000$ repeats, $T = 4,800$ timepoints

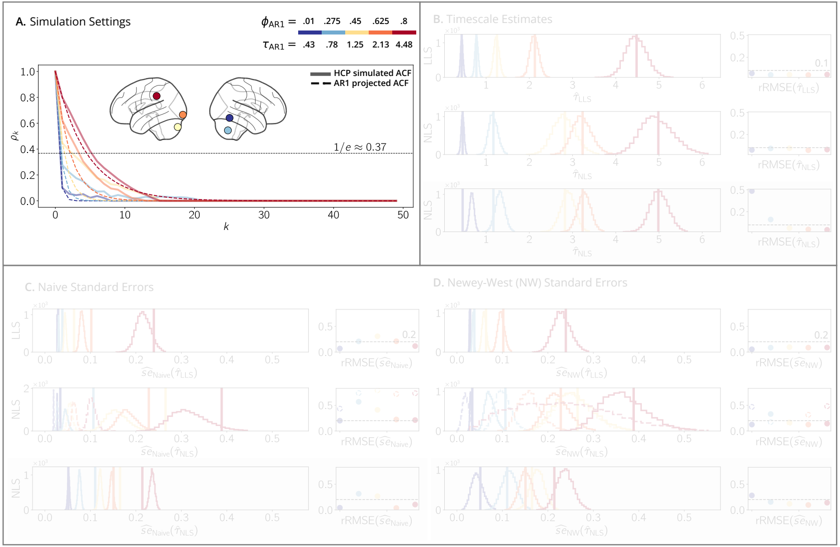
$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + e_t,\ e_t \sim \mathcal{N}(0,1)$ $\quad \rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + e_k,\ e_k \sim \mathcal{N}(0,1)$



Realistic rfMRI Simulations

$N = 10,000$ repeats, $T = 4,800$ timepoints

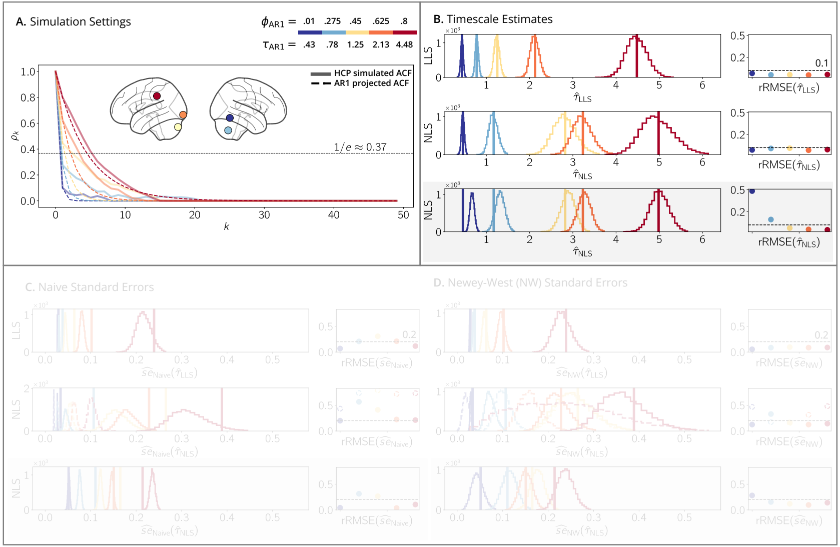
$x_t \sim \mathcal{N}(0, \hat{\Sigma}) \quad \rho_k = \hat{\rho}_k + e_k, \; e_k \sim \mathcal{N}(0, 1)$



Realistic rfMRI Simulations

$N = 10,000$ repeats, $T = 4,800$ timepoints

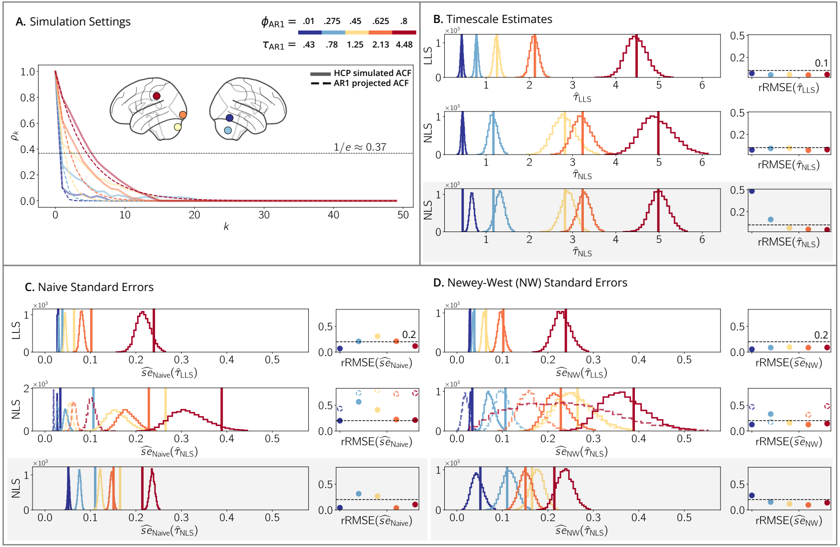
$x_t \sim \mathcal{N}(0, \hat{\Sigma}) \quad \rho_k = \hat{\rho}_k + e_k, \; e_k \sim \mathcal{N}(0, 1)$



Realistic rfMRI Simulations

$N = 10,000$ repeats, $T = 4,800$ timepoints

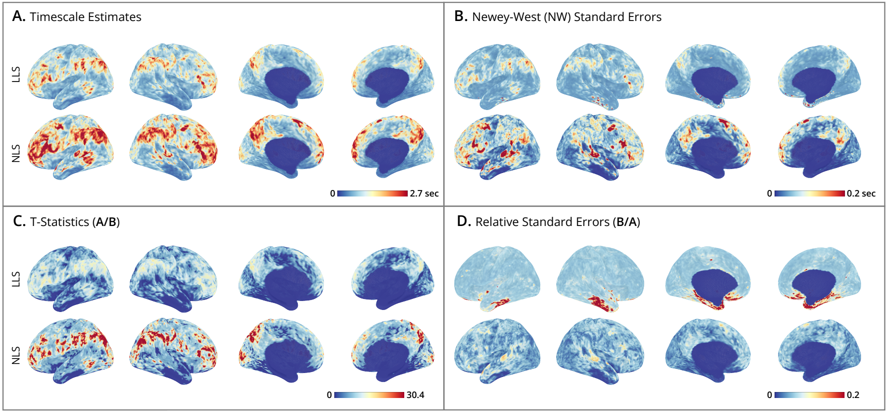
$x_t \sim \mathcal{N}(0, \hat{\Sigma}) \quad \rho_k = \hat{\rho}_k + e_k,\ e_k \sim \mathcal{N}(0,1)$



Subject-Level Results

$N = 1$ subject, $T = 4,800$ timepoints, $V = 64,984$ vertices

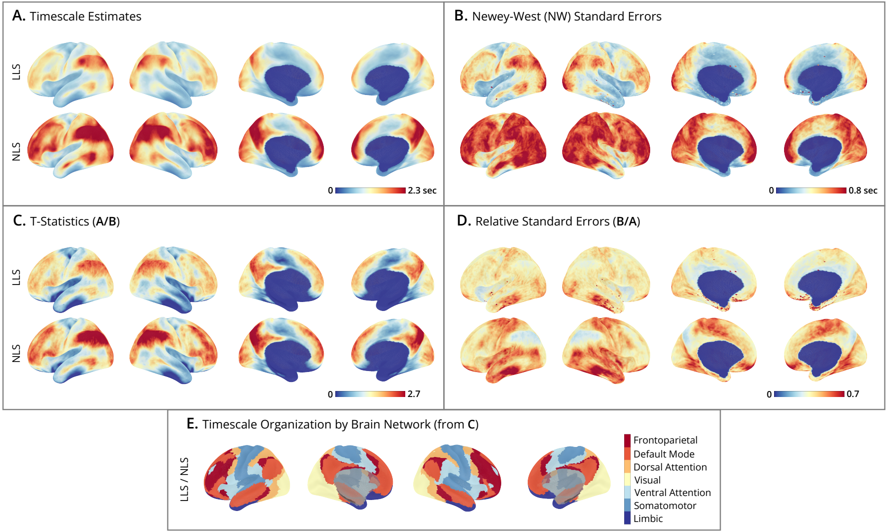
Human Connectome Project (HCP)



Group-Level Results

$N = 180$ subjects, $T = 4,800$ timepoints, $V = 64,984$ vertices   Human Connectome Project (HCP)

$$\bar{\tau} = \frac{1}{N}\sum_{n=1}^{N}\hat{\tau}_n, \qquad \widehat{\text{se}}(\bar{\tau}) = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\widehat{\text{se}}(\hat{\tau}_n)^2 + \frac{1}{N}\sum_{n=1}^{N}(\hat{\tau}_n - \bar{\tau})^2}$$



Conclusions

1. We formalized and compared time- and autocorrelation-domain methods for fMRI timescale mapping.
2. Both methods are consistent under broad conditions (stationary, mixing), though they estimate distinct timescale values.
3. We introduced robust standard errors to enable valid statistical inference, addressing a key limitation of prior work.
4. Comparatively, the time-domain method is more accurate under model misspecification and is more computationally efficient for high-dimensional fMRI data.
5. Both methods produce timescale maps that align with known functional brain organization.
6. Our work provides a foundation for future research on the role of timescales in brain structure and function.

References

B. E. Hansen. *Econometrics*. Princeton University Press, Princeton, 2022. ISBN 978-0-691-23589-9.

W. Newey and K. West. A simple, positive semi-definite, heteroskedasticity and autocorrelationconsistent covariance matrix. *Econometrica*, 55(3):703–08, 1987.

H. White and I. Domowitz. Nonlinear Regression with Dependent Observations. *Econometrica*, 52(1): 143, Jan. 1984.

Thank you!

paper:

https://doi.org/10.1101/2025.04.23.650300

code:

https://github.com/griegner/fmri-timescales