# Final Project Appendix

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```
library(car)
library(lattice)
econ <- read.csv("https://raw.githubusercontent.com/griesea/Math245/master/finaldata.csv", header = TRU
#Creating ratio of imports to exports
econ$TradeRatio <- econ$Imports/econ$Exports
econ$Population <- econ$Population/1000000</pre>
econ
##
               GDP_Rate Inflation IntRate PrimEducRate SecEducRate TerEducRate
## 1
      1975 -0.19767854
                              9.13
                                      -1.28
                                                   89.24
                                                                 85.19
                                                                              51.01
## 2
      1976
            5.38609005
                              5.74
                                       1.28
                                                   90.11
                                                                 85.36
                                                                              54.67
## 3
      1977
                                                                              52.84
             4.60859741
                              6.49
                                      0.58
                                                   90.35
                                                                 85.41
## 4
      1978
             5.56168493
                              7.65
                                      1.90
                                                   90.82
                                                                 85.10
                                                                              53.29
## 5
      1979
                             11.27
                                      4.07
                                                                78.57
                                                                              52.50
            3.17569075
                                                   90.45
      1980 -0.24459623
                             13.51
                                      5.73
                                                   98.85
                                                                 90.84
                                                                              53.45
## 7
      1981
                             10.32
                                      8.72
                                                                             55.52
             2.59447039
                                                   98.70
                                                                89.53
                              6.16
      1982 -1.91089107
                                      8.15
                                                  100.68
                                                                 92.75
                                                                              56.77
## 9
      1983
            4.63245718
                              3.21
                                      6.59
                                                   99.30
                                                                92.92
                                                                             57.32
## 10 1984
            7.25908696
                              4.32
                                      8.20
                                                   98.13
                                                                 94.92
                                                                             58.13
## 11 1985
                              3.56
                                                                93.96
                                                                             57.99
             4.23873752
                                      6.52
                                                   98.35
## 12 1986
            3.51161450
                              1.86
                                      6.19
                                                   99.82
                                                                95.05
                                                                             59.16
## 13 1987
             3.46174769
                              3.74
                                      5.51
                                                  100.83
                                                                95.25
                                                                              61.80
## 14 1990
            1.91937030
                              5.40
                                      6.08
                                                  105.16
                                                                91.01
                                                                             70.75
                                                                 90.88
                                                                             72.55
## 15 1991 -0.07408453
                              4.23
                                      4.97
                                                  104.42
## 16 1993
                              2.95
                                                                             78.68
            2.74585672
                                      3.54
                                                  102.96
                                                                 95.36
## 17 1994
            4.03764342
                              2.61
                                      4.91
                                                  102.44
                                                                 96.47
                                                                             78.31
## 18 1995
             2.71897579
                              2.81
                                      6.61
                                                  103.88
                                                                 95.06
                                                                             78.31
## 19 1996
             3.79588123
                              2.93
                                      6.33
                                                  103.92
                                                                 94.72
                                                                             77.78
## 20 1998
                              1.55
                                                                             70.63
             4.44991096
                                      7.19
                                                  103.94
                                                                 94.83
             4.68519961
## 21 1999
                              2.19
                                       6.37
                                                  103.57
                                                                 94.04
                                                                             72.16
## 22 2000
             4.09217645
                              3.38
                                      6.80
                                                  102.52
                                                                 93.17
                                                                              68.14
## 23 2001
            0.97598183
                              2.83
                                      4.54
                                                  103.00
                                                                 93.95
                                                                              68.98
## 24 2002
            1.78612769
                              1.59
                                      3.09
                                                  100.59
                                                                 93.29
                                                                             79.33
## 25 2003
                                                                             81.31
            2.80677596
                              2.27
                                      2.09
                                                  100.58
                                                                 95.01
## 26 2004
            3.78574285
                              2.68
                                      1.55
                                                  100.00
                                                                95.50
                                                                             81.46
## 27 2005
             3.34521606
                              3.39
                                      2.88
                                                  100.70
                                                                95.67
                                                                             82.08
## 28 2006
            2.66662583
                              3.23
                                      4.74
                                                  101.60
                                                                95.40
                                                                             82.05
## 29 2007
             1.77857024
                              2.85
                                      5.25
                                                  102.79
                                                                96.07
                                                                             83.03
## 30 2008 -0.29162146
                              3.84
                                      3.07
                                                  103.38
                                                                95.98
                                                                             85.01
## 31 2009 -2.77552957
                             -0.36
                                      2.47
                                                  102.34
                                                                95.42
                                                                             88.58
## 32 2010
            2.53192062
                              1.64
                                      2.00
                                                  100.60
                                                                 94.31
                                                                             94.23
## 33 2011
             1.60145467
                              3.16
                                      1.16
                                                   99.90
                                                                 94.68
                                                                              96.32
## 34 2012
             2.22403085
                              2.07
                                      1.38
                                                   99.56
                                                                 94.98
                                                                              94.84
## 35 2013
            1.67733153
                              1.46
                                                   99.42
                                                                 95.93
                                                                              88.81
                                       1.61
##
  36 2014
            2.37045767
                              1.62
                                                   99.53
                                                                 97.56
                                                                              86.66
                                       1.43
##
       MCapRate Population Unemployment
                                               Imports
                                                            Exports
                                                                      S.P500
```

8.46 1.22729e+11 1.38707e+11

## 1

41.67153

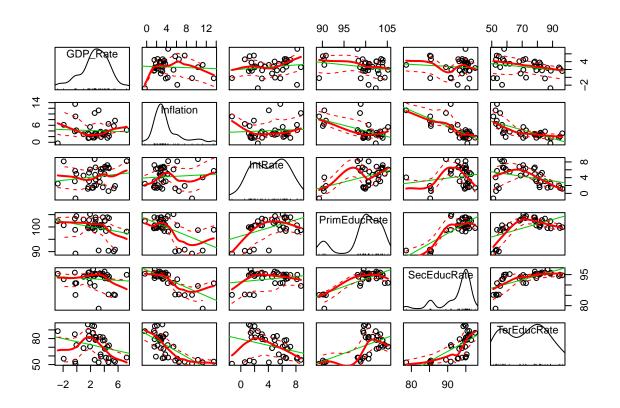
215.9730

```
## 2
       47.03377
                   218.0350
                                    7.70 1.51146e+11 1.49515e+11 0.2383
## 3
                                    7.06 1.82443e+11 1.59350e+11 -0.0698
       39.99317
                   220.2390
## 4
                                                                    0.0651
       36.57692
                   222.5850
                                    6.07 2.12251e+11 1.86885e+11
       37.74836
                                    5.85 2.52675e+11 2.30129e+11
## 5
                  225.0550
                                                                    0.1852
## 6
       47.50377
                   227.2250
                                    7.14 2.93829e+11 2.80773e+11
                                                                    0.3174
## 7
       39.35154
                  229.4660
                                    7.61 3.17759e+11 3.05239e+11 -0.0470
## 8
       43.55364
                   231.6640
                                    9.69 3.03183e+11 2.83209e+11
                                                                    0.2042
                                    9.61 3.28638e+11 2.76996e+11
## 9
       49.72420
                  233.7920
                                                                    0.2234
## 10
       39.65062
                   235.8250
                                    7.52 4.05107e+11 3.02383e+11
                                                                    0.0615
## 11
       52.93303
                   237.9240
                                    7.20 4.17228e+11 3.03209e+11
                                                                    0.3124
## 12
       55.29099
                   240.1330
                                    6.99 4.52867e+11 3.21000e+11
                                                                    0.1849
                                    6.19 5.08714e+11 3.63944e+11
## 13
       51.98470
                   242.2890
                                                                    0.0581
## 14
       51.73346
                  249,6230
                                    5.60 6.29729e+11 5.51874e+11 -0.0306
## 15
       67.37231
                   252.9810
                                    6.83 6.23547e+11 5.94932e+11
                                                                    0.3023
       76.33806
                   259.9190
                                    6.92 7.19974e+11 6.54800e+11
## 16
                                                                    0.0997
## 17
       70.29569
                   263.1260
                                    6.10 8.13425e+11 7.20939e+11
                                                                    0.0133
## 18
       90.70944
                   266.2780
                                    5.60 9.02571e+11 8.12813e+11
                                                                    0.3720
## 19 104.69490
                   269.3940
                                    5.40 9.63966e+11 8.67590e+11
                                                                    0.2268
                                    4.51 1.11569e+12 9.52981e+11
## 20 142.17561
                  275.8540
                                                                    0.2834
## 21 152.96514
                  279.0400
                                    4.22 1.24861e+12 9.91980e+11
                                                                    0.2089
## 22 146.89427
                  282.1624
                                    3.99 1.47263e+12 1.09684e+12 -0.0903
## 23 131.65033
                   284.9690
                                    4.73 1.39540e+12 1.02671e+12 -0.1185
## 24 100.70067
                  287.6252
                                    5.78 1.42897e+12 1.00251e+12 -0.2197
## 25 123.93949
                                    5.99 1.54393e+12 1.04028e+12
                   290.1079
                                                                    0.2836
## 26 132.98429
                  292.8053
                                    5.53 1.80068e+12 1.18151e+12
                                                                    0.1074
## 27 129.83978
                   295.5166
                                    5.08 2.03009e+12 1.30890e+12
                                                                    0.0483
## 28 141.23218
                  298.3799
                                    4.62 2.24726e+12 1.47632e+12
                                                                    0.1561
## 29 137.60728
                   301.2312
                                    4.62 2.38317e+12 1.66463e+12
                                                                    0.0548
## 30
      78.74589
                   304.0940
                                    5.78 2.56501e+12 1.84194e+12 -0.3655
## 31 104.56730
                   306.7715
                                    9.25 1.98318e+12 1.58774e+12
                                                                    0.2594
## 32 115.49734
                   309.3469
                                    9.63 2.36499e+12 1.85234e+12
                                                                    0.1482
## 33 100.79122
                   311.7189
                                    8.95 2.68637e+12 2.10637e+12
                                                                    0.0210
## 34 115.55579
                   314.1026
                                    8.07 2.76384e+12 2.19818e+12
                                                                    0.1589
## 35 143.99442
                  316.4274
                                    7.38 2.76861e+12 2.27661e+12
                                                                    0.3215
   36 151.38523
                  318.9074
                                    6.17 2.88407e+12 2.37528e+12 0.1352
##
##
      Poverty President Senate House TradeRatio
## 1
         12.3
                       0
                              1
                                       0.8848075
## 2
         11.8
                       0
                              1
                                        1.0109086
                                     1
## 3
         11.6
                       1
                              1
                                    1
                                        1.1449200
## 4
         11.4
                       1
                              1
                                        1.1357305
                                    1
## 5
         11.7
                       1
                              1
                                    1
                                        1.0979711
## 6
         13.0
                       1
                              1
                                    1
                                        1.0465002
## 7
         14.0
                       0
                              0
                                    1
                                       1.0410170
## 8
                       0
                              0
         15.0
                                    1
                                       1.0705274
## 9
         15.2
                       0
                              0
                                    1
                                       1.1864359
## 10
         14.4
                       0
                              0
                                    1
                                       1.3397149
## 11
         14.0
                       0
                              0
                                    1
                                       1.3760409
                       0
## 12
         13.6
                              0
                                    1
                                        1.4108006
## 13
         13.4
                       0
                              1
                                       1.3977810
                                    1
## 14
         13.5
                       0
                              1
                                        1.1410739
## 15
                       0
         14.2
                              1
                                       1.0480979
                                    1
## 16
         15.1
                       1
                              1
                                    1
                                       1.0995327
## 17
         14.5
                       1
                              1
                                       1.1282855
                                    1
## 18
         13.8
                       1
                              0
                                       1.1104288
```

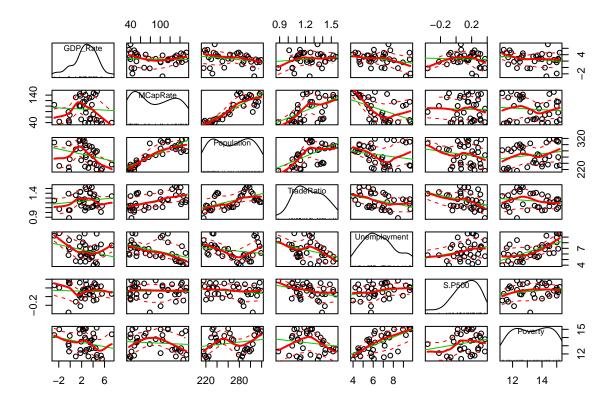
```
13.7
                                      0 1.1110847
## 19
         12.7
                                         1.1707369
## 20
                        1
                               0
## 21
         11.9
                                         1.2587048
## 22
         11.3
                               0
                                         1.3426115
                        1
## 23
         11.7
                        0
                               1
                                         1.3590985
## 24
         12.1
                        0
                               1
                                         1.4253923
## 25
         12.5
                        0
                               0
                                         1.4841485
         12.7
## 26
                       0
                               0
                                         1.5240497
## 27
         12.6
                        0
                               0
                                         1.5509894
## 28
         12.3
                        0
                               0
                                         1.5222039
## 29
         12.5
                        0
                               1
                                         1.4316515
         13.2
                       0
## 30
                               1
                                         1.3925589
  31
         14.3
                                         1.2490584
##
                        1
                               1
## 32
         15.1
                        1
                               1
                                         1.2767580
## 33
         15.0
                        1
                               1
                                         1.2753552
                                      1
## 34
         15.0
                        1
                               1
                                         1.2573311
## 35
         14.5
                        1
                               1
                                      0
                                         1.2161108
## 36
         14.8
                                         1.2142021
```

# Performing EDA:

scatterplotMatrix(~ GDP\_Rate + Inflation + IntRate + PrimEducRate +SecEducRate + TerEducRate, data = ec



scatterplotMatrix(~ GDP\_Rate + MCapRate+ Population + TradeRatio + Unemployment + S.P500 + Poverty, dat

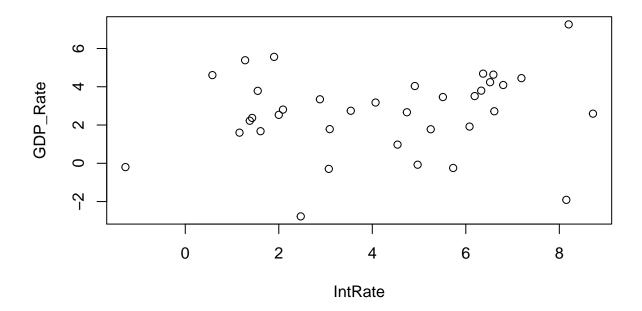


We see that some of our variables (specifically MCapRate and Population) appear to be highly colinear (we will deal with this later).

# Transformations

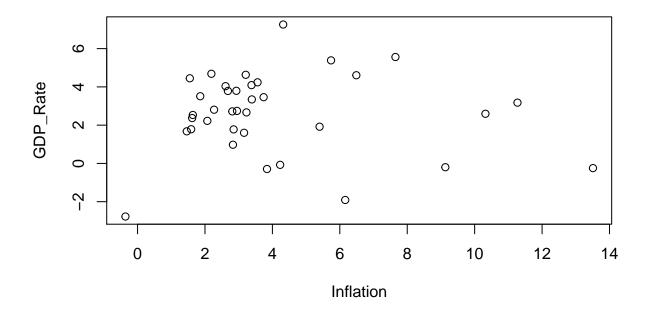
We dont appear to need a tranformation to IntRate (no convincing trend appears).

```
plot(GDP_Rate ~ (IntRate) , data = econ)
```



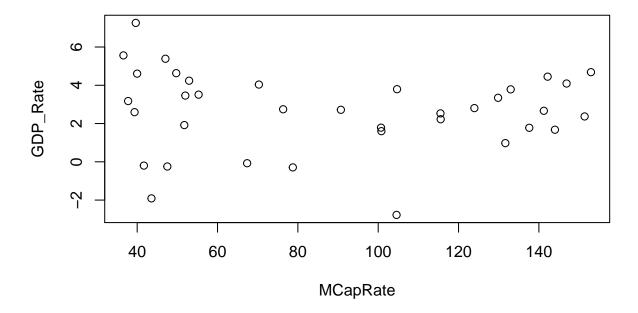
Inflation looks like it needs a transformation. However, economic theory suggests a peak benefit inflation rate (with too low or too high inflation rates being detrimental to the economy). As such, we will try to make inflation a quadratic term in the model, and so do not transform it here.

```
plot(GDP_Rate ~ (Inflation) , data = econ)
```



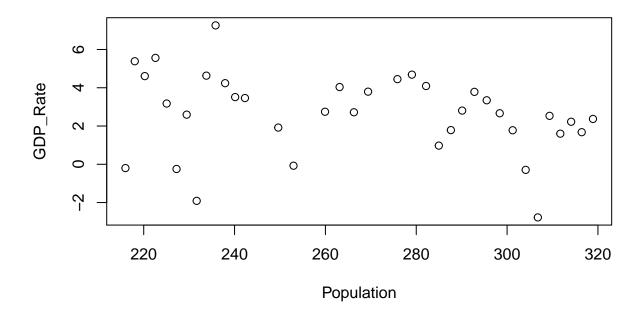
From the matrix above, it appears that MCapRate might both be curved. We examine this trend further, below. There looks to be a parabolic relationship between GDP\_Rate and MCapRate. To determine if this is the correct transformation, we must fit a linear model and look at the residuals. To be done after other transformations.

```
plot(GDP_Rate ~ MCapRate , data = econ)
```



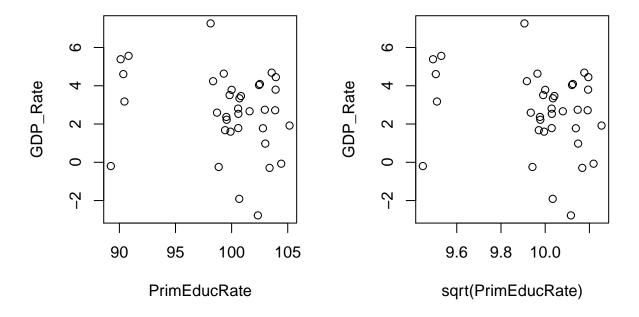
Next, we check if there is a transformation needed for the population explanatory variable. There does not seem any clear indication that there needs to be a transformation based on the below plot.

```
plot(GDP_Rate ~ Population , data = econ)
```

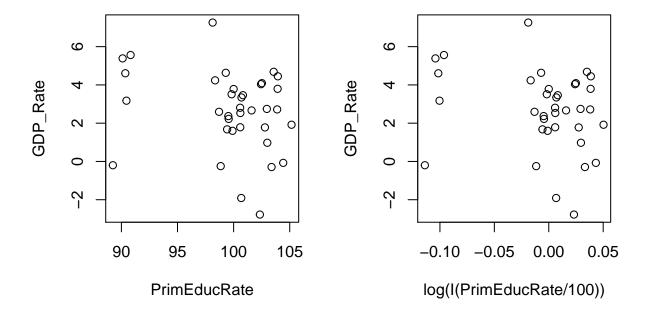


Next, we check if there is a transformation needed for the primary education explanatory variable. There is some clustering we would like to get rid of. However, no transformation seems to make the relationship between GDP\_Rate and PrimEducRate any better.

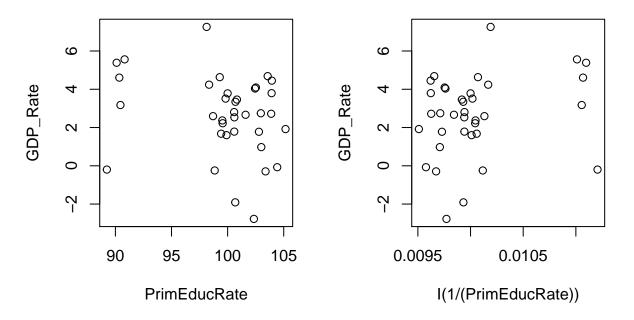
```
par(mfrow=c(1,2))
plot(GDP_Rate ~ PrimEducRate , data = econ)
plot(GDP_Rate ~ sqrt(PrimEducRate) , data = econ)
```



```
par(mfrow=c(1,1))
par(mfrow=c(1,2))
plot(GDP_Rate ~ PrimEducRate , data = econ)
plot(GDP_Rate ~ log(I(PrimEducRate/100)) , data = econ)
```



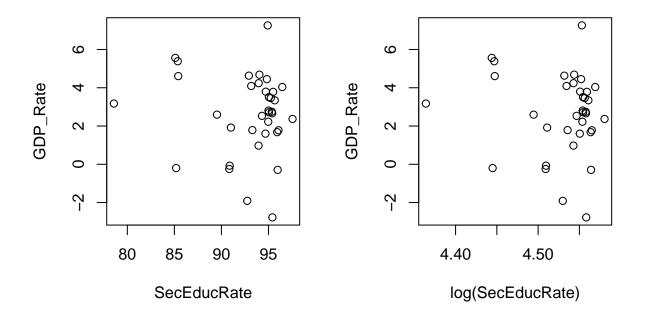
```
par(mfrow=c(1,1))
par(mfrow=c(1,2))
plot(GDP_Rate ~ PrimEducRate , data = econ)
plot(GDP_Rate ~ I(1/(PrimEducRate)) , data = econ)
```



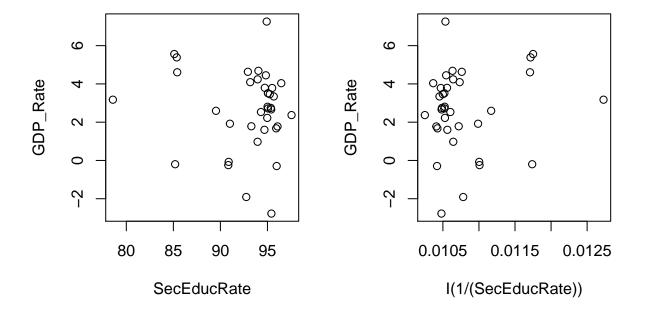
```
par(mfrow=c(1,1))
```

Next, we check if there is a transformation needed for the secondary education explanatory variable. There is some clustering we would like to get rid of. However, no transformations seem to improve the fit.

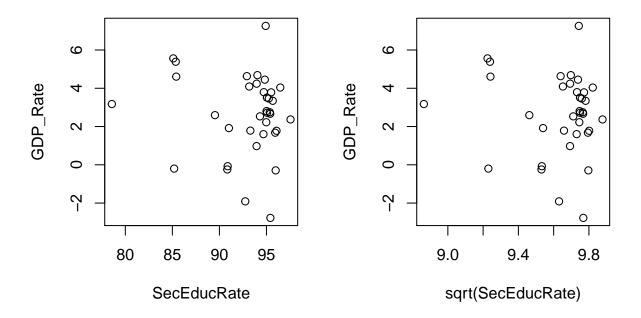
```
par(mfrow=c(1,2))
plot(GDP_Rate ~ SecEducRate , data = econ)
plot(GDP_Rate ~ log(SecEducRate) , data = econ)
```



```
par(mfrow=c(1,1))
par(mfrow=c(1,2))
plot(GDP_Rate ~ SecEducRate , data = econ)
plot(GDP_Rate ~ I(1/(SecEducRate)) , data = econ)
```



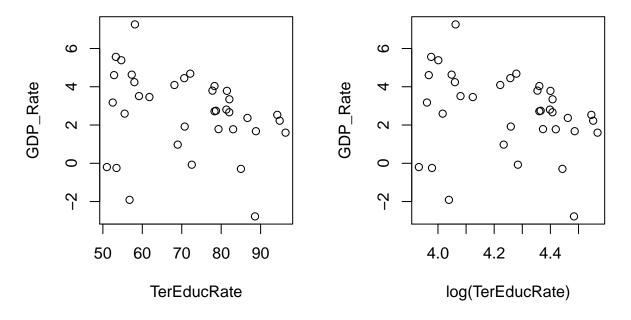
```
par(mfrow=c(1,1))
par(mfrow=c(1,2))
plot(GDP_Rate ~ SecEducRate , data = econ)
plot(GDP_Rate ~ sqrt(SecEducRate) , data = econ)
```



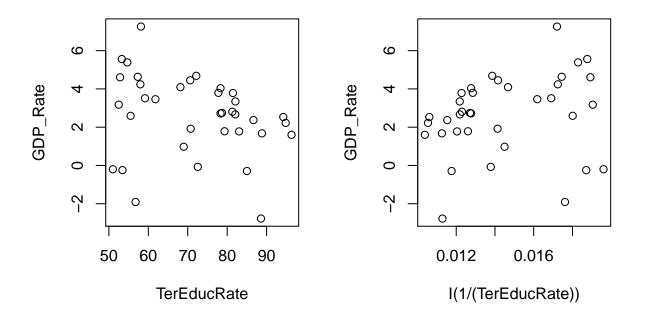
```
par(mfrow=c(1,1))
```

Finally, we check if there is a transformation needed for the tertiary education explanatory variable. There is some clustering we would like to get rid of. However, no transformations seem to improve the fit.

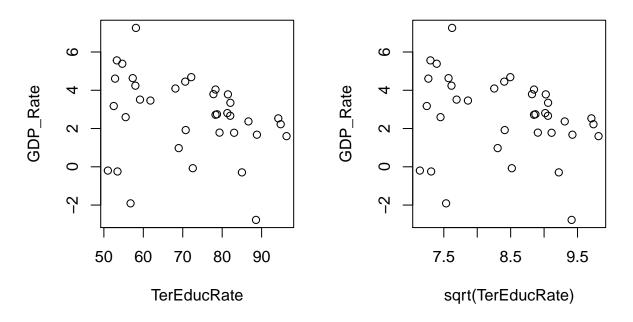
```
par(mfrow=c(1,2))
plot(GDP_Rate ~ TerEducRate , data = econ)
plot(GDP_Rate ~ log(TerEducRate) , data = econ)
```



```
par(mfrow=c(1,1))
par(mfrow=c(1,2))
plot(GDP_Rate ~ TerEducRate , data = econ)
plot(GDP_Rate ~ I(1/(TerEducRate)) , data = econ)
```



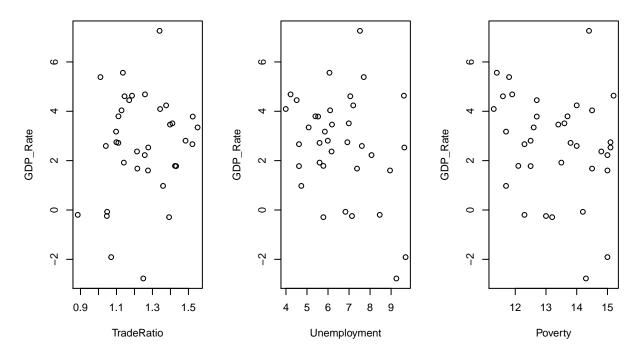
```
par(mfrow=c(1,1))
par(mfrow=c(1,2))
plot(GDP_Rate ~ TerEducRate , data = econ)
plot(GDP_Rate ~ sqrt(TerEducRate) , data = econ)
```



```
par(mfrow=c(1,1))
```

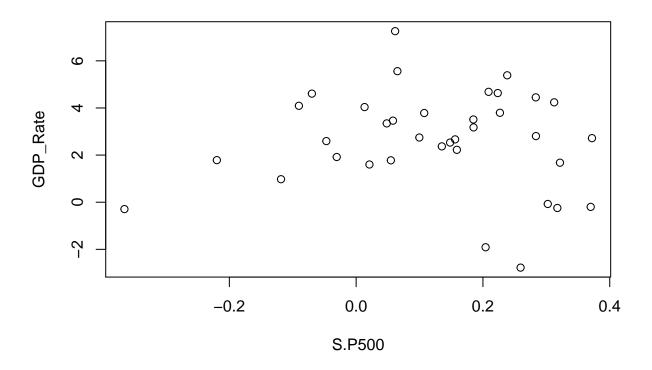
Trade ratio, poverty rate, and unemployment show no convincing needs for transformations:

```
par(mfrow=c(1,3))
plot(GDP_Rate ~ (TradeRatio), data=econ)
plot(GDP_Rate ~ Unemployment, data=econ)
plot(GDP_Rate ~ Poverty, data=econ)
```



# par(mfrow=c(1,1))

However, S.P500 might have a quadratic effect on GDP\_Rate. We will test this as we fit our model. plot(GDP\_Rate ~ (S.P500) , data = econ)



test.lm1 <- lm(GDP\_Rate ~ Inflation + IntRate + PrimEducRate +SecEducRate + TerEducRate + MCapRate+ Pop

#### Preliminary Linear Model

Fitting only linear terms:

```
summary(test.lm1)
##
## Call:
## lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
##
       SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
       Unemployment + S.P500 + Poverty + President + Senate + House,
##
##
       data = econ)
##
## Residuals:
##
        Min
                  1Q
                       Median
                                     3Q
                                             Max
## -2.81006 -0.68182 0.05817 0.64397
                                         1.66891
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                73.117083
                           13.163184
                                        5.555 1.64e-05 ***
## Inflation
                -0.280117
                            0.123364
                                       -2.271 0.033813 *
## IntRate
                 0.445717
                            0.230647
                                        1.932 0.066903 .
## PrimEducRate -0.652083
                            0.157576
                                       -4.138 0.000467 ***
## SecEducRate
                                        0.036 0.971878
                 0.005268
                            0.147668
## TerEducRate
                 0.227245
                            0.081951
                                        2.773 0.011400 *
```

```
## MCapRate
                 0.063367
                            0.027339
                                       2.318 0.030635 *
## Population
                -0.170352
                            0.045579
                                      -3.737 0.001215 **
## TradeRatio
                 5.809755
                            2.788127
                                       2.084 0.049586 *
## Unemployment -0.795322
                            0.305231
                                      -2.606 0.016511 *
## S.P500
                -4.354906
                            1.758571
                                      -2.476 0.021864 *
## Poverty
                 1.128384
                            0.505880
                                       2.231 0.036750 *
## President
                 0.736001
                            0.585801
                                       1.256 0.222768
## Senate
                -0.177714
                            0.725315
                                      -0.245 0.808822
## House
                 0.292669
                            1.063025
                                       0.275 0.785763
## ---
## Signif. codes:
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.195 on 21 degrees of freedom
## Multiple R-squared: 0.8047, Adjusted R-squared: 0.6746
## F-statistic: 6.182 on 14 and 21 DF, p-value: 0.0001102
```

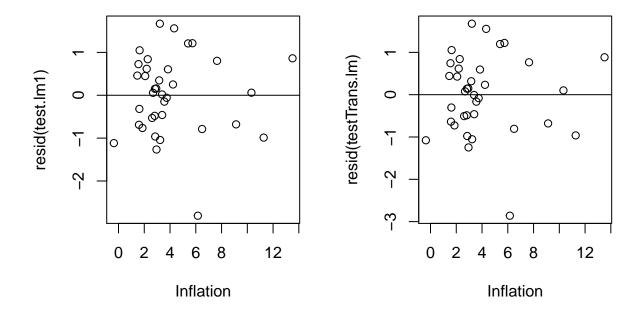
# Checking Residual Plots

First, we check the residual plot of our linear model against inflation. From before, using economic theory we would expect there to be a parabolic relationship. However, transforming inflation does not improve the fit and we proceed with the original model.

```
testTrans.lm <- lm(GDP_Rate ~ I(Inflation^2)+Inflation + IntRate + PrimEducRate +SecEducRate + TerEducRsummary(testTrans.lm)
```

```
##
## Call:
  lm(formula = GDP_Rate ~ I(Inflation^2) + Inflation + IntRate +
##
       PrimEducRate + SecEducRate + TerEducRate + MCapRate + Population +
##
       TradeRatio + Unemployment + S.P500 + Poverty + President +
##
##
       Senate + House, data = econ)
##
## Residuals:
       Min
                1Q
                   Median
                                 3Q
                                        Max
  -2.8602 -0.6481 0.0901
##
                            0.6480
                                     1.6771
##
## Coefficients:
##
                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                  71.986565
                             16.816573
                                          4.281 0.000365 ***
## I(Inflation^2) -0.003476
                              0.030896
                                        -0.113 0.911547
## Inflation
                  -0.229912
                              0.463805
                                        -0.496 0.625504
## IntRate
                   0.447178
                              0.236624
                                         1.890 0.073365
## PrimEducRate
                  -0.646941
                              0.167763
                                        -3.856 0.000984 ***
## SecEducRate
                   0.008270
                              0.153602
                                         0.054 0.957595
## TerEducRate
                   0.224264
                              0.088030
                                          2.548 0.019176 *
## MCapRate
                   0.062486
                              0.029082
                                          2.149 0.044092 *
## Population
                  -0.168294
                              0.050146
                                        -3.356 0.003144 **
## TradeRatio
                                         2.016 0.057441
                   5.874948
                              2.914272
## Unemployment
                  -0.781539
                              0.335815
                                        -2.327 0.030565 *
## S.P500
                  -4.219904
                              2.164512
                                        -1.950 0.065387
## Poverty
                   1.109681
                              0.544223
                                          2.039 0.054884
## President
                   0.779213
                              0.712476
                                         1.094 0.287099
## Senate
                  -0.154616
                              0.770839 -0.201 0.843052
## House
                   0.268278
                              1.110304
                                          0.242 0.811530
```

```
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.224 on 20 degrees of freedom
## Multiple R-squared: 0.8049, Adjusted R-squared: 0.6585
## F-statistic: 5.499 on 15 and 20 DF, p-value: 0.0002859
par(mfrow=c(1,2))
plot(resid(test.lm1) ~ Inflation , data = econ)
abline(h=0)
plot(resid(testTrans.lm) ~ Inflation, data = econ)
abline(h=0)
```



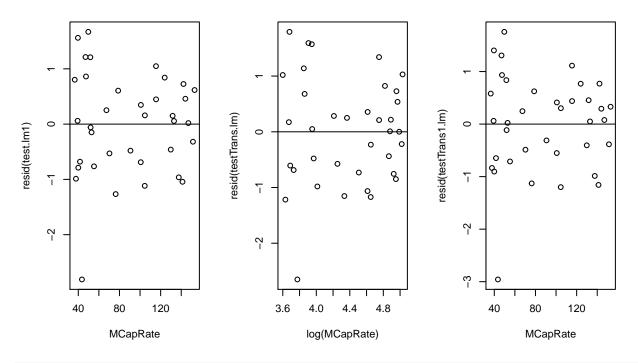
# par(mfrow=c(1,1))

Next, we check the residual plot of our linear model against MCapRate. We noticed a parabolic relationship previously and check for this. However, on the basis of the residual plot this seems to not improve the model significantly. As such, we check for a log transformation instead. However, this does not improve the fit and we move forward with no transformation.

```
testTrans.lm <- lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRate +log(MCapR testTrans1.lm <- lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRate +I(MCapRa summary(testTrans.lm)
```

```
##
## Call:
## lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
## SecEducRate + TerEducRate + log(MCapRate) + Population +
## TradeRatio + Unemployment + S.P500 + Poverty + President +
## Senate + House, data = econ)
##
```

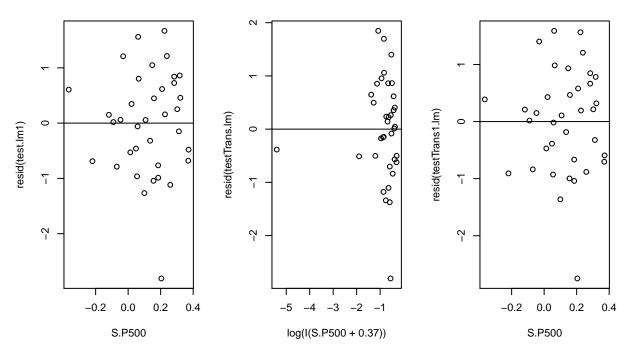
```
## Residuals:
##
       Min
                   Median
                1Q
                                30
                                       Max
## -2.65056 -0.69738 0.02891 0.69223 1.79384
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
               61.827216 12.930488 4.782 0.000101 ***
## (Intercept)
## Inflation
               1.967 0.062572 .
## IntRate
                0.481393 0.244771
## PrimEducRate -0.711043 0.175585 -4.050 0.000577 ***
## SecEducRate -0.005494 0.155495 -0.035 0.972150
                0.178853 0.080301
## TerEducRate
                                    2.227 0.036998 *
## log(MCapRate) 4.734898 2.618040
                                   1.809 0.084861 .
## Population
               ## TradeRatio
                5.257694
                          2.880437
                                   1.825 0.082212 .
## Unemployment -0.875314 0.315684 -2.773 0.011404 *
## S.P500
               -4.127519 1.872064 -2.205 0.038756 *
## Poverty
                1.184997 0.556851
                                    2.128 0.045342 *
## President
                0.795511 0.611622
                                   1.301 0.207468
## Senate
               -0.303267
                          0.752074 -0.403 0.690848
## House
                0.023939 1.101748 0.022 0.982870
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.246 on 21 degrees of freedom
## Multiple R-squared: 0.7878, Adjusted R-squared: 0.6464
## F-statistic: 5.57 on 14 and 21 DF, p-value: 0.0002355
par(mfrow=c(1,3))
plot(resid(test.lm1) ~ MCapRate, data = econ)
abline(h=0)
plot(resid(testTrans.lm) ~ log(MCapRate), data = econ)
abline(h=0)
plot(resid(testTrans1.lm) ~ MCapRate, data = econ)
abline(h=0)
```



# par(mfrow=c(1,1))

Next, we check the residual plot of our linear model against S.P500. We noted a potential parabolic relationship previously, as such we check the residual plot. On the basis of the residual plot, a parabolic transformation seems inappropriate. However, there may be a need for a log transformation. This is wildly inappropriate and we move forward with no transformation.

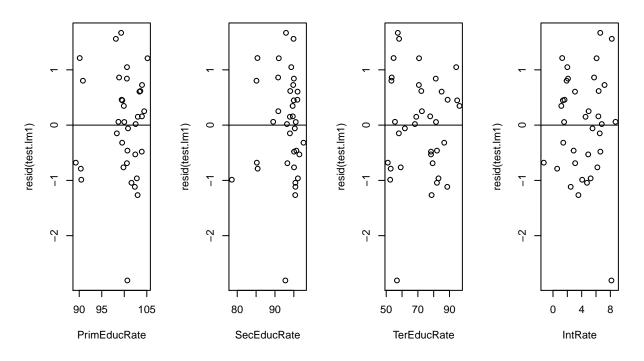
```
testTrans.lm <- lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRate + MCapRate+
testTrans1.lm <- lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRate + MCapRate
par(mfrow=c(1,3))
plot(resid(test.lm1) ~ S.P500, data = econ)
abline(h=0)
plot(resid(testTrans.lm) ~ log(I(S.P500+.37)), data = econ)
abline(h=0)
plot(resid(testTrans1.lm) ~ S.P500, data = econ)
abline(h=0)</pre>
```



```
par(mfrow=c(1,1))
```

Next, we check the residual plot of our linear model against IntRate, PrimEducRate, SecEducRate and TerEducRate. There does not seem to be any obvious transformations that will improve the fit.

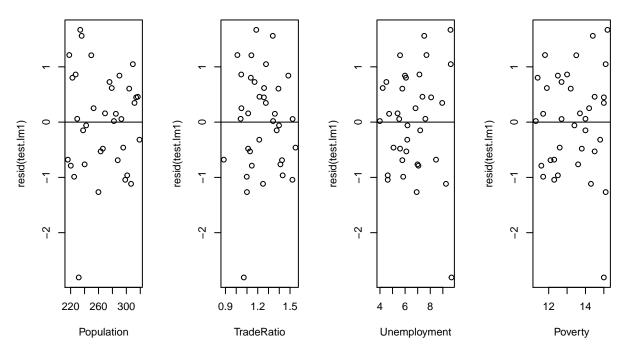
```
par(mfrow=c(1,4))
plot(resid(test.lm1) ~ (PrimEducRate) , data = econ)
abline(h=0)
plot(resid(test.lm1) ~ (SecEducRate) , data = econ)
abline(h=0)
plot(resid(test.lm1) ~ (TerEducRate) , data = econ)
abline(h=0)
plot(resid(test.lm1) ~ (IntRate) , data = econ)
abline(h=0)
```



# par(mfrow=c(1,1))

Next, we check the residual plot of our linear model against Population, Poverty, TradeRatio and Unemployment. As expected, we conclude there is no transformation to be performed.

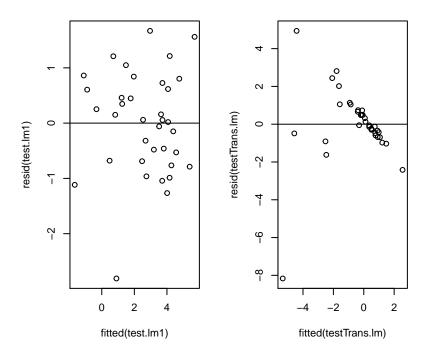
```
par(mfrow=c(1,4))
plot(resid(test.lm1) ~ Population, data = econ)
abline(h=0)
plot(resid(test.lm1) ~ TradeRatio, data = econ)
abline(h=0)
plot(resid(test.lm1) ~ Unemployment, data = econ)
abline(h=0)
plot(resid(test.lm1) ~ Poverty, data = econ)
abline(h=0)
```



```
par(mfrow=c(1,1))
```

Finally, we check for a log-linear model. However, clearly the fit is not improved.

```
testTrans.lm <- lm(I(1/GDP_Rate) ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRate + MCap
par(mfrow=c(1,3))
plot(resid(test.lm1) ~ fitted(test.lm1), data = econ)
abline(h=0)
plot(resid(testTrans.lm) ~ fitted(testTrans.lm), data = econ)
abline(h=0)
par(mfrow=c(1,1))</pre>
```

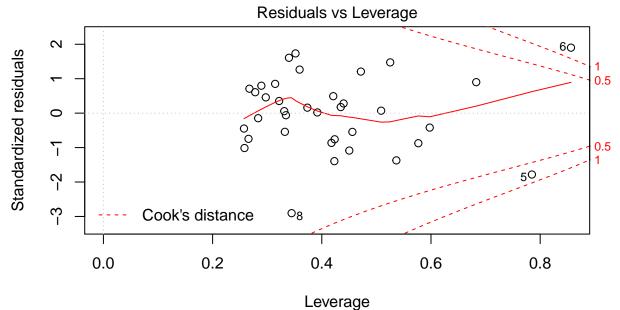


# Case Influence Stats

Now we check for case influence stats. We notice that case 6, which is the year 1980 is exerting large influence on our model as it has a Cook's distance of 1.5. This is due to the very high inflation during this year. As such, we remove the point from our data set due to really high inflation.

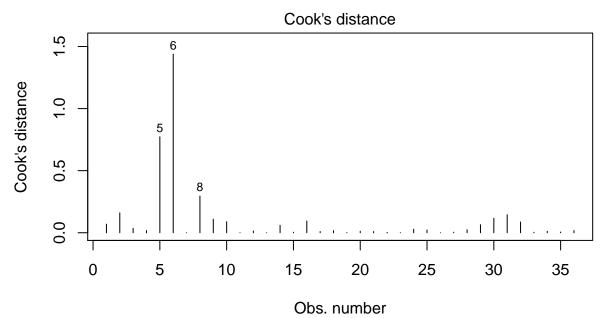
```
cooks.distance(test.lm1)
```

```
2
                                           3
                                                         4
                                                                       5
##
   6.939291e-02 1.601746e-01 3.568674e-02 1.722600e-02 7.727511e-01
##
                             7
                                           8
##
               6
                                                         9
   1.438605e+00 3.525260e-04
                               2.957976e-01 1.088504e-01 8.867965e-02
##
##
                            12
                                          13
                                                        14
                                                                      15
   5.707494e-04
                 1.343123e-02 1.237445e-04 5.977915e-02 4.143389e-03
##
                                          18
##
              16
                            17
                                                        19
                                                                      20
##
   9.497188e-02
                 9.766577e-03
                               1.656814e-02
                                             1.595921e-03
                                                           1.224736e-02
##
              21
                            22
                                          23
                                                        24
                                                                      25
                 1.785535e-05
                               1.005579e-03
##
   9.464610e-03
                                            2.823630e-02 2.210881e-02
##
              26
                            27
                                          28
                                                        29
                                                                      30
                 4.632716e-03
                               2.385480e-02 6.450649e-02 1.161886e-01
##
   1.102038e-04
##
              31
                            32
                                          33
                                                        34
                                                                      35
   1.453173e-01 8.663293e-02 3.933193e-03 1.174797e-02 5.914685e-03
##
## 1.747451e-02
plot(test.lm1, which=5)
```



Im(GDP\_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRat .

plot(test.lm1, which=4)



 $Im(GDP\_Rate \sim Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRat \; .$ 

econ[6,]

## YEAR GDP\_Rate Inflation IntRate PrimEducRate SecEducRate TerEducRate

```
## 6 1980 -0.2445962 13.51 5.73 98.85 90.84 53.45

## MCapRate Population Unemployment Imports Exports S.P500 Poverty

## 6 47.50377 227.225 7.14 2.93829e+11 2.80773e+11 0.3174 13

## President Senate House TradeRatio

## 6 1 1 1 1.0465

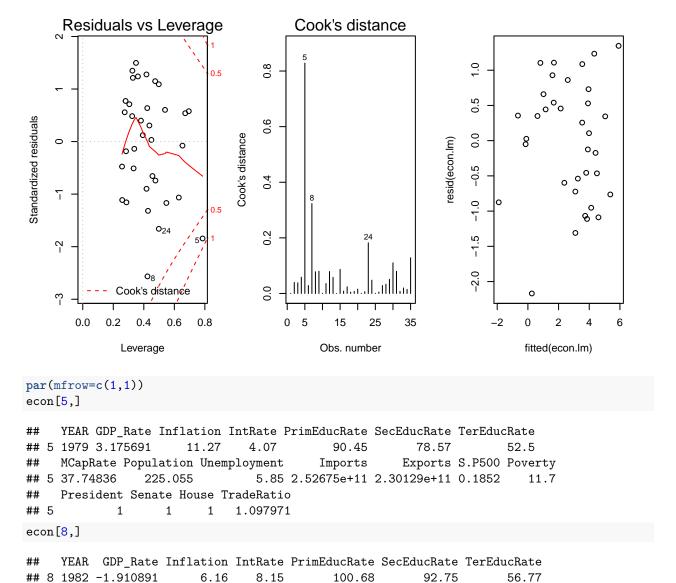
econ1<-econ[-6,]
```

Cases 5 and 8 (corresponding to 1979 and 1982) have relatively large Cook's distances but leverage less than 2\*p/n. However, case 8 does have a studentized residual close to -3, as well. However, since they both do not have a Cook's distance greater than 1 and residuals less than  $\pm 2$  we include both cases. The high Cook's distance value is most likely due to high inflation in both years, especially in 1979.

```
econ.lm <- lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate +SecEducRate + TerEducRate + MCapRate+ Popusummary(econ.lm)
```

```
##
## Call:
##
  lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
       SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
       Unemployment + S.P500 + Poverty + President + Senate + House,
##
##
       data = econ1)
##
##
  Residuals:
##
       Min
                1Q
                   Median
                                3Q
                                        Max
##
   -2.1691 -0.6604 0.1059
                            0.5997
                                    1.3457
##
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
                108.72889
                            21.33741
                                        5.096 5.52e-05 ***
## (Intercept)
## Inflation
                 -0.69789
                             0.23486
                                      -2.971 0.007544 **
## IntRate
                             0.21635
                                        2.286 0.033300 *
                  0.49458
## PrimEducRate
                 -0.80467
                             0.16485
                                       -4.881 9.03e-05 ***
## SecEducRate
                 -0.24819
                             0.18544
                                      -1.338 0.195774
## TerEducRate
                  0.24413
                             0.07685
                                        3.177 0.004740 **
## MCapRate
                  0.07054
                             0.02573
                                        2.742 0.012573 *
## Population
                 -0.16521
                             0.04257
                                       -3.881 0.000929 ***
## TradeRatio
                  3.48477
                             2.83813
                                        1.228 0.233762
## Unemployment
                 -1.20213
                             0.34747
                                       -3.460 0.002476 **
## S.P500
                 -6.57091
                              1.96663
                                       -3.341 0.003253 **
                             0.54882
## Poverty
                  1.70100
                                        3.099 0.005653 **
## President
                 -0.20704
                             0.71548
                                      -0.289 0.775280
## Senate
                 -0.95317
                              0.77570
                                       -1.229 0.233411
## House
                  0.91721
                              1.03723
                                        0.884 0.387047
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.114 on 20 degrees of freedom
## Multiple R-squared: 0.8289, Adjusted R-squared: 0.7091
## F-statistic: 6.919 on 14 and 20 DF, p-value: 6.287e-05
cooks.distance(econ.lm)
                           2
                                         3
##
              1
                                                                    5
## 7.411507e-04 3.963056e-02 3.861500e-02 5.846268e-02 8.283927e-01
##
              7
                           8
                                         9
                                                     10
                                                                   11
```

```
## 2.838591e-02 3.232067e-01 7.798020e-02 7.999290e-02 8.944040e-04
##
             12
                          13
                                       14
                                                    15
                                                                  16
## 3.572345e-02 7.873449e-02 5.801370e-02 5.653847e-05 8.654959e-02
                                       19
                                                    20
             17
                          18
## 8.664144e-03 2.411120e-02 4.851060e-03 7.861004e-03 1.553772e-02
##
             22
                          23
                                       24
                                                    25
## 6.436578e-04 6.550056e-03 1.823283e-01 4.790699e-02 6.411177e-04
             27
                          28
                                       29
                                                    30
##
## 5.227425e-03 2.872956e-02 3.332848e-02 5.080429e-02 1.102186e-01
             32
                          33
                                       34
                                                    35
## 7.949393e-02 7.481949e-03 1.981564e-02 1.477847e-02 1.283808e-01
2*15/35
## [1] 0.8571429
hatvalues(econ.lm)
                     2
                               3
                                                   5
                                                             7
                                         4
## 0.6534008 0.6713739 0.4178134 0.3252946 0.7851173 0.5391322 0.4241982
           9
                    10
                              11
                                        12
                                                  13
## 0.4174453 0.3488510 0.2835487 0.2861331 0.4981672 0.3615191 0.4497381
##
          16
                    17
                              18
                                        19
                                                  20
                                                             21
## 0.4276190 0.3332171 0.4566926 0.4366109 0.2748658 0.2805311 0.3932930
                    24
                                                             28
          23
                              25
                                        26
                                                  27
                                                                       29
## 0.3812859 0.4982299 0.3282978 0.3375798 0.2574540 0.2584734 0.4761270
         30
                    31
                              32
                                        33
                                                  34
                                                             35
## 0.6949405 0.5478322 0.4743564 0.3236772 0.4226045 0.3052576 0.6293216
par(mfrow=c(1,3))
plot(econ.lm, which=5)
plot(econ.lm, which=4)
plot(resid(econ.lm) ~ fitted(econ.lm))
```



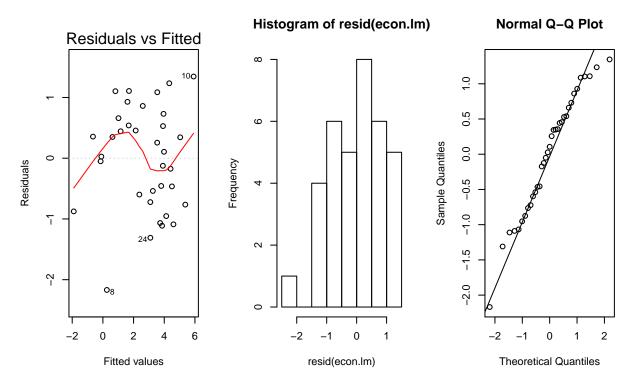
```
## MCapRate Population Unemployment Imports Exports S.P500 Poverty ## 8 43.55364 231.664 9.69 3.03183e+11 2.83209e+11 0.2042 15 ## President Senate House TradeRatio ## 8 0 0 1 1.070527
```

#### Assumptions

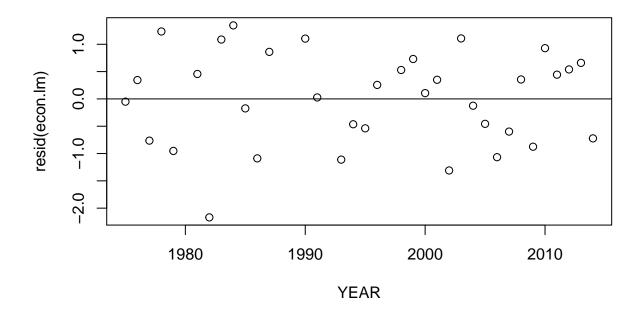
Before we can proceed with backwards selection, we must check the assumptions of our model. The QQ plot has some skewedness in both tails. However, the distribution of the residuals seems to be pretty normal. There is a small amount of left skewedness but it is not too bad. The plots of residuals vs. year, however, support the idea that our model satisfies the independence assumption. It doesn't really appear nearby years are clustered close to each other, possibly as a result of using growth rates instead of raw values. Hence, the independence assumption appears to hold.

```
par(mfrow=c(1,3))
plot(econ.lm, which=1)
```

```
hist(resid(econ.lm))
qqnorm(resid(econ.lm))
qqline(resid(econ.lm))
```



```
par(mfrow=c(1,1))
plot(resid(econ.lm)~YEAR, data=econ1)
abline(h=0)
```



#### **Backwards Selection**

Not surprisingly, we notice that our categorical political categorical variables are insignificant. Due to the presence of some collinearity, we conduct an ANOVA F test on all three to determine if any are worth including in our model. With a p-value of .5791, we fail to reject the null hypothesis that the reduced model is adequate at a 5% level of significance. As such, we exclude all the political categorical variables.

```
summary(econ.lm)
```

```
##
## Call:
   lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
##
       SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
##
       Unemployment + S.P500 + Poverty + President + Senate + House,
       data = econ1)
##
##
##
   Residuals:
##
       Min
                1Q
                    Median
                                 3Q
                                         Max
##
   -2.1691 -0.6604
                     0.1059
                             0.5997
                                     1.3457
##
   Coefficients:
##
##
                  Estimate Std. Error t value Pr(>|t|)
                108.72889
                             21.33741
                                         5.096 5.52e-05 ***
##
   (Intercept)
## Inflation
                  -0.69789
                              0.23486
                                        -2.971 0.007544 **
                                         2.286 0.033300 *
## IntRate
                  0.49458
                              0.21635
## PrimEducRate
                  -0.80467
                              0.16485
                                        -4.881 9.03e-05 ***
## SecEducRate
                  -0.24819
                              0.18544
                                        -1.338 0.195774
## TerEducRate
                  0.24413
                              0.07685
                                         3.177 0.004740 **
## MCapRate
                  0.07054
                              0.02573
                                         2.742 0.012573 *
```

```
## Population
                 -0.16521
                              0.04257 -3.881 0.000929 ***
## TradeRatio
                              2.83813
                                        1.228 0.233762
                  3.48477
## Unemployment
                 -1.20213
                              0.34747
                                       -3.460 0.002476 **
## S.P500
                 -6.57091
                              1.96663
                                      -3.341 0.003253 **
## Poverty
                  1.70100
                              0.54882
                                        3.099 0.005653 **
                             0.71548
## President
                 -0.20704
                                       -0.289 0.775280
                 -0.95317
## Senate
                              0.77570
                                       -1.229 0.233411
## House
                  0.91721
                              1.03723
                                        0.884 0.387047
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.114 on 20 degrees of freedom
## Multiple R-squared: 0.8289, Adjusted R-squared: 0.7091
## F-statistic: 6.919 on 14 and 20 DF, p-value: 6.287e-05
vif(econ.lm)
                                                          TerEducRate
##
      Inflation
                     IntRate PrimEducRate
                                            SecEducRate
       9.924518
##
                    8.226730
                                 14.335413
                                              16.543879
                                                            30.914312
##
       MCapRate
                  Population
                                TradeRatio Unemployment
                                                               S.P500
      30.724187
                                               8.630038
##
                   55.210966
                                  6.016254
                                                             2.969426
##
        Poverty
                   President
                                    Senate
                                                  House
##
      13.124616
                    3.580850
                                  4.153552
                                               7.277964
econ1.lm<-lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRate + MCapRate+ Popul
anova(econ1.lm,econ.lm)
## Analysis of Variance Table
##
## Model 1: GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate +
##
       TerEducRate + MCapRate + Population + TradeRatio + Unemployment +
       S.P500 + Poverty
##
## Model 2: GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate +
##
       TerEducRate + MCapRate + Population + TradeRatio + Unemployment +
       S.P500 + Poverty + President + Senate + House
##
##
               RSS Df Sum of Sq
## 1
         23 27.338
         20 24.834 3
## 2
                         2.5041 0.6722 0.5791
We know that high collinearity suggests that the standard errors for coefficients may be larger than usual.
```

We know that high collinearity suggests that the standard errors for coefficients may be larger than usual. Thus, the high VIF value (>5) for SecEducRate may explain its insignificance. Despite all the very large VIF values, every coefficient/explanatory variable is otherwise significant - i.e. the standard errors are relatively small despite the high levels of collinearity. To check for the significance of SecEducRate, we theorize that only PrimEducRate significantly impacts GDP. However, the ANOVA F-test shows that this theory is very wrong, as the p-value of excluding both TerEducRate and SecEducRate is .00159. Thus, we reject the null hypothesis at a 5% level of significance that the reduced model is adequate. With no theoretical reason to remove SecEducRate, we seek to include it in our model despite the high t-statistic of 3.812. We attribute this high t-statistic value to the corresponding high VIF value. Thus, this is our final model.

```
summary(econ1.lm)
```

```
##
## Call:
## lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
## SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
## Unemployment + S.P500 + Poverty, data = econ1)
##
```

```
## Residuals:
##
       Min
                1Q Median
                                30
                                       Max
  -2.3958 -0.6067 0.0982
                           0.6568
                                    1.3301
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                100.53594
                            14.31160
                                       7.025 3.70e-07 ***
## Inflation
                 -0.58182
                             0.17360
                                      -3.352 0.002764 **
## IntRate
                  0.60274
                             0.17612
                                       3.422 0.002329 **
## PrimEducRate
                -0.78982
                             0.12964
                                      -6.092 3.25e-06 ***
## SecEducRate
                 -0.15591
                             0.15516
                                      -1.005 0.325440
## TerEducRate
                  0.26747
                             0.07017
                                       3.812 0.000896 ***
## MCapRate
                  0.06811
                             0.01907
                                       3.572 0.001617 **
## Population
                 -0.18218
                             0.03838
                                      -4.746 8.76e-05 ***
## TradeRatio
                  4.80290
                             2.13529
                                       2.249 0.034367 *
## Unemployment
                 -0.97990
                             0.26751
                                      -3.663 0.001294 **
## S.P500
                 -5.56625
                                      -3.274 0.003328 **
                             1.69989
## Poverty
                  1.47626
                             0.48650
                                       3.034 0.005893 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.09 on 23 degrees of freedom
## Multiple R-squared: 0.8116, Adjusted R-squared: 0.7215
## F-statistic: 9.007 on 11 and 23 DF, p-value: 5.728e-06
vif(econ1.lm)
##
      Inflation
                     IntRate PrimEducRate SecEducRate TerEducRate
##
       5.664226
                    5.695029
                                 9.262676
                                              12.099923
                                                           26.923969
##
                  Population
                               TradeRatio Unemployment
       MCapRate
                                                              S.P500
##
      17.627397
                   46.896672
                                 3.557571
                                              5.343653
                                                            2.317630
##
        Poverty
      10.773957
##
econ2.lm<-lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate + MCapRate+ Population + TradeRatio + Unempl
anova(econ1.lm,econ2.lm)
## Analysis of Variance Table
##
## Model 1: GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate +
##
       TerEducRate + MCapRate + Population + TradeRatio + Unemployment +
       S.P500 + Poverty
##
## Model 2: GDP_Rate ~ Inflation + IntRate + PrimEducRate + MCapRate + Population +
##
       TradeRatio + Unemployment + S.P500 + Poverty
     Res.Df
               RSS Df Sum of Sq
##
                                     F Pr(>F)
## 1
         23 27.338
## 2
         25 47.876 -2
                        -20.538 8.6395 0.00159 **
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

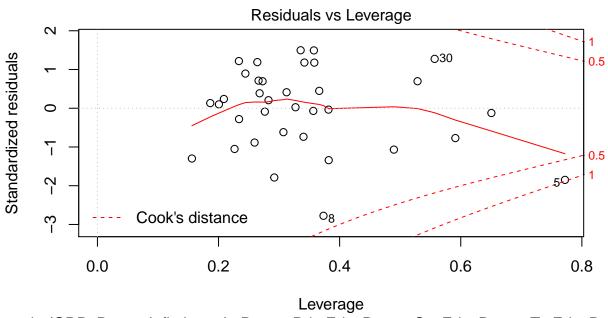
#### Case Influence Stats

Due to a studentized residual of about -2 and a Cook's distance of nearly 1, we exclude case 5 or the year 1979 from the data set and refit. We feel comfortable excluding this data point due to high inflation that year. There are no other significant cases and we move forward this data set and model. Additionally, with

the exclusion of this data point we now find SecEducRate is significant.

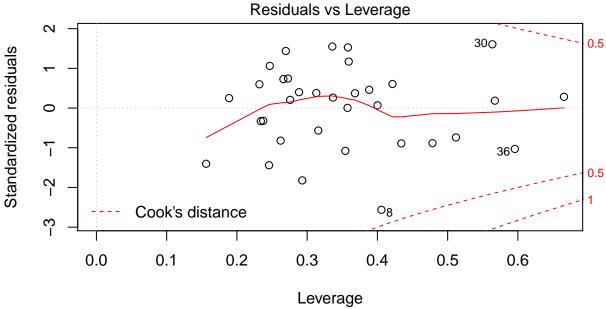
plot(econ1.lm, which=5)

cooks.distance(econ1.lm)



Im(GDP\_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRat .

```
2
                                         3
   2.342698e-03 9.678346e-03 2.335933e-02 3.767469e-02 9.661790e-01
##
              7
                            8
                                         9
                                                      10
   4.548597e-02 3.825515e-01 1.033420e-01 9.428882e-02 2.464054e-04
##
             12
                           13
   2.290960e-02 1.255797e-03 6.019361e-02 6.326758e-05 9.258033e-02
             17
                           18
                                        19
                                                      20
    .409505e-02
                              3.378423e-04
                                           1.534912e-02 2.158366e-02
##
                2.200247e-04
##
             22
                           23
                                        24
                                                      25
   6.442513e-03 2.690603e-05
                             1.099303e-01 4.237163e-02 1.410776e-03
             27
## 2.000723e-03 2.693562e-02 2.594091e-02 1.700269e-01 9.116704e-02
             32
                           33
                                        34
                                                      35
## 6.442722e-02 1.517257e-02 2.123367e-04 4.540390e-03 7.152057e-02
econ1[5,]
     YEAR GDP_Rate Inflation IntRate PrimEducRate SecEducRate TerEducRate
## 5 1979 3.175691
                        11.27
                                                          78.57
                                                                       52.5
                                 4.07
                                             90.45
     MCapRate Population Unemployment
                                           Imports
                                                        Exports S.P500 Poverty
                 225.055
                                  5.85 2.52675e+11 2.30129e+11 0.1852
## 5 37.74836
     President Senate House TradeRatio
## 5
                               1.097971
econ2 < -econ1[-5,]
econ1.lm<-lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate +SecEducRate + TerEducRate + MCapRate+ Popul
```



Im(GDP\_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRat .

#### summary(econ1.lm)

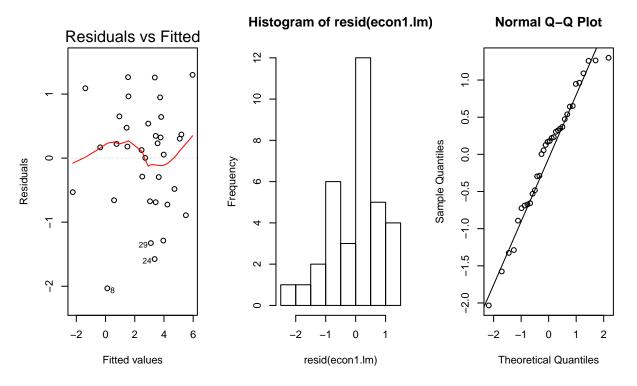
```
##
## Call:
   lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
       SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
##
##
       Unemployment + S.P500 + Poverty, data = econ2)
##
##
   Residuals:
##
       Min
                1Q
                    Median
                                 3Q
                                        Max
   -2.0329 -0.6260
                    0.1742
                             0.5227
                                     1.2992
##
##
  Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                119.04515
                             16.47302
                                        7.227 3.06e-07 ***
## Inflation
                 -0.63586
                              0.16606
                                       -3.829 0.000914 ***
                              0.17069
## IntRate
                  0.67946
                                        3.981 0.000632 ***
                              0.12267
## PrimEducRate
                 -0.80852
                                       -6.591 1.25e-06 ***
## SecEducRate
                 -0.47958
                              0.22061
                                       -2.174 0.040750 *
## TerEducRate
                  0.23841
                              0.06783
                                        3.515 0.001953 **
## MCapRate
                  0.06047
                              0.01840
                                        3.286 0.003375 **
## Population
                 -0.15190
                              0.03936
                                       -3.859 0.000850 ***
## TradeRatio
                              2.26237
                                        3.016 0.006359 **
                  6.82267
## Unemployment
                 -1.33420
                              0.31036
                                       -4.299 0.000291 ***
## S.P500
                 -4.64101
                              1.67156
                                       -2.776 0.011008 *
## Poverty
                  2.06225
                              0.54765
                                        3.766 0.001066 **
## ---
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.028 on 22 degrees of freedom
## Multiple R-squared: 0.8394, Adjusted R-squared: 0.7591
## F-statistic: 10.45 on 11 and 22 DF, p-value: 2.229e-06
```

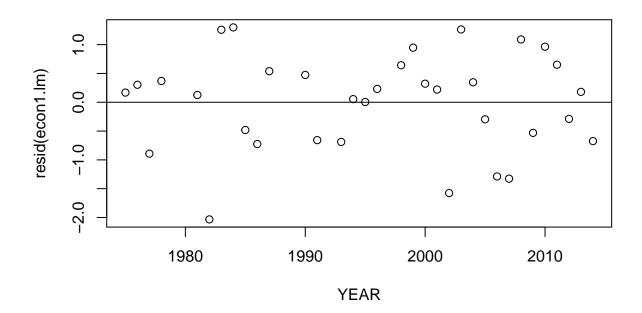
#### Assumptions

The distribution of the residuals does seem to be relatively normal. After removing the insignificant terms and influential cases, normality seems to be violated on the basis of the qqplot. On the basis of the residual plot, normality and constant variance seems to be satisfied. Once again, our data shows no obvious signs of clustering in groups of nearby years. Hence, independence also seems to be satisfied.

```
par(mfrow=c(1,3))
plot(econ1.lm, which=1)
hist(resid(econ1.lm))
qqnorm(resid(econ1.lm))
qqline(resid(econ1.lm))
```



```
par(mfrow=c(1,1))
plot(resid(econ1.lm)~YEAR, data=econ2)
abline(h=0)
```



#### Final model

After removing the influential cases, we have found our final model and can confirm this with an ANOVA F-test.

```
\verb| econ2.lm < -lm (GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRate + MCapRate + Populsummary (econ2.lm)|
```

```
##
## Call:
  lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
##
       SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
##
       Unemployment + S.P500 + Poverty + President + House + Senate,
##
##
       data = econ2
##
##
   Residuals:
##
       Min
                1Q
                                 3Q
                    Median
                                        Max
  -1.7956 -0.6252
##
                    0.1586
                             0.5363
                                     1.3629
##
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                122.60327
                             21.14719
                                        5.798 1.38e-05 ***
## Inflation
                  -0.69577
                              0.21953
                                       -3.169 0.005048 **
## IntRate
                  0.56366
                              0.20523
                                        2.746 0.012833 *
## PrimEducRate
                 -0.79852
                              0.15411
                                       -5.182 5.31e-05 ***
## SecEducRate
                 -0.55480
                              0.23279
                                       -2.383 0.027752 *
## TerEducRate
                  0.20853
                              0.07406
                                        2.816 0.011041 *
## MCapRate
                  0.06432
                              0.02425
                                        2.652 0.015741 *
## Population
                 -0.13621
                              0.04241
                                       -3.212 0.004594 **
```

```
## TradeRatio
                 6.32590
                            3.01839 2.096 0.049728 *
## Unemployment -1.51640
                            0.36173 -4.192 0.000494 ***
## S.P500
                -5.33703
                            1.94165 -2.749 0.012771 *
## Poverty
                 2.25010
                            0.58360
                                      3.856 0.001066 **
## President
                 0.04190
                            0.68055
                                      0.062 0.951545
                            0.97047
## House
                 1.00381
                                      1.034 0.313961
                -0.80791
## Senate
                            0.72876 -1.109 0.281434
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.042 on 19 degrees of freedom
## Multiple R-squared: 0.8577, Adjusted R-squared: 0.7529
## F-statistic: 8.183 on 14 and 19 DF, p-value: 2.511e-05
anova(econ1.lm,econ2.lm)
## Analysis of Variance Table
## Model 1: GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate +
##
       TerEducRate + MCapRate + Population + TradeRatio + Unemployment +
##
       S.P500 + Poverty
## Model 2: GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate +
##
       TerEducRate + MCapRate + Population + TradeRatio + Unemployment +
##
       S.P500 + Poverty + President + House + Senate
##
    Res.Df
              RSS Df Sum of Sq
                                    F Pr(>F)
         22 23.271
## 1
         19 20.611 3
                        2.6594 0.8172 0.5003
summary(econ1.lm)
##
## Call:
## lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
##
       SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
##
       Unemployment + S.P500 + Poverty, data = econ2)
##
## Residuals:
##
      Min
                10 Median
                                3Q
                                       Max
  -2.0329 -0.6260 0.1742 0.5227
                                   1.2992
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
                                      7.227 3.06e-07 ***
## (Intercept) 119.04515
                          16.47302
## Inflation
                -0.63586
                            0.16606 -3.829 0.000914 ***
## IntRate
                 0.67946
                            0.17069
                                      3.981 0.000632 ***
## PrimEducRate -0.80852
                            0.12267 -6.591 1.25e-06 ***
## SecEducRate
                -0.47958
                            0.22061 -2.174 0.040750 *
## TerEducRate
                 0.23841
                            0.06783
                                      3.515 0.001953 **
## MCapRate
                            0.01840
                                      3.286 0.003375 **
                 0.06047
## Population
                -0.15190
                            0.03936 -3.859 0.000850 ***
## TradeRatio
                 6.82267
                            2.26237
                                      3.016 0.006359 **
## Unemployment
                -1.33420
                            0.31036 -4.299 0.000291 ***
## S.P500
                -4.64101
                            1.67156 -2.776 0.011008 *
## Poverty
                 2.06225
                            0.54765 3.766 0.001066 **
## ---
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.028 on 22 degrees of freedom
## Multiple R-squared: 0.8394, Adjusted R-squared: 0.7591
## F-statistic: 10.45 on 11 and 22 DF, p-value: 2.229e-06
```

#### Tables and Graphs for Paper

Model summary:

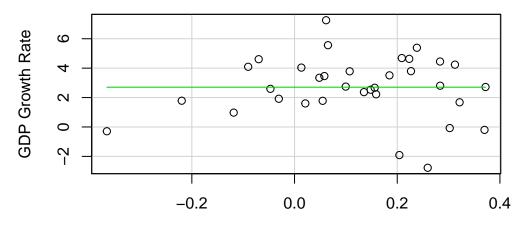
```
library(stargazer)
## Please cite as:
  Hlavac, Marek (2015). stargazer: Well-Formatted Regression and Summary Statistics Tables.
## R package version 5.2. http://CRAN.R-project.org/package=stargazer
stargazer(econ1.lm, single.row=TRUE)
## % Table created by stargazer v.5.2 by Marek Hlavac, Harvard University. E-mail: hlavac at fas.harvar
## % Date and time: Sun, Jun 04, 2017 - 12:41:16
## \begin{table}[!htbp] \centering
     \caption{}
##
     \label{}
## \begin{tabular}{@{\extracolsep{5pt}}lc}
## \\[-1.8ex]\hline
## \hline \\[-1.8ex]
## & \multicolumn{1}{c}{\textit{Dependent variable:}} \\
## \cline{2-2}
## \\[-1.8ex] & GDP\_Rate \\
## \hline \\[-1.8ex]
## Inflation & $-$0.636$^{***}$ (0.166) \\
    IntRate & 0.679\$^{***}$ (0.171) \\
##
##
    PrimEducRate & $-$0.809$^{***}$ (0.123) \\
    SecEducRate & $-$0.480$^{**}$ (0.221) \\
##
    TerEducRate & 0.238$^{***}$ (0.068) \\
##
##
    MCapRate & 0.060\$^{***} (0.018) \\
##
    Population & $-$0.152$^{***}$ (0.039) \\
##
    TradeRatio & 6.823$^{***}$ (2.262) \\
    Unemployment & -\$1.334^{***} (0.310) \\
    S.P500 & $-$4.641$^{**}$ (1.672) \\
##
    Poverty & 2.062$^{***}$ (0.548) \\
    Constant & 119.045$^{***}$ (16.473) \\
##
## \hline \\[-1.8ex]
## Observations & 34 \\
## R$^{2}$ & 0.839 \\
## Adjusted R$^{2}$ & 0.759 \\
## Residual Std. Error & 1.028 (df = 22) \\
## F Statistic & 10.453$^{***}$ (df = 11; 22) \\
## \hline
## \hline \\[-1.8ex]
## \textit{Note:} & \multicolumn{1}{r}{$^{*}$p$<$0.1; $^{**}$p$<$0.05; $^{***}$p$<$0.01} \\
## \end{tabular}
```

# ## \end{table}

Plot of GDP growth vs. S&P 500 returns.

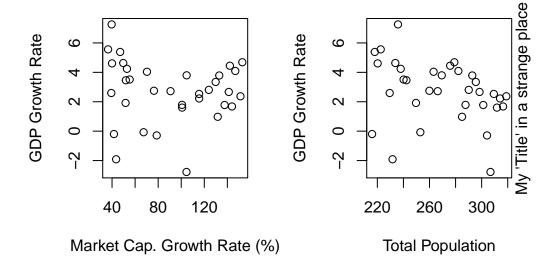
scatterplot(GDP\_Rate ~ S.P500 , data=econ2, boxplots = F, smoother=F, xlab = "S&P 500 Annual Total Retu

# Plot of GDP growth rate against annual returns of the S&P 500



S&P 500 Annual Total Returns

```
par(mfrow =c(1,2))
plot(GDP_Rate ~ MCapRate , data=econ2, xlab = "Market Cap. Growth Rate (%)", ylab = "GDP Growth Rate",
plot(GDP_Rate ~ Population , data=econ2, xlab = "Total Population", ylab = "GDP Growth Rate", cex.main = mtext("My 'Title' in a strange place", side = 4, outer = F)
```



par(mfrow = c(1,2))