Programming Project

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1 Introduction

1.1 Project Objective

The goal of this project is to implement a state estimator to track the position and heading of a kinematic bicycle system. The state estimator used is up to the user and the estimator chosen must be efficient.

This project implements a state estimator to track the position and heading angle of a bicycle using and Extended Kalman Filter(EKF). The estimator combines noisy GPS measurements with known control inputs (steering angle and pedal speed) to estimate the bicycle's rear-wheel position (x, y) and heading θ.

2 Modeling

The bicycle is modeled using a kinematic model:

$$\begin{aligned} \dot{x}_1(t) &= v(t)\cos(\theta(t)) \\ \dot{y}_1(t) &= v(t)\sin(\theta(t)) \\ \dot{\theta}(t) &= \frac{v(t)}{B}\tan(\gamma(t)) \end{aligned} \tag{1}$$

where the inputs are:

- $v = 5r$
- ω is the linear speed (with $r = 0.425m$, gear ratio = 5)
- γ: steering angle
- $B = 0.8m$: wheelbase
- θ: heading angle

The measurement provides the position of the bicycle's center, modeled as:

$$z = \begin{bmatrix} x + 0.5B\cos(\theta) \\ y + 0.5B\sin(\theta) \end{bmatrix}$$

3 Implementation

An Extended Kalman filter was selected due to the nonlinear motion and measurement model.

- State Vector: $[x, y, \theta]$
- Covariance Matrix: P
- Process Noise Q:
- Measurement noise R

The EKF is an extension of the Kalman Filter to nonlinear systems. The EKF operates by linearizing the nonlinear system equations about the latest state estimate. While an Unscented Kalman Filter generally performs better than an EKF for nonlinear systems, the nonlinearity of the bicycle model is moderate, and faster runtime was prioritized. Faster runtime is the same reason why the EKF was selected over the Particle Filter, which is useful for non-gaussian and multi modal distributions, but computationally intensive

Initial Parameters

- Initial state: $x_0 = [0, 0, \pi/4]$
- $Q = \text{diag}(0.2, 0.1, 0.05)$
- $R = \begin{bmatrix} 1.0 & 1.50 \\ 1.50 & 3.00 \end{bmatrix}$ (empirically estimated from run_000.csv)

The initial state x_0 assumes that the bicycle starts at 0,0 and faces exactly north east. The process noise and and measurement noise matrices were initially set arbitrarily and were then tuned by hand to improve results. By increasing Q the filter becomes more responsive and adapts more to measurement updates, which is ideal for uncertain models. By increasing R the EKF assumes the measurement data is noisy and the filter relies more on the model, which is helpful when the GPS in unreliable

Estimator Implementation

The EKF is implemented in the function estRun, which executes a predict-update cycle at each time step. The estRun function uses the bicycle's kinematic model to predict the next state (x, y, θ) based on control inputs (pedal speed and steering angle) and then updates the uncertainty using the linearized motion model. Next is the update step which computes the expected GPS measurement from the predicted state and compares it to the actual measurement. Then it uses the kalman gain to update the state estimate and reduce uncertainty. In the case of no valid GPS data, the function skips the update and uses the prediction only. The function then returns the best current estimates for x,y, and θ, as well as the updated internal state for the next time step.

4 Results and Discussion

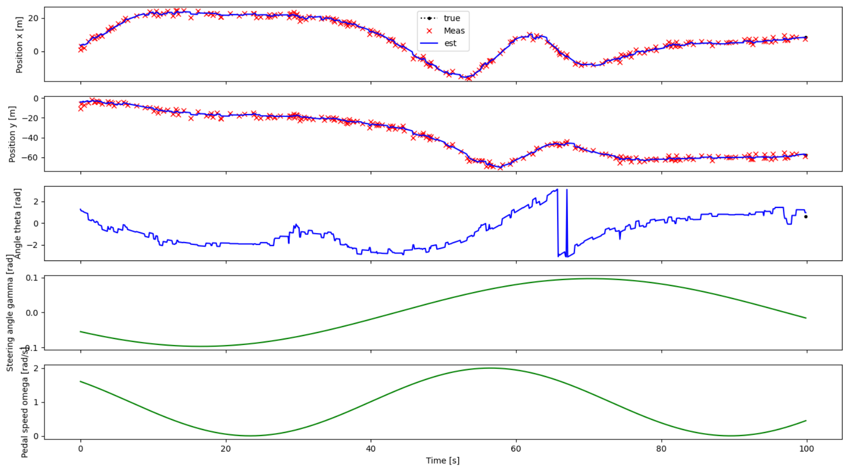

Figure 1: Extended Kalman Filter tracking comparison with individual states

4.1 Observation

The extended kalman filter accurately tracks the data in both the x and y trajectories. The heading angle also correctly reflects turns in the trajectory and the EKF is able

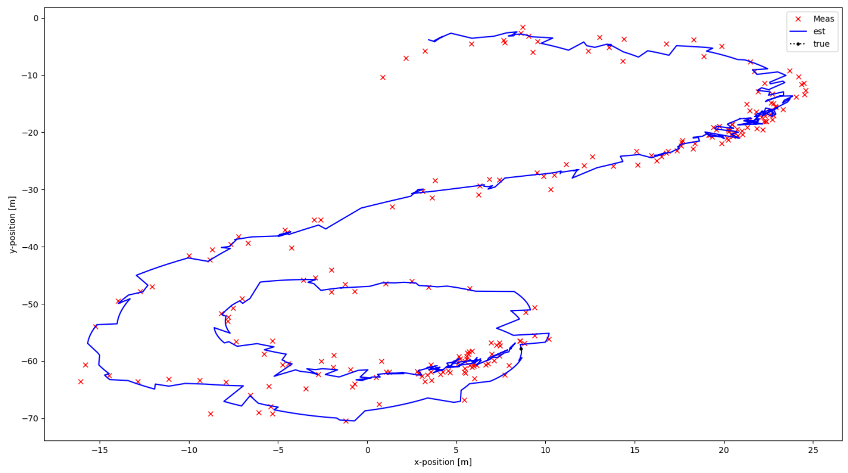

Figure 2: 2D trajectory plot

Table 1: Estimator Error

Run #	x [m]	y [m]	θ [rad]
1	-0.547	0.0028	0.359

to mitigate the transient spike near t = 70s (see Figure 1). The 2D trajectory plot shows the estimated path matches closely with the true final point (see Figure 2). The filter parameters Q and R provide a good balance between filter responsiveness and noise rejection.