

# Final Project Appendix

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```
library(car)
```

```
## Warning: package 'car' was built under R version 3.2.5
```

```
library(lattice)
```

```
## Warning: package 'lattice' was built under R version 3.2.5
```

```
econ <- read.csv("https://raw.githubusercontent.com/griesea/Math245/master/finaldata.csv", header = TRUE)
#Creating ratio of imports to exports
econ$TradeRatio <- econ$Imports/econ$Exports
econ$Population <- econ$Population/1000000
econ
```

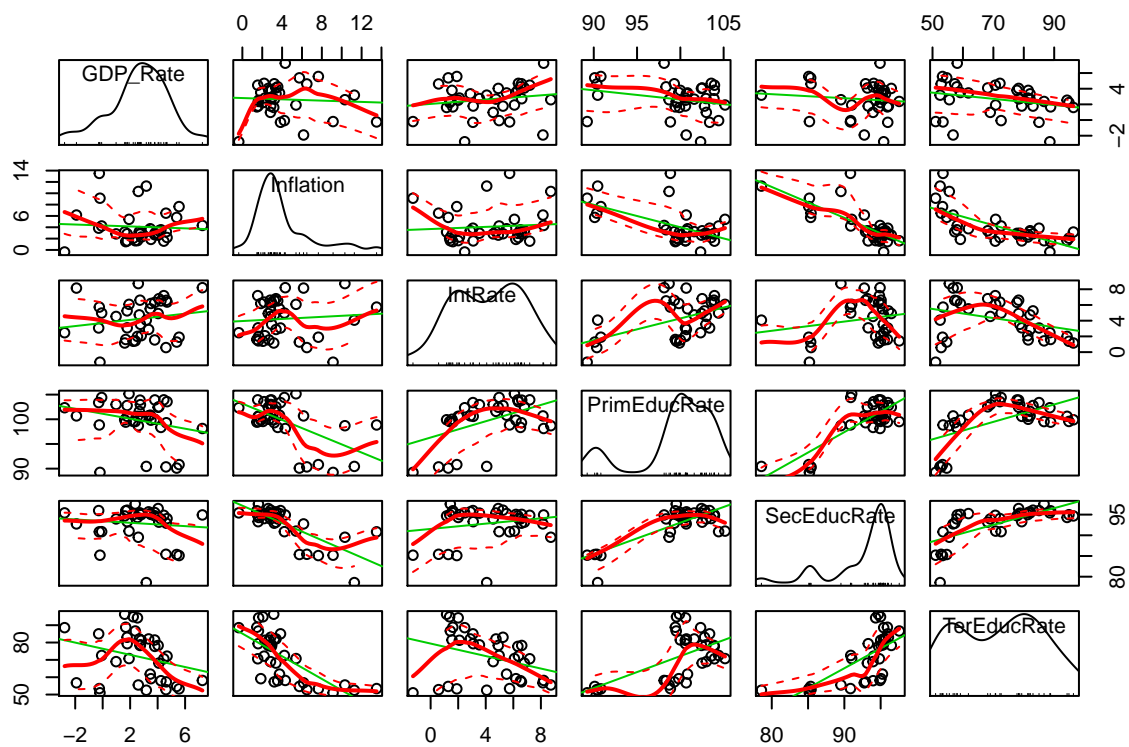
##	YEAR	GDP_Rate	Inflation	IntRate	PrimEducRate	SecEducRate	TerEducRate
## 1	1975	-0.19767854	9.13	-1.28	89.24	85.19	51.01
## 2	1976	5.38609005	5.74	1.28	90.11	85.36	54.67
## 3	1977	4.60859741	6.49	0.58	90.35	85.41	52.84
## 4	1978	5.56168493	7.65	1.90	90.82	85.10	53.29
## 5	1979	3.17569075	11.27	4.07	90.45	78.57	52.50
## 6	1980	-0.24459623	13.51	5.73	98.85	90.84	53.45
## 7	1981	2.59447039	10.32	8.72	98.70	89.53	55.52
## 8	1982	-1.91089107	6.16	8.15	100.68	92.75	56.77
## 9	1983	4.63245718	3.21	6.59	99.30	92.92	57.32
## 10	1984	7.25908696	4.32	8.20	98.13	94.92	58.13
## 11	1985	4.23873752	3.56	6.52	98.35	93.96	57.99
## 12	1986	3.51161450	1.86	6.19	99.82	95.05	59.16
## 13	1987	3.46174769	3.74	5.51	100.83	95.25	61.80
## 14	1990	1.91937030	5.40	6.08	105.16	91.01	70.75
## 15	1991	-0.07408453	4.23	4.97	104.42	90.88	72.55
## 16	1993	2.74585672	2.95	3.54	102.96	95.36	78.68
## 17	1994	4.03764342	2.61	4.91	102.44	96.47	78.31
## 18	1995	2.71897579	2.81	6.61	103.88	95.06	78.31
## 19	1996	3.79588123	2.93	6.33	103.92	94.72	77.78
## 20	1998	4.44991096	1.55	7.19	103.94	94.83	70.63
## 21	1999	4.68519961	2.19	6.37	103.57	94.04	72.16
## 22	2000	4.09217645	3.38	6.80	102.52	93.17	68.14
## 23	2001	0.97598183	2.83	4.54	103.00	93.95	68.98
## 24	2002	1.78612769	1.59	3.09	100.59	93.29	79.33
## 25	2003	2.80677596	2.27	2.09	100.58	95.01	81.31
## 26	2004	3.78574285	2.68	1.55	100.00	95.50	81.46
## 27	2005	3.34521606	3.39	2.88	100.70	95.67	82.08
## 28	2006	2.66662583	3.23	4.74	101.60	95.40	82.05
## 29	2007	1.77857024	2.85	5.25	102.79	96.07	83.03
## 30	2008	-0.29162146	3.84	3.07	103.38	95.98	85.01
## 31	2009	-2.77552957	-0.36	2.47	102.34	95.42	88.58
## 32	2010	2.53192062	1.64	2.00	100.60	94.31	94.23
## 33	2011	1.60145467	3.16	1.16	99.90	94.68	96.32
## 34	2012	2.22403085	2.07	1.38	99.56	94.98	94.84

##	35	2013	1.67733153	1.46	1.61	99.42	95.93	88.81
##	36	2014	2.37045767	1.62	1.43	99.53	97.56	86.66
##		MCapRate	Population	Unemployment		Imports	Exports	S.P500
##	1	41.67153	215.9730	8.46	1.22729e+11	1.38707e+11	0.3700	
##	2	47.03377	218.0350	7.70	1.51146e+11	1.49515e+11	0.2383	
##	3	39.99317	220.2390	7.06	1.82443e+11	1.59350e+11	-0.0698	
##	4	36.57692	222.5850	6.07	2.12251e+11	1.86885e+11	0.0651	
##	5	37.74836	225.0550	5.85	2.52675e+11	2.30129e+11	0.1852	
##	6	47.50377	227.2250	7.14	2.93829e+11	2.80773e+11	0.3174	
##	7	39.35154	229.4660	7.61	3.17759e+11	3.05239e+11	-0.0470	
##	8	43.55364	231.6640	9.69	3.03183e+11	2.83209e+11	0.2042	
##	9	49.72420	233.7920	9.61	3.28638e+11	2.76996e+11	0.2234	
##	10	39.65062	235.8250	7.52	4.05107e+11	3.02383e+11	0.0615	
##	11	52.93303	237.9240	7.20	4.17228e+11	3.03209e+11	0.3124	
##	12	55.29099	240.1330	6.99	4.52867e+11	3.21000e+11	0.1849	
##	13	51.98470	242.2890	6.19	5.08714e+11	3.63944e+11	0.0581	
##	14	51.73346	249.6230	5.60	6.29729e+11	5.51874e+11	-0.0306	
##	15	67.37231	252.9810	6.83	6.23547e+11	5.94932e+11	0.3023	
##	16	76.33806	259.9190	6.92	7.19974e+11	6.54800e+11	0.0997	
##	17	70.29569	263.1260	6.10	8.13425e+11	7.20939e+11	0.0133	
##	18	90.70944	266.2780	5.60	9.02571e+11	8.12813e+11	0.3720	
##	19	104.69490	269.3940	5.40	9.63966e+11	8.67590e+11	0.2268	
##	20	142.17561	275.8540	4.51	1.11569e+12	9.52981e+11	0.2834	
##	21	152.96514	279.0400	4.22	1.24861e+12	9.91980e+11	0.2089	
##	22	146.89427	282.1624	3.99	1.47263e+12	1.09684e+12	-0.0903	
##	23	131.65033	284.9690	4.73	1.39540e+12	1.02671e+12	-0.1185	
##	24	100.70067	287.6252	5.78	1.42897e+12	1.00251e+12	-0.2197	
##	25	123.93949	290.1079	5.99	1.54393e+12	1.04028e+12	0.2836	
##	26	132.98429	292.8053	5.53	1.80068e+12	1.18151e+12	0.1074	
##	27	129.83978	295.5166	5.08	2.03009e+12	1.30890e+12	0.0483	
##	28	141.23218	298.3799	4.62	2.24726e+12	1.47632e+12	0.1561	
##	29	137.60728	301.2312	4.62	2.38317e+12	1.66463e+12	0.0548	
##	30	78.74589	304.0940	5.78	2.56501e+12	1.84194e+12	-0.3655	
##	31	104.56730	306.7715	9.25	1.98318e+12	1.58774e+12	0.2594	
##	32	115.49734	309.3469	9.63	2.36499e+12	1.85234e+12	0.1482	
##	33	100.79122	311.7189	8.95	2.68637e+12	2.10637e+12	0.0210	
##	34	115.55579	314.1026	8.07	2.76384e+12	2.19818e+12	0.1589	
##	35	143.99442	316.4274	7.38	2.76861e+12	2.27661e+12	0.3215	
##	36	151.38523	318.9074	6.17	2.88407e+12	2.37528e+12	0.1352	
##		Poverty	President	Senate	House	TradeRatio		
##	1	12.3	0	1	1	0.8848075		
##	2	11.8	0	1	1	1.0109086		
##	3	11.6	1	1	1	1.1449200		
##	4	11.4	1	1	1	1.1357305		
##	5	11.7	1	1	1	1.0979711		
##	6	13.0	1	1	1	1.0465002		
##	7	14.0	0	0	1	1.0410170		
##	8	15.0	0	0	1	1.0705274		
##	9	15.2	0	0	1	1.1864359		
##	10	14.4	0	0	1	1.3397149		
##	11	14.0	0	0	1	1.3760409		
##	12	13.6	0	0	1	1.4108006		
##	13	13.4	0	1	1	1.3977810		
##	14	13.5	0	1	1	1.1410739		

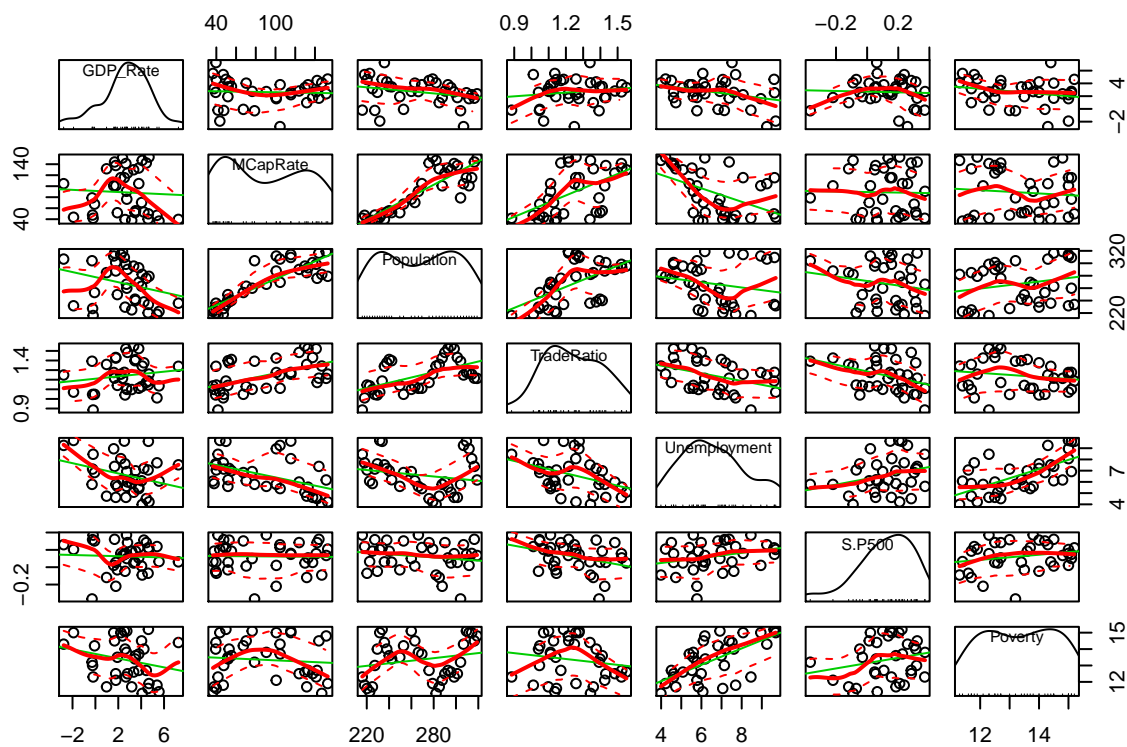
## 15	14.2	0	1	1	1.0480979
## 16	15.1	1	1	1	1.0995327
## 17	14.5	1	1	1	1.1282855
## 18	13.8	1	0	1	1.1104288
## 19	13.7	1	0	0	1.1110847
## 20	12.7	1	0	0	1.1707369
## 21	11.9	1	0	0	1.2587048
## 22	11.3	1	0	0	1.3426115
## 23	11.7	0	1	0	1.3590985
## 24	12.1	0	1	0	1.4253923
## 25	12.5	0	0	0	1.4841485
## 26	12.7	0	0	0	1.5240497
## 27	12.6	0	0	0	1.5509894
## 28	12.3	0	0	0	1.5222039
## 29	12.5	0	1	0	1.4316515
## 30	13.2	0	1	1	1.3925589
## 31	14.3	1	1	1	1.2490584
## 32	15.1	1	1	1	1.2767580
## 33	15.0	1	1	1	1.2753552
## 34	15.0	1	1	0	1.2573311
## 35	14.5	1	1	0	1.2161108
## 36	14.8	1	1	0	1.2142021

Performing EDA:

```
scatterplotMatrix(~ GDP_Rate + Inflation + IntRate + PrimEducRate +SecEducRate + TerEducRate, data = ec
```



```
scatterplotMatrix(~ GDP_Rate + MCapRate+ Population + TradeRatio + Unemployment + S.P500 + Poverty, data = data)
```

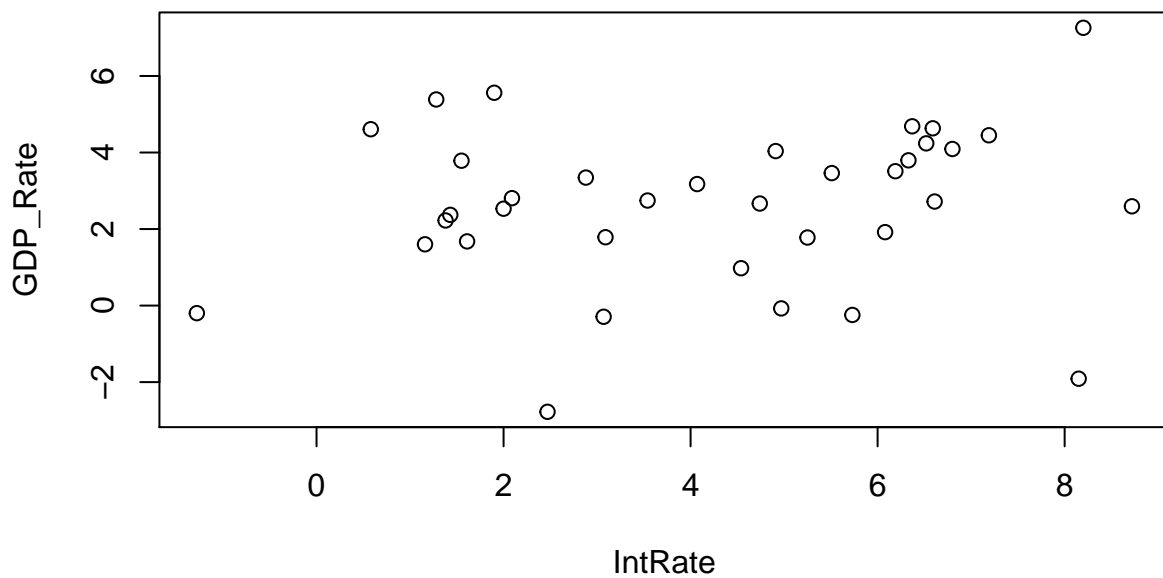


We see that some of our variables (specifically MCapRate and Population) appear to be highly colinear (we will deal with this later).

## Transformations

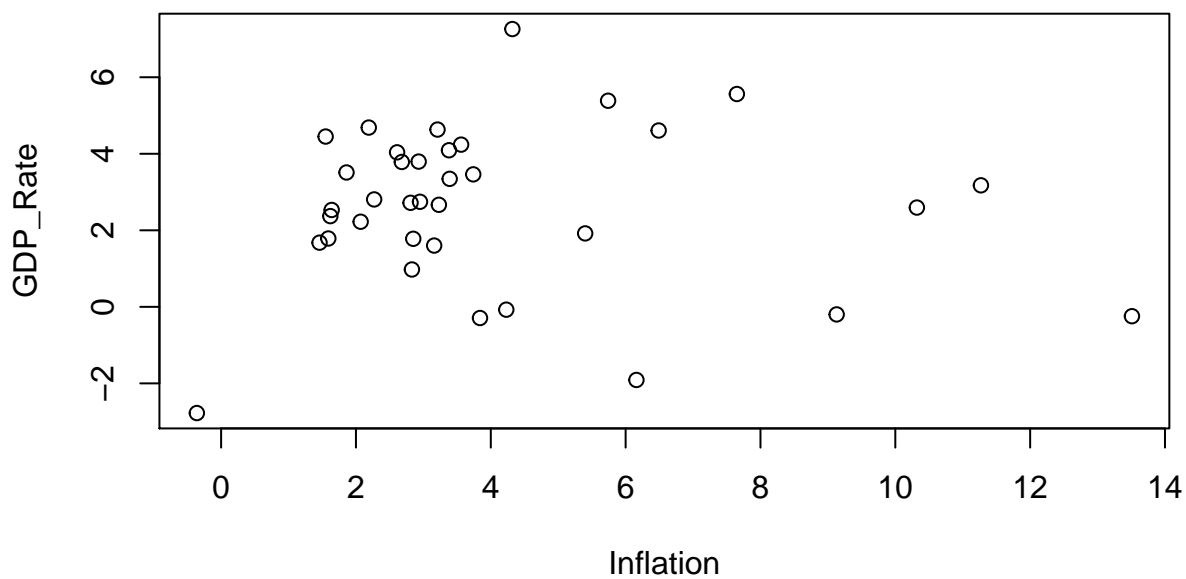
We don't appear to need a transformation to IntRate (no convincing trend appears).

```
plot(GDP_Rate ~ (IntRate) , data = econ)
```



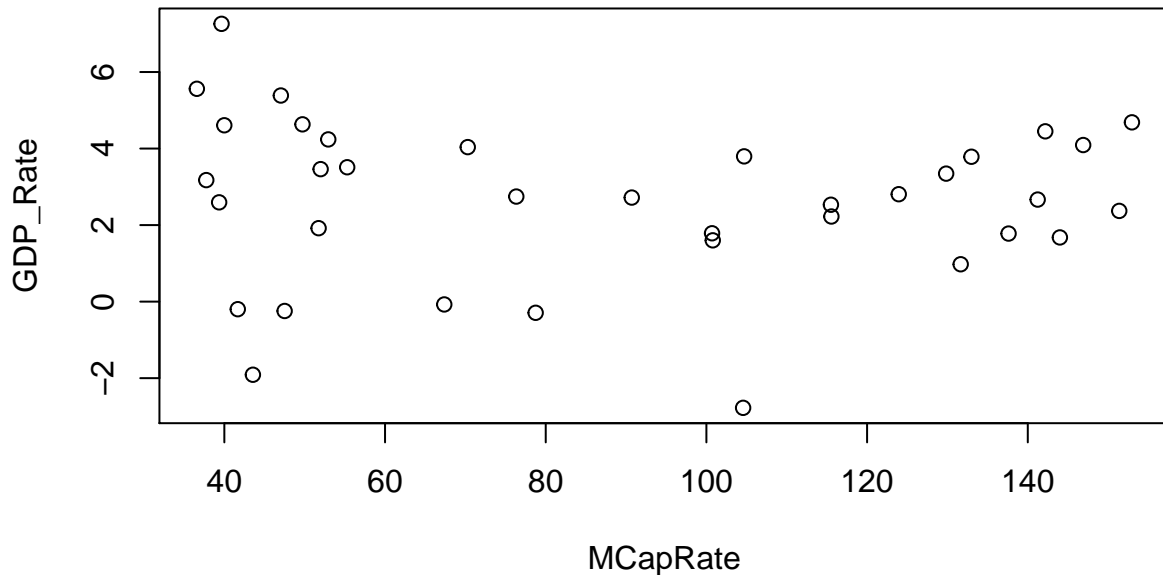
Inflation looks like it needs a transformation. However, economic theory suggests a peak benefit inflation rate (with too low or too high inflation rates being detrimental to the economy). As such, we will try to make inflation a quadratic term in the model, and so do not transform it here.

```
plot(GDP_Rate ~ (Inflation) , data = econ)
```



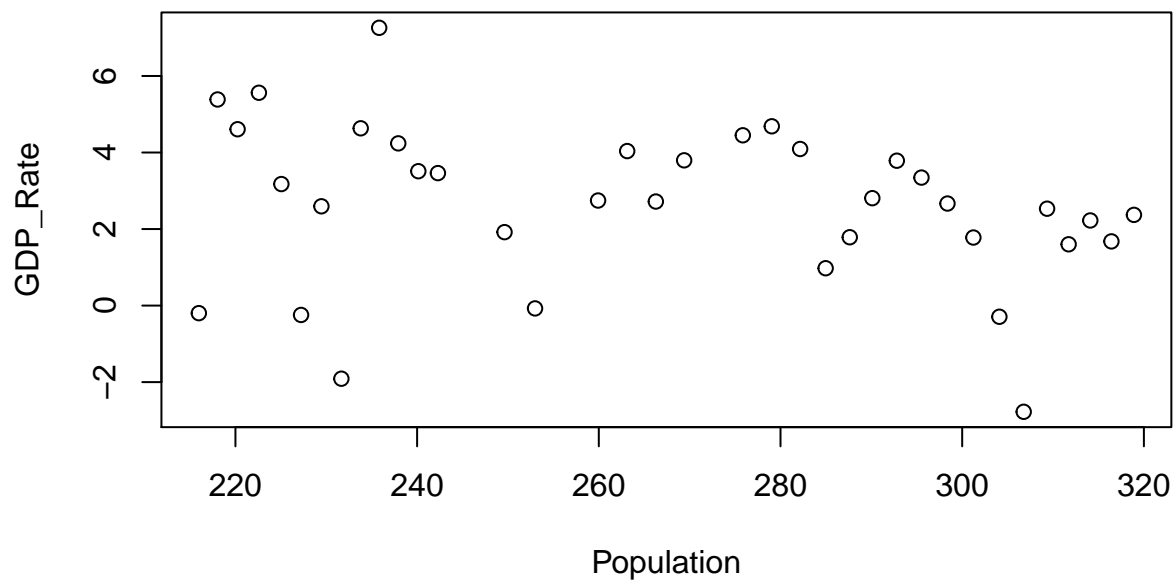
From the matrix above, it appears that MCapRate might both be curved. We examine this trend further, below. There looks to be a parabolic relationship between GDP\_Rate and MCapRate. To determine if this is the correct transformation, we must fit a linear model and look at the residuals. *To be done after other transformations.*

```
plot(GDP_Rate ~ MCapRate , data = econ)
```



Next, we check if there is a transformation needed for the population explanatory variable. There does not seem any clear indication that there needs to be a transformation based on the below plot.

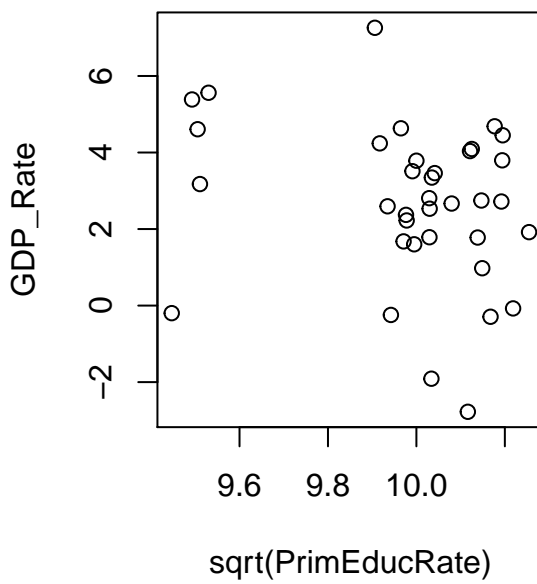
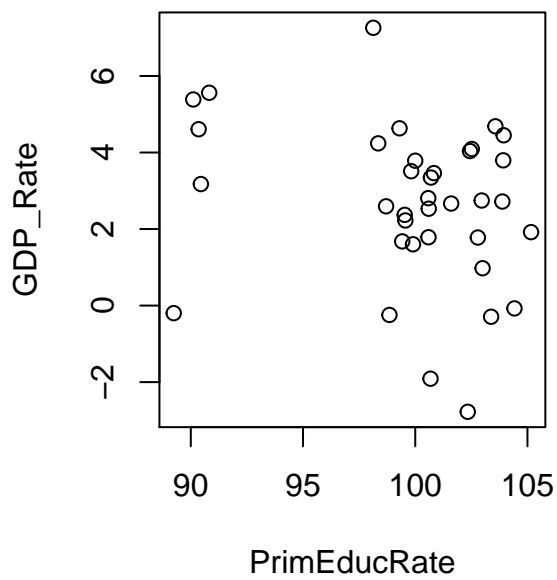
```
plot(GDP_Rate ~ Population , data = econ)
```



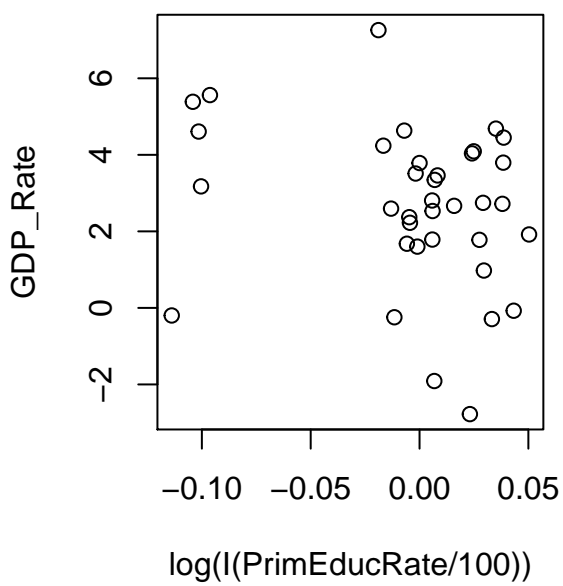
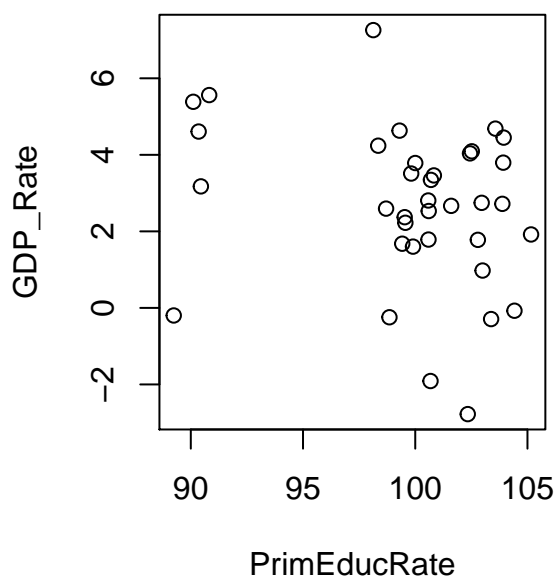
Next, we check if there is a transformation needed for the primary education explanatory variable. There is some clustering we would like to get rid of. However, no transformation seems to make the relationship between GDP\_Rate and PrimEducRate any better.

```
par(mfrow=c(1,2))
plot(GDP_Rate ~ PrimEducRate , data = econ)
plot(GDP_Rate ~ sqrt(PrimEducRate) , data = econ)
```





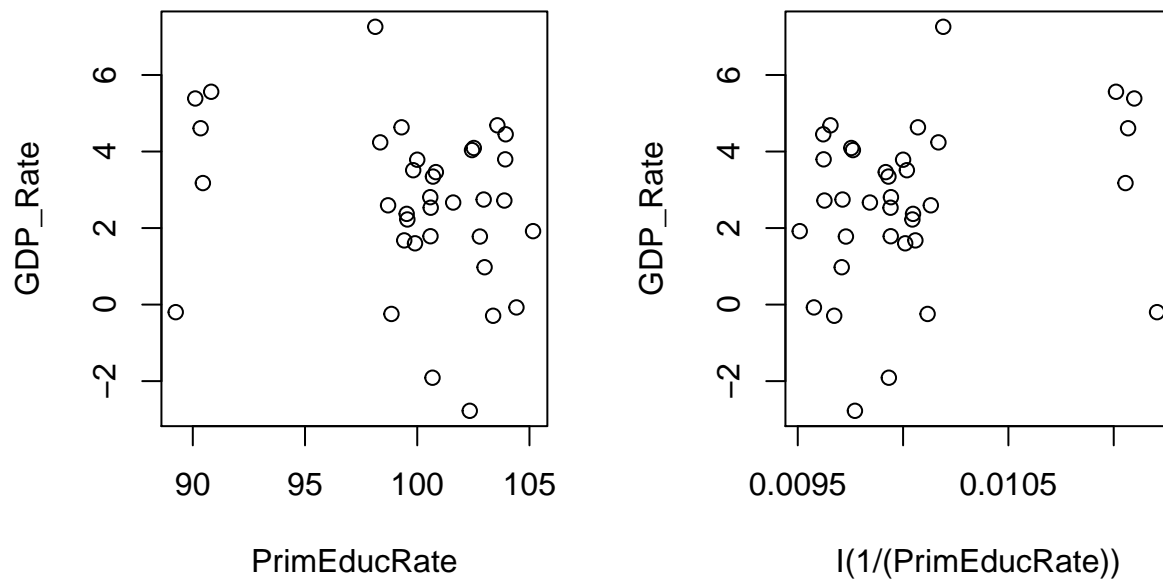
```
par(mfrow=c(1,1))
par(mfrow=c(1,2))
plot(GDP_Rate ~ PrimEducRate , data = econ)
plot(GDP_Rate ~ log(I(PrimEducRate/100)) , data = econ)
```



```

par(mfrow=c(1,1))
par(mfrow=c(1,2))
plot(GDP_Rate ~ PrimEducRate , data = econ)
plot(GDP_Rate ~ I(1/(PrimEducRate)) , data = econ)

```



```

par(mfrow=c(1,1))

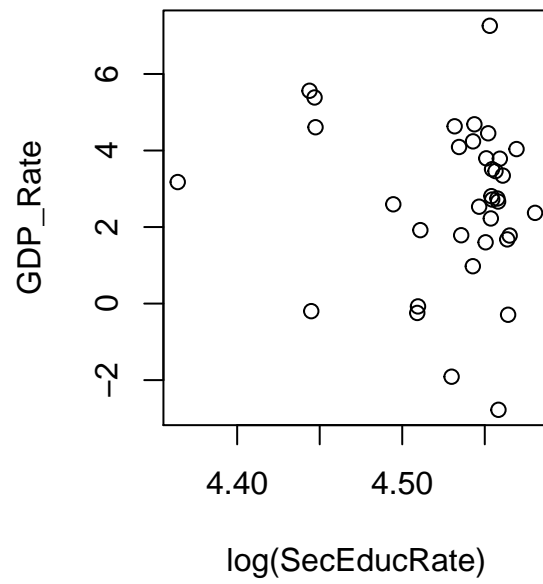
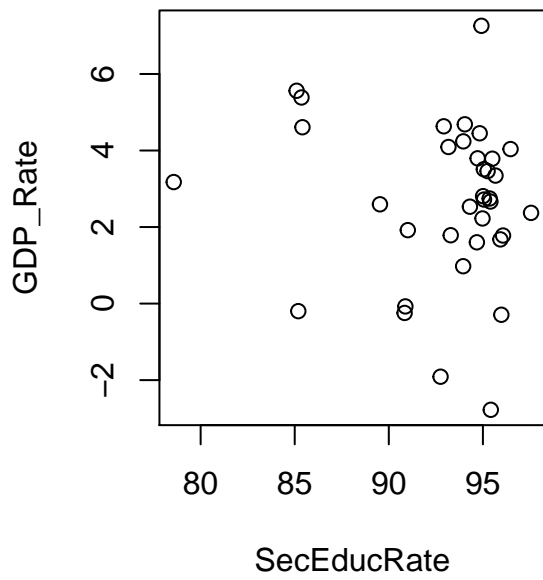
```

Next, we check if there is a transformation needed for the secondary education explanatory variable. There is some clustering we would like to get rid of. However, no transformations seem to improve the fit.

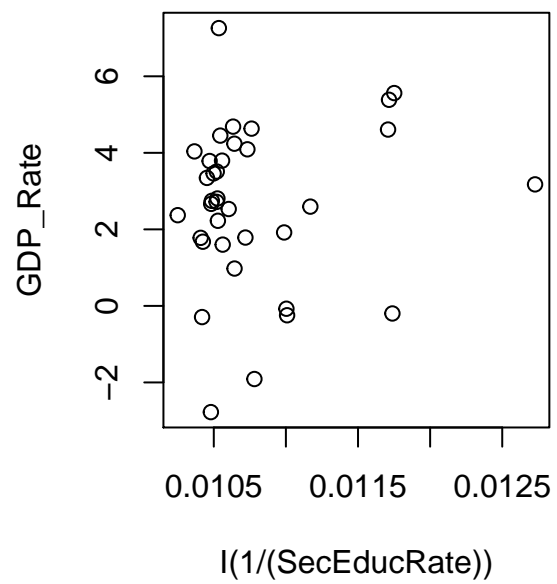
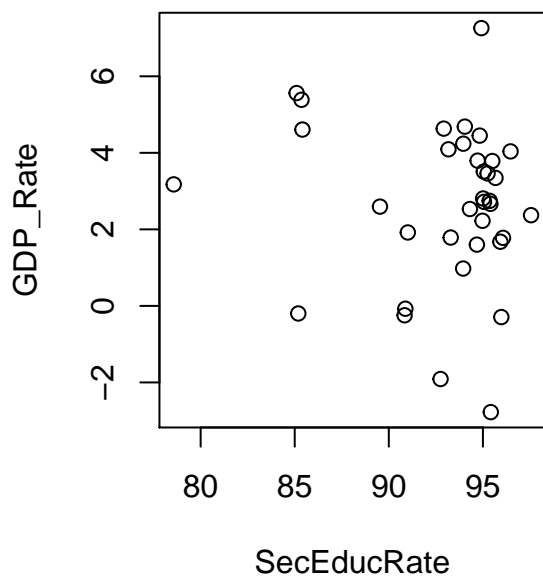
```

par(mfrow=c(1,2))
plot(GDP_Rate ~ SecEducRate , data = econ)
plot(GDP_Rate ~ log(SecEducRate) , data = econ)

```



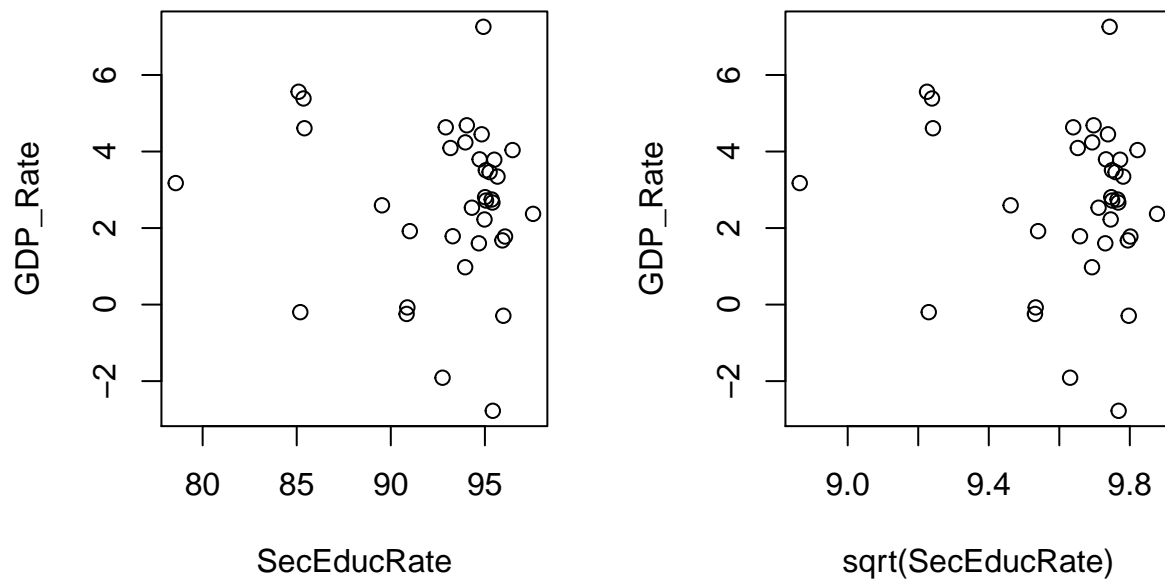
```
par(mfrow=c(1,1))
par(mfrow=c(1,2))
plot(GDP_Rate ~ SecEducRate , data = econ)
plot(GDP_Rate ~ I(1/(SecEducRate)) , data = econ)
```



```

par(mfrow=c(1,1))
par(mfrow=c(1,2))
plot(GDP_Rate ~ SecEducRate , data = econ)
plot(GDP_Rate ~ sqrt(SecEducRate) , data = econ)

```



```

par(mfrow=c(1,1))

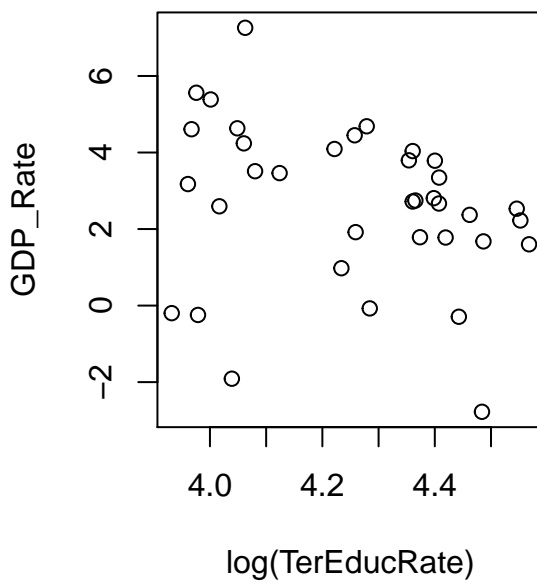
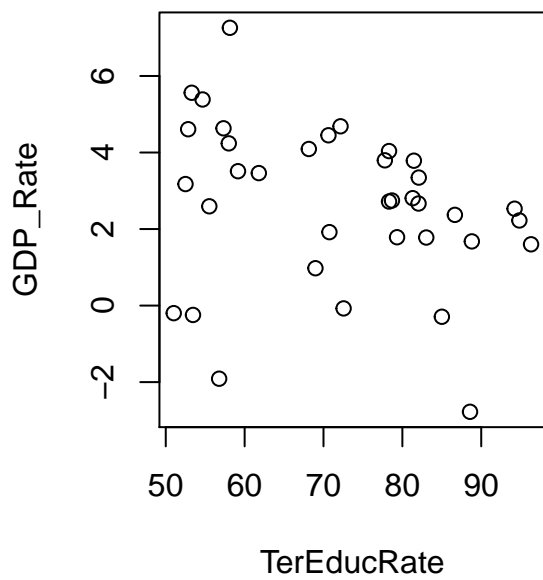
```

Finally, we check if there is a transformation needed for the tertiary education explanatory variable. There is some clustering we would like to get rid of. However, no transformations seem to improve the fit.

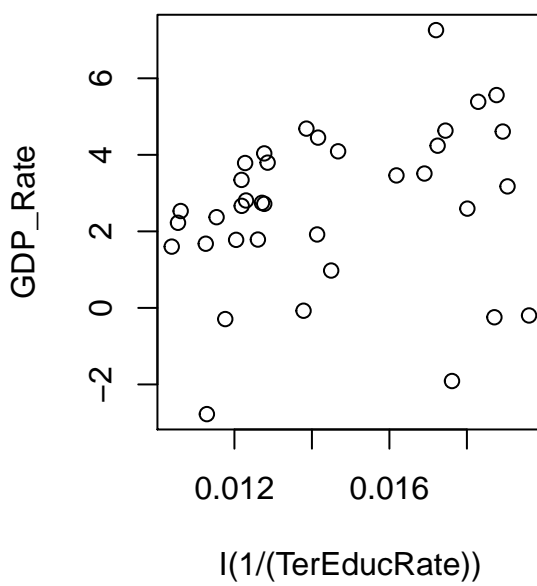
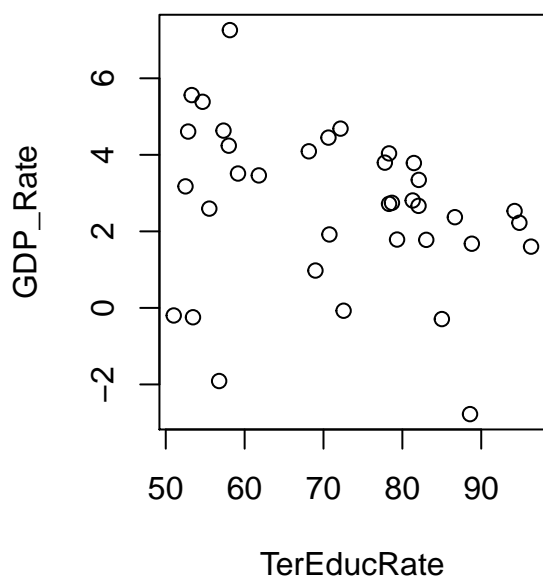
```

par(mfrow=c(1,2))
plot(GDP_Rate ~ TerEducRate , data = econ)
plot(GDP_Rate ~ log(TerEducRate) , data = econ)

```



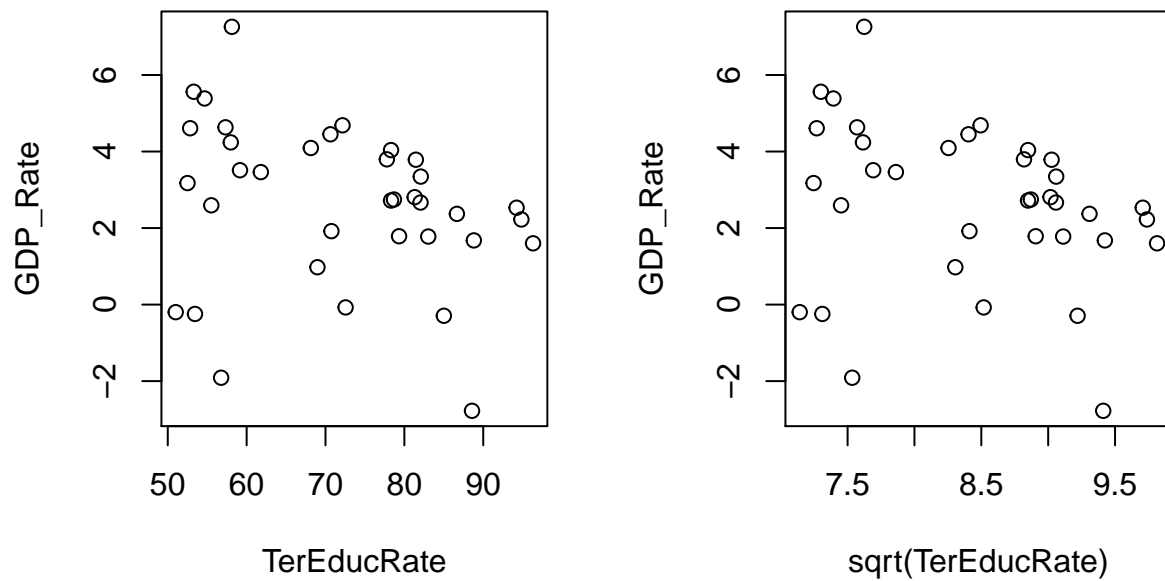
```
par(mfrow=c(1,1))
par(mfrow=c(1,2))
plot(GDP_Rate ~ TerEducRate , data = econ)
plot(GDP_Rate ~ I(1/(TerEducRate)) , data = econ)
```



```

par(mfrow=c(1,1))
par(mfrow=c(1,2))
plot(GDP_Rate ~ TerEducRate , data = econ)
plot(GDP_Rate ~ sqrt(TerEducRate) , data = econ)

```



```

par(mfrow=c(1,1))

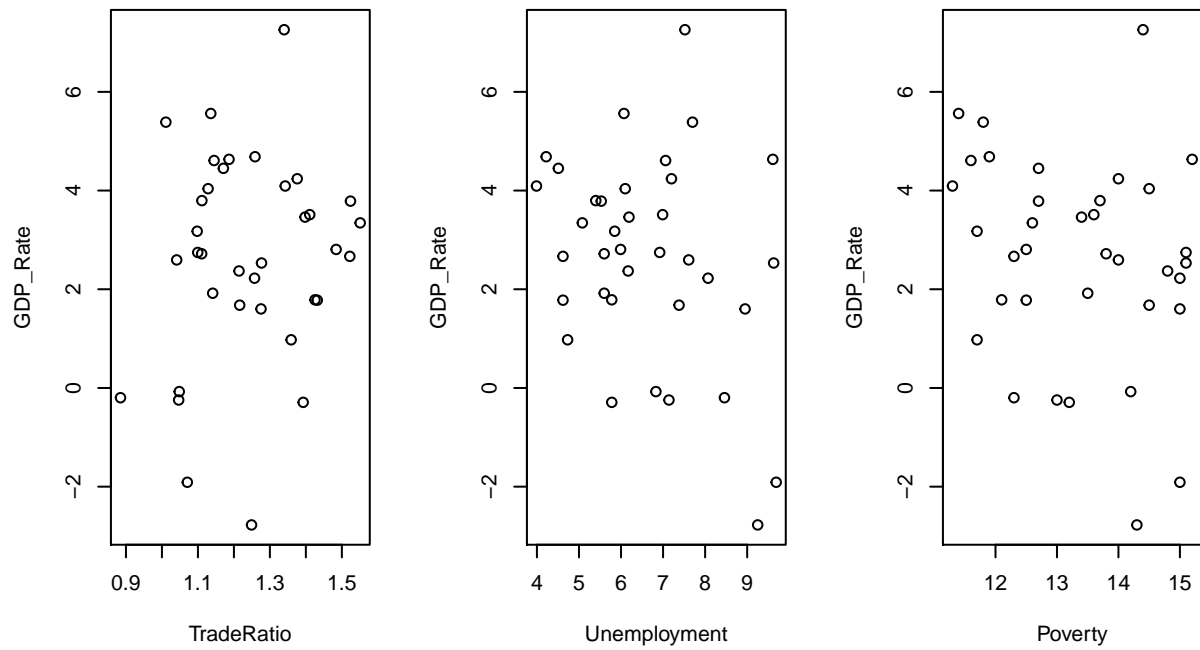
```

Trade ratio, poverty rate, and unemployment show no convincing needs for transformations:

```

par(mfrow=c(1,3))
plot(GDP_Rate ~ (TradeRatio), data=econ)
plot(GDP_Rate ~ Unemployment, data=econ)
plot(GDP_Rate ~ Poverty, data=econ)

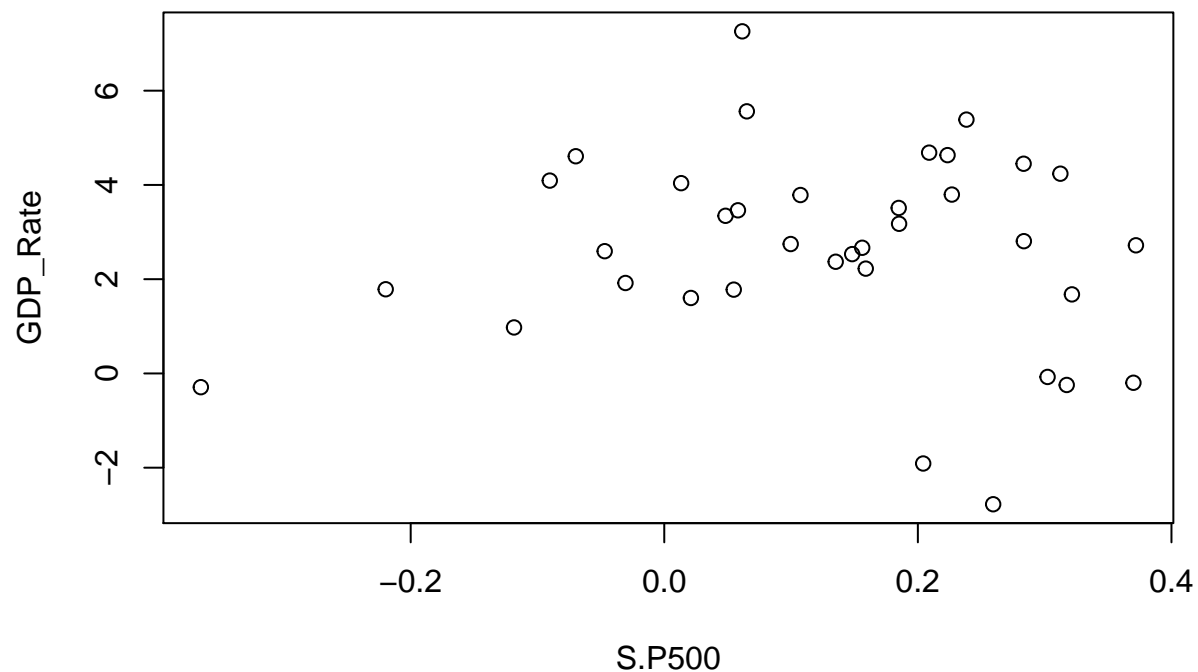
```



```
par(mfrow=c(1,1))
```

We also see no reason to transform S.P500.

```
plot(GDP_Rate ~ (S.P500) , data = econ)
```



### Preliminary Linear Model

Fitting only linear terms:

```
test.lm1 <- lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRate + MCapRate + Population + TradeRatio + Unemployment + S.P500 + Poverty + President + Senate + House, data = econ)
summary(test.lm1)
```

```
##
## Call:
## lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
##      SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
##      Unemployment + S.P500 + Poverty + President + Senate + House,
##      data = econ)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
	-2.81006	-0.68182	0.05817	0.64397	1.66891

```
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	73.117083	13.163184	5.555	1.64e-05	***
Inflation	-0.280117	0.123364	-2.271	0.033813	*
IntRate	0.445717	0.230647	1.932	0.066903	.
PrimEducRate	-0.652083	0.157576	-4.138	0.000467	***
SecEducRate	0.005268	0.147668	0.036	0.971878	
TerEducRate	0.227245	0.081951	2.773	0.011400	*



```
## MCapRate      0.063367    0.027339    2.318 0.030635 *
## Population    -0.170352    0.045579   -3.737 0.001215 **
## TradeRatio     5.809755    2.788127    2.084 0.049586 *
## Unemployment  -0.795322    0.305231   -2.606 0.016511 *
## S.P500        -4.354906    1.758571   -2.476 0.021864 *
## Poverty        1.128384    0.505880    2.231 0.036750 *
## President      0.736001    0.585801    1.256 0.222768
## Senate        -0.177714    0.725315   -0.245 0.808822
## House         0.292669    1.063025    0.275 0.785763
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.195 on 21 degrees of freedom
## Multiple R-squared:  0.8047, Adjusted R-squared:  0.6746
## F-statistic: 6.182 on 14 and 21 DF,  p-value: 0.0001102
```

## Checking Residual Plots

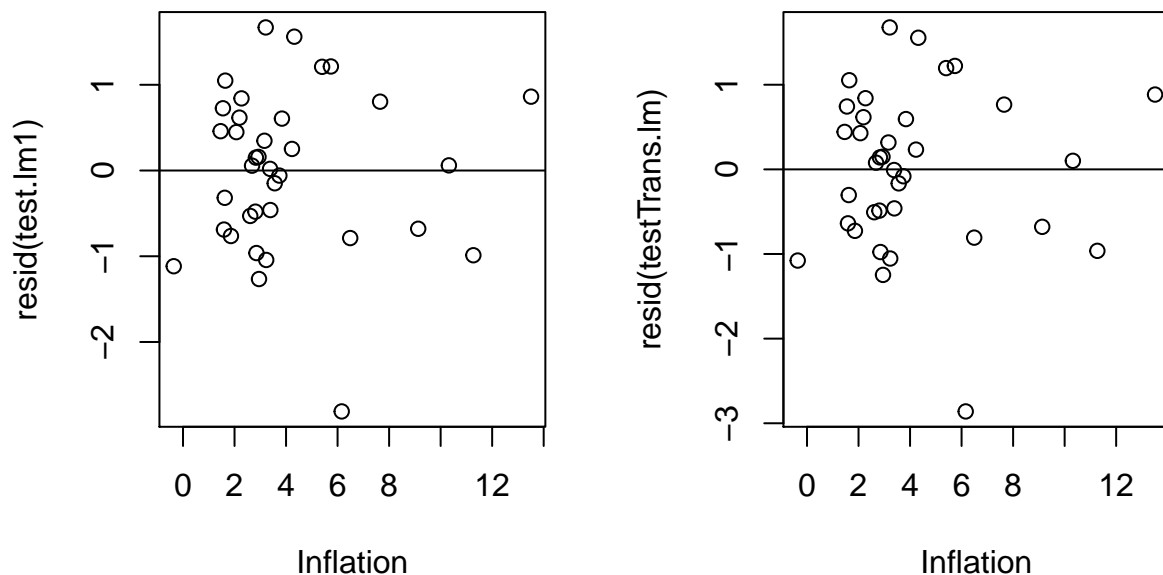
First, we check the residual plot of our linear model against inflation. From before, using economic theory we would expect there to be a parabolic relationship. However, transforming inflation does not improve the fit and we proceed with the original model.

```
testTrans.lm <- lm(GDP_Rate ~ I(Inflation^2)+Inflation + IntRate + PrimEducRate +SecEducRate + TerEducRate, data = econ)
summary(testTrans.lm)
```

```
##
## Call:
## lm(formula = GDP_Rate ~ I(Inflation^2) + Inflation + IntRate +
##     PrimEducRate + SecEducRate + TerEducRate + MCapRate + Population +
##     TradeRatio + Unemployment + S.P500 + Poverty + President +
##     Senate + House, data = econ)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.8602 -0.6481  0.0901  0.6480  1.6771
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   71.986565   16.816573   4.281 0.000365 ***
## I(Inflation^2) -0.003476    0.030896  -0.113 0.911547
## Inflation     -0.229912    0.463805  -0.496 0.625504
## IntRate        0.447178    0.236624   1.890 0.073365 .
## PrimEducRate  -0.646941    0.167763  -3.856 0.000984 ***
## SecEducRate    0.008270    0.153602   0.054 0.957595
## TerEducRate    0.224264    0.088030   2.548 0.019176 *
## MCapRate       0.062486    0.029082   2.149 0.044092 *
## Population    -0.168294    0.050146  -3.356 0.003144 **
## TradeRatio     5.874948    2.914272   2.016 0.057441 .
## Unemployment  -0.781539    0.335815  -2.327 0.030565 *
## S.P500        -4.219904    2.164512  -1.950 0.065387 .
## Poverty        1.109681    0.544223   2.039 0.054884 .
## President      0.779213    0.712476   1.094 0.287099
## Senate        -0.154616    0.770839  -0.201 0.843052
## House         0.268278    1.110304   0.242 0.811530
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.224 on 20 degrees of freedom
## Multiple R-squared:  0.8049, Adjusted R-squared:  0.6585
## F-statistic: 5.499 on 15 and 20 DF,  p-value: 0.0002859

par(mfrow=c(1,2))
plot(resid(test.lm1) ~ Inflation, data = econ)
abline(h=0)
plot(resid(testTrans.lm) ~ Inflation, data = econ)
abline(h=0)
```



```
par(mfrow=c(1,1))
```

Next, we check the residual plot of our linear model against MCapRate. We noticed a parabolic relationship previously and check for this. However, on the basis of the residual plot this seems to not improve the model significantly. As such, we check for a log transformation instead. However, this does not improve the fit and we move forward with no transformation.

```
testTrans.lm <- lm(GDP_Rate ~ Inflation + IntrRate + PrimEducRate + SecEducRate + TerEducRate + log(MCapRate))
testTrans1.lm <- lm(GDP_Rate ~ Inflation + IntrRate + PrimEducRate + SecEducRate + TerEducRate + I(MCapRate))
summary(testTrans.lm)
```

```
##
## Call:
## lm(formula = GDP_Rate ~ Inflation + IntrRate + PrimEducRate +
##     SecEducRate + TerEducRate + log(MCapRate) + Population +
##     TradeRatio + Unemployment + S.P500 + Poverty + President +
##     Senate + House, data = econ)
##
```

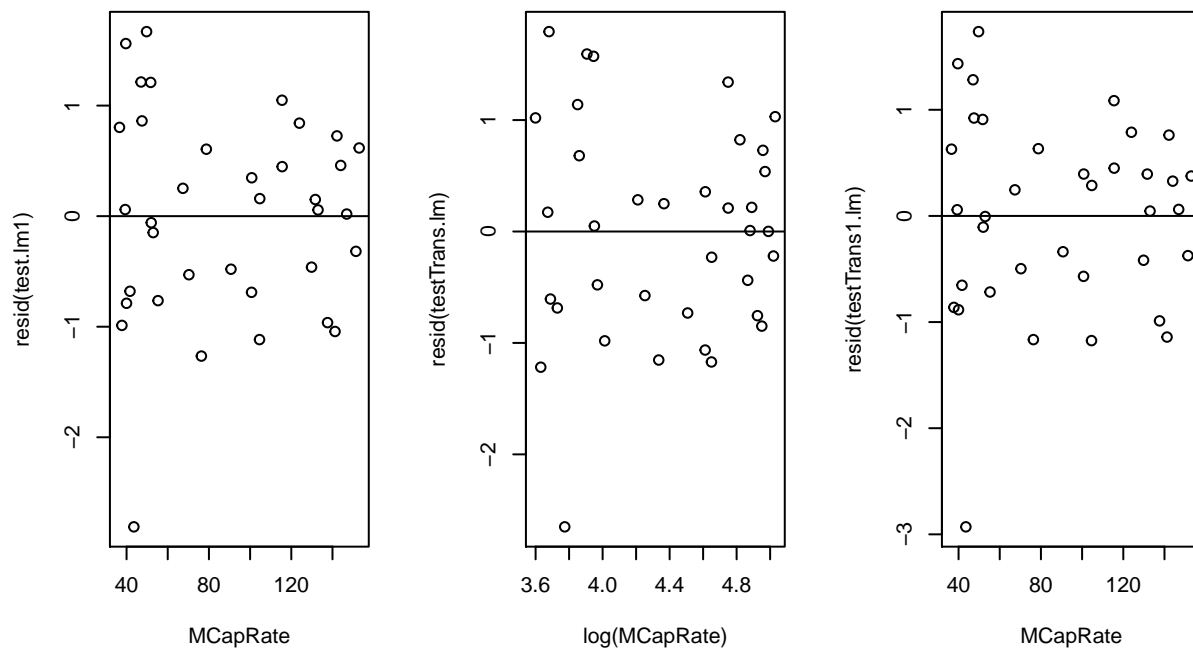
```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.65056 -0.69738  0.02891  0.69223  1.79384
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   61.827216  12.930488   4.782 0.000101 ***
## Inflation     -0.275910   0.132226  -2.087 0.049297 *
## IntRate        0.481393   0.244771   1.967 0.062572 .
## PrimEducRate  -0.711043   0.175585  -4.050 0.000577 ***
## SecEducRate   -0.005494   0.155495  -0.035 0.972150
## TerEducRate    0.178853   0.080301   2.227 0.036998 *
## log(MCapRate)  4.734898   2.618040   1.809 0.084861 .
## Population    -0.143914   0.042892  -3.355 0.002996 **
## TradeRatio     5.257694   2.880437   1.825 0.082212 .
## Unemployment  -0.875314   0.315684  -2.773 0.011404 *
## S.P500        -4.127519   1.872064  -2.205 0.038756 *
## Poverty        1.184997   0.556851   2.128 0.045342 *
## President      0.795511   0.611622   1.301 0.207468
## Senate        -0.303267   0.752074  -0.403 0.690848
## House          0.023939   1.101748   0.022 0.982870
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.246 on 21 degrees of freedom
## Multiple R-squared:  0.7878, Adjusted R-squared:  0.6464
## F-statistic:  5.57 on 14 and 21 DF, p-value: 0.0002355
```

```
summary(testTrans1.lm)
```

```
##
## Call:
## lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
##      SecEducRate + TerEducRate + I(MCapRate^2) + MCapRate + Population +
##      TradeRatio + Unemployment + S.P500 + Poverty + President +
##      Senate + House, data = econ)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.92934 -0.59043  0.05969  0.63005  1.73635
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   70.8068033  14.0415376   5.043 6.23e-05 ***
## Inflation     -0.2946268   0.1282621  -2.297  0.03254 *
## IntRate        0.4217079   0.2386864   1.767  0.09252 .
## PrimEducRate  -0.6067904   0.1804723  -3.362  0.00310 **
## SecEducRate    0.0163646   0.1515678   0.108  0.91510
## TerEducRate    0.2532434   0.0959978   2.638  0.01577 *
## I(MCapRate^2)  0.0002264   0.0004147   0.546  0.59111
## MCapRate       0.0127096   0.0968579   0.131  0.89691
## Population    -0.1739547   0.0468277  -3.715  0.00137 **
## TradeRatio     5.8171326   2.8359552   2.051  0.05358 .
## Unemployment  -0.7319452   0.3314530  -2.208  0.03906 *
## S.P500        -4.1746603   1.8189254  -2.295  0.03268 *
```

```
## Poverty      0.9784930  0.5832055   1.678  0.10895
## President    0.7868336  0.6030731   1.305  0.20681
## Senate      -0.2254454  0.7429103  -0.303  0.76467
## House        0.1430462  1.1154335   0.128  0.89924
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.216 on 20 degrees of freedom
## Multiple R-squared:  0.8076, Adjusted R-squared:  0.6633
## F-statistic: 5.597 on 15 and 20 DF,  p-value: 0.000253
```

```
par(mfrow=c(1,3))
plot(resid(test.lm1) ~ MCapRate, data = econ)
abline(h=0)
plot(resid(testTrans.lm) ~ log(MCapRate), data = econ)
abline(h=0)
plot(resid(testTrans1.lm) ~ MCapRate, data = econ)
abline(h=0)
```

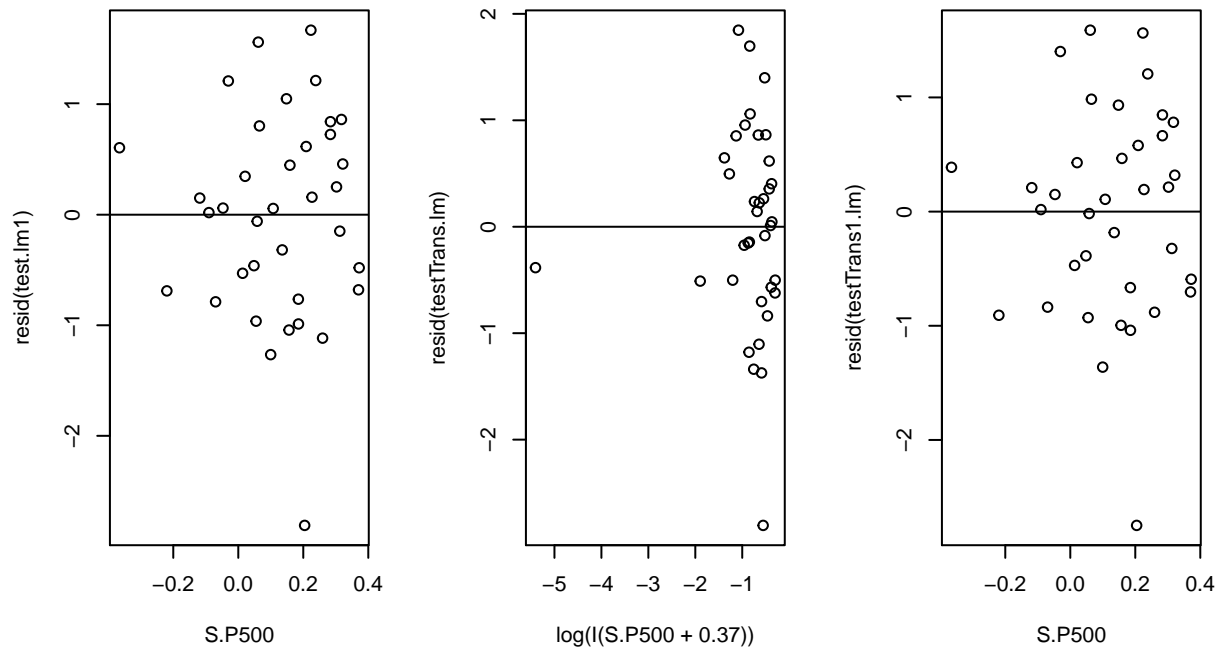


```
par(mfrow=c(1,1))
```

Next, we check the residual plot of our linear model against S.P500. We noted a potential parabolic relationship previously, as such we check the residual plot. On the basis of the residual plot, a parabolic transformation seems inappropriate. However, there may be a need for a log transformation. This is wildly inappropriate and we move forward with no transformation.

```
testTrans.lm <- lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRate + MCapRate +
testTrans1.lm <- lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRate + MCapRate +
par(mfrow=c(1,3))
plot(resid(test.lm1) ~ S.P500, data = econ)
abline(h=0)
```

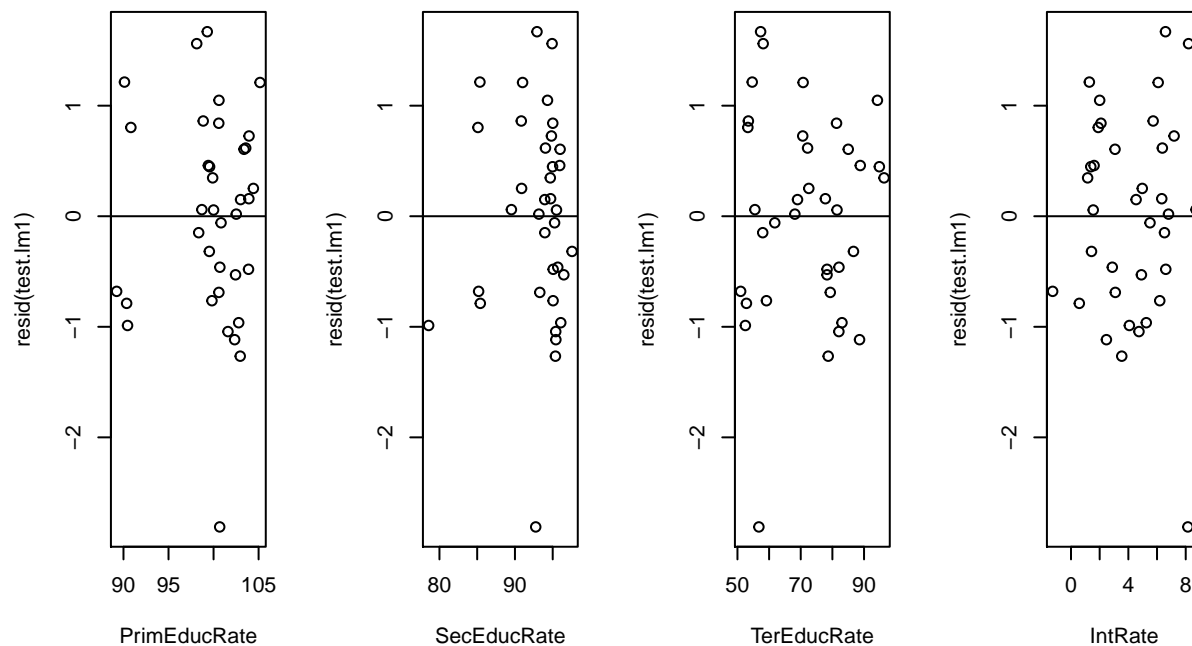
```
plot(resid(testTrans.lm) ~ log(I(S.P500+.37)), data = econ)
abline(h=0)
plot(resid(testTrans1.lm) ~ S.P500, data = econ)
abline(h=0)
```



```
par(mfrow=c(1,1))
```

Next, we check the residual plot of our linear model against `IntRate`, `PrimEducRate`, `SecEducRate` and `TerEducRate`. There does not seem to be any obvious transformations that will improve the fit.

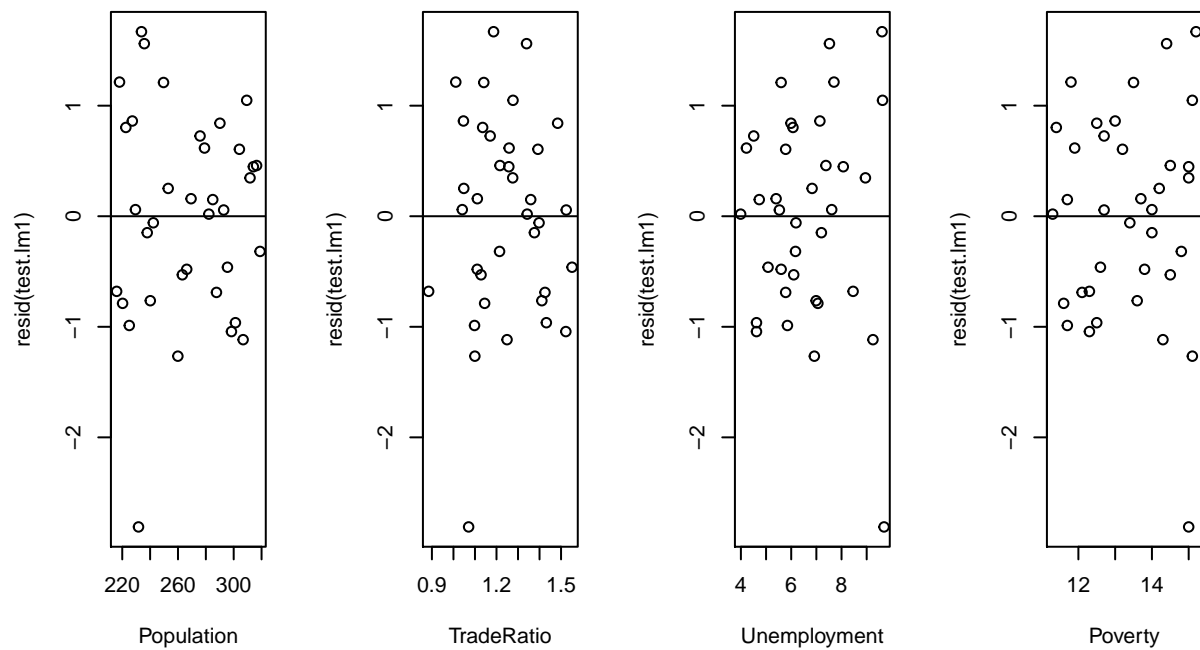
```
par(mfrow=c(1,4))
plot(resid(test.lm1) ~ (PrimEducRate) , data = econ)
abline(h=0)
plot(resid(test.lm1) ~ (SecEducRate) , data = econ)
abline(h=0)
plot(resid(test.lm1) ~ (TerEducRate) , data = econ)
abline(h=0)
plot(resid(test.lm1) ~ (IntRate) , data = econ)
abline(h=0)
```



```
par(mfrow=c(1,1))
```

Next, we check the residual plot of our linear model against Population, Poverty, TradeRatio and Unemployment. As expected, we conclude there is no transformation to be performed.

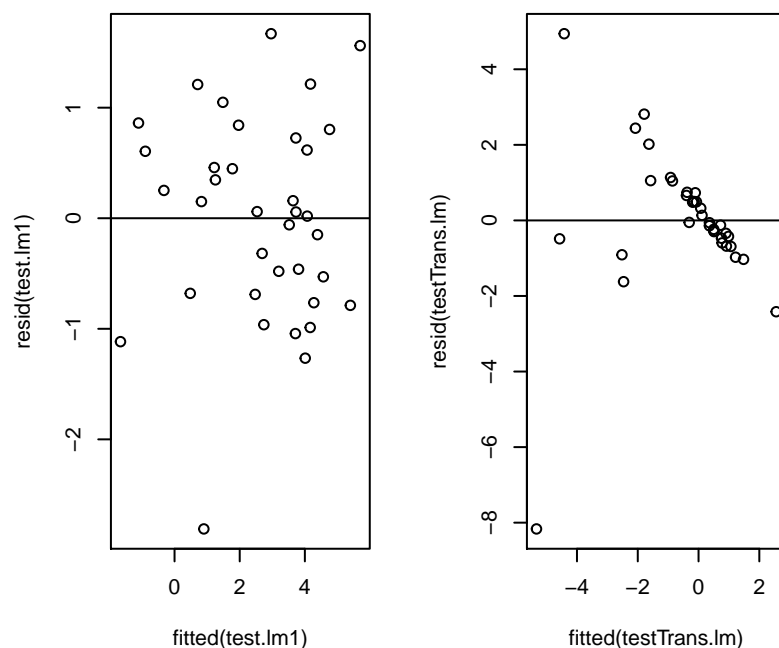
```
par(mfrow=c(1,4))
plot(resid(test.lm1) ~ Population, data = econ)
abline(h=0)
plot(resid(test.lm1) ~ TradeRatio, data = econ)
abline(h=0)
plot(resid(test.lm1) ~ Unemployment, data = econ)
abline(h=0)
plot(resid(test.lm1) ~ Poverty, data = econ)
abline(h=0)
```



```
par(mfrow=c(1,1))
```

Finally, we check for a log-linear model. However, clearly the fit is not improved.

```
testTrans.lm <- lm(I(1/GDP_Rate) ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRate + MCap
par(mfrow=c(1,3))
plot(resid(test.lm1) ~ fitted(test.lm1), data = econ)
abline(h=0)
plot(resid(testTrans.lm) ~ fitted(testTrans.lm), data = econ)
abline(h=0)
par(mfrow=c(1,1))
```



## Case Influence Stats

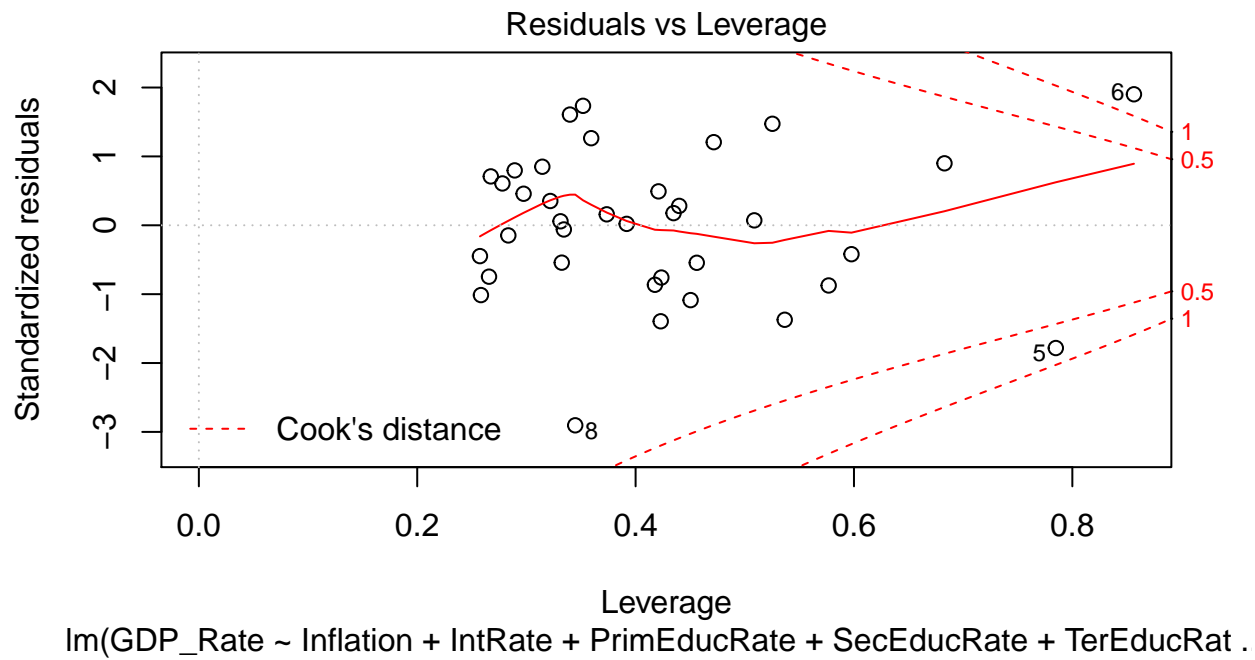
Now we check for case influence stats. We notice that case 6, which is the year 1980 is exerting large influence on our model as it has a Cook's distance of 1.5. This is due to the very high inflation during this year. As such, we remove the point from our data set due to really high inflation.

```
cooks.distance(test.lm1)
```

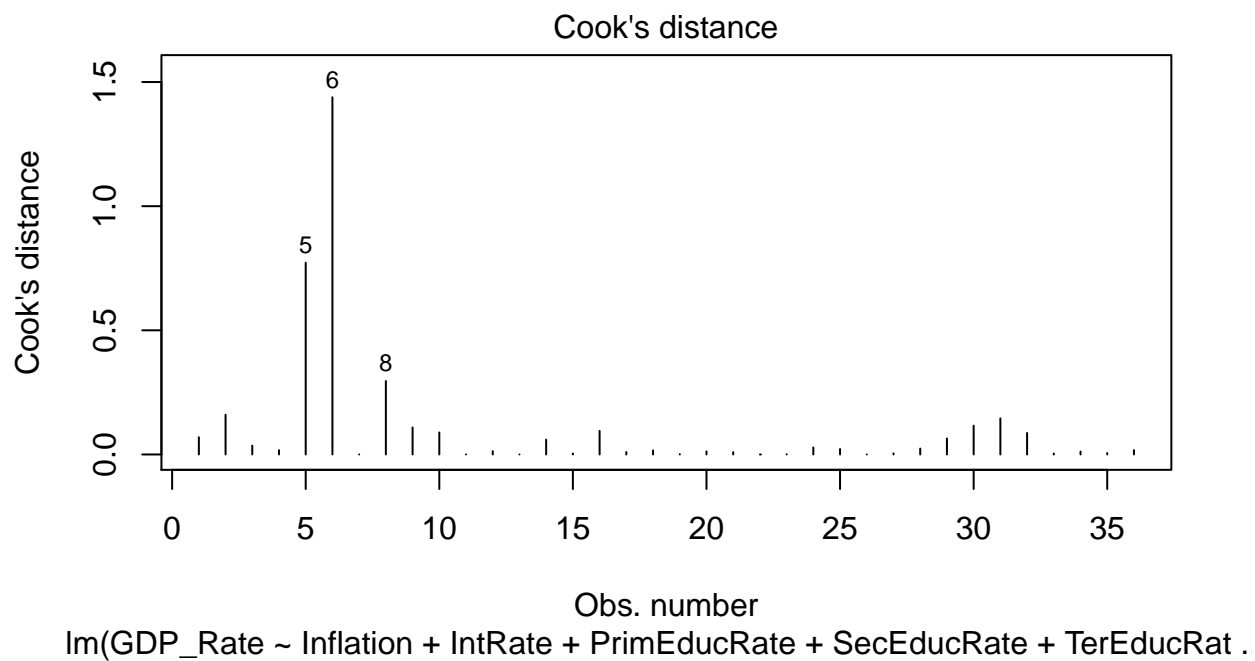
```
##           1           2           3           4           5
## 6.939291e-02 1.601746e-01 3.568674e-02 1.722600e-02 7.727511e-01
##           6           7           8           9          10
## 1.438605e+00 3.525260e-04 2.957976e-01 1.088504e-01 8.867965e-02
##          11          12          13          14          15
## 5.707494e-04 1.343123e-02 1.237445e-04 5.977915e-02 4.143389e-03
##          16          17          18          19          20
## 9.497188e-02 9.766577e-03 1.656814e-02 1.595921e-03 1.224736e-02
##          21          22          23          24          25
## 9.464610e-03 1.785535e-05 1.005579e-03 2.823630e-02 2.210881e-02
##          26          27          28          29          30
## 1.102038e-04 4.632716e-03 2.385480e-02 6.450649e-02 1.161886e-01
##          31          32          33          34          35
## 1.453173e-01 8.663293e-02 3.933193e-03 1.174797e-02 5.914685e-03
##          36
## 1.747451e-02
```

```
plot(test.lm1,which=5)
```





```
plot(test.lm1, which=4)
```



```
econ[6,]
```

```
##   YEAR   GDP_Rate Inflation IntRate PrimEducRate SecEducRate TerEducRate
```

```
## 6 1980 -0.2445962      13.51    5.73      98.85      90.84      53.45
##   MCapRate Population Unemployment      Imports      Exports S.P500 Poverty
## 6 47.50377      227.225      7.14 2.93829e+11 2.80773e+11 0.3174      13
##   President Senate House TradeRatio
## 6      1      1      1      1.0465
```

```
econ1<-econ[-6,]
```

Cases 5 and 8 (corresponding to 1979 and 1982) have relatively large Cook's distances but leverage less than  $2 \cdot p/n$ . However, case 8 does have a studentized residual close to -3, as well. However, since they both do not have a Cook's distance greater than 1 and residuals less than  $\pm 2$  we include both cases. The high Cook's distance value is most likely due to high inflation in both years, especially in 1979.

```
econ.lm <- lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate +SecEducRate + TerEducRate + MCapRate+ Popu
summary(econ.lm)
```

```
##
## Call:
## lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
##   SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
##   Unemployment + S.P500 + Poverty + President + Senate + House,
##   data = econ1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.1691 -0.6604  0.1059  0.5997  1.3457
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  108.72889    21.33741   5.096 5.52e-05 ***
## Inflation     -0.69789     0.23486  -2.971 0.007544 **
## IntRate        0.49458     0.21635   2.286 0.033300 *
## PrimEducRate  -0.80467     0.16485  -4.881 9.03e-05 ***
## SecEducRate   -0.24819     0.18544  -1.338 0.195774
## TerEducRate    0.24413     0.07685   3.177 0.004740 **
## MCapRate       0.07054     0.02573   2.742 0.012573 *
## Population    -0.16521     0.04257  -3.881 0.000929 ***
## TradeRatio     3.48477     2.83813   1.228 0.233762
## Unemployment  -1.20213     0.34747  -3.460 0.002476 **
## S.P500        -6.57091     1.96663  -3.341 0.003253 **
## Poverty        1.70100     0.54882   3.099 0.005653 **
## President     -0.20704     0.71548  -0.289 0.775280
## Senate        -0.95317     0.77570  -1.229 0.233411
## House         0.91721     1.03723   0.884 0.387047
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.114 on 20 degrees of freedom
## Multiple R-squared:  0.8289, Adjusted R-squared:  0.7091
## F-statistic: 6.919 on 14 and 20 DF, p-value: 6.287e-05
```

```
cooks.distance(econ.lm)
```

```
##           1           2           3           4           5
## 7.411507e-04 3.963056e-02 3.861500e-02 5.846268e-02 8.283927e-01
##           7           8           9          10          11
```

```
## 2.838591e-02 3.232067e-01 7.798020e-02 7.999290e-02 8.944040e-04
##          12          13          14          15          16
## 3.572345e-02 7.873449e-02 5.801370e-02 5.653847e-05 8.654959e-02
##          17          18          19          20          21
## 8.664144e-03 2.411120e-02 4.851060e-03 7.861004e-03 1.553772e-02
##          22          23          24          25          26
## 6.436578e-04 6.550056e-03 1.823283e-01 4.790699e-02 6.411177e-04
##          27          28          29          30          31
## 5.227425e-03 2.872956e-02 3.332848e-02 5.080429e-02 1.102186e-01
##          32          33          34          35          36
## 7.949393e-02 7.481949e-03 1.981564e-02 1.477847e-02 1.283808e-01
```

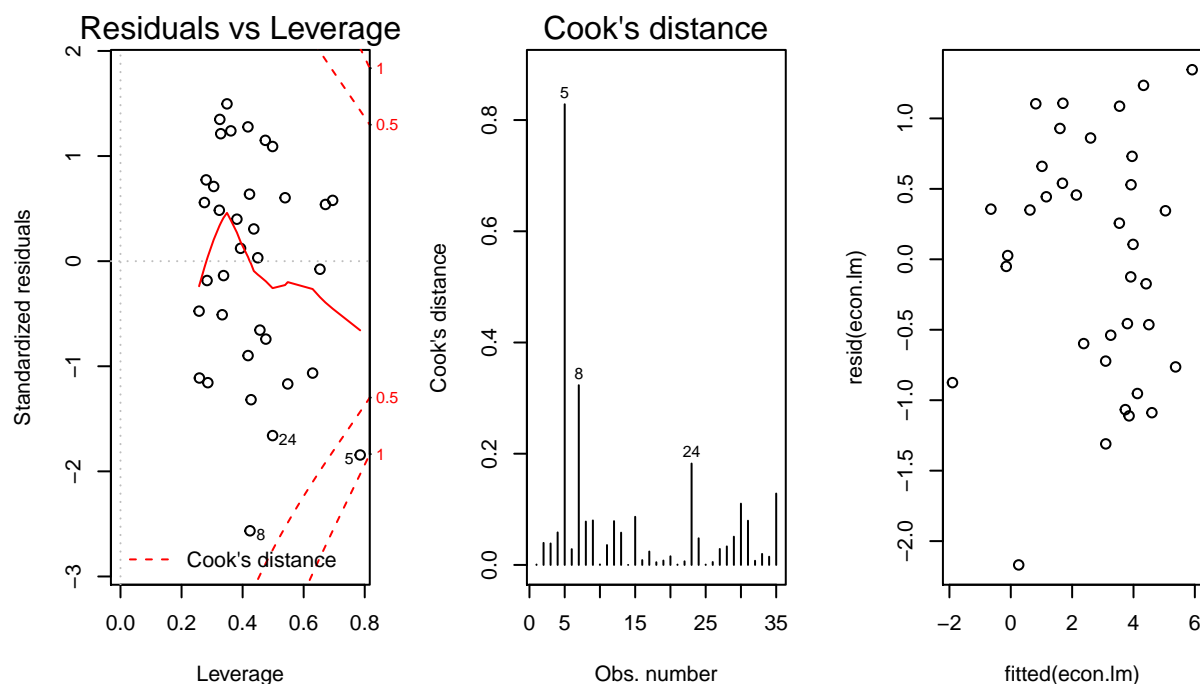
```
2*15/35
```

```
## [1] 0.8571429
```

```
hatvalues(econ.lm)
```

```
##          1          2          3          4          5          7          8
## 0.6534008 0.6713739 0.4178134 0.3252946 0.7851173 0.5391322 0.4241982
##          9         10         11         12         13         14         15
## 0.4174453 0.3488510 0.2835487 0.2861331 0.4981672 0.3615191 0.4497381
##         16         17         18         19         20         21         22
## 0.4276190 0.3332171 0.4566926 0.4366109 0.2748658 0.2805311 0.3932930
##         23         24         25         26         27         28         29
## 0.3812859 0.4982299 0.3282978 0.3375798 0.2574540 0.2584734 0.4761270
##         30         31         32         33         34         35         36
## 0.6949405 0.5478322 0.4743564 0.3236772 0.4226045 0.3052576 0.6293216
```

```
par(mfrow=c(1,3))
plot(econ.lm,which=5)
plot(econ.lm,which=4)
plot(resid(econ.lm) ~ fitted(econ.lm))
```



```
par(mfrow=c(1,1))
econ[5,]
```

```
##   YEAR GDP_Rate Inflation IntrRate PrimEducRate SecEducRate TerEducRate
## 5 1979 3.175691    11.27    4.07          90.45         78.57         52.5
##   MCapRate Population Unemployment Imports Exports S.P500 Poverty
## 5 37.74836    225.055          5.85 2.52675e+11 2.30129e+11 0.1852    11.7
##   President Senate House TradeRatio
## 5         1         1         1    1.097971
```

```
econ[8,]
```

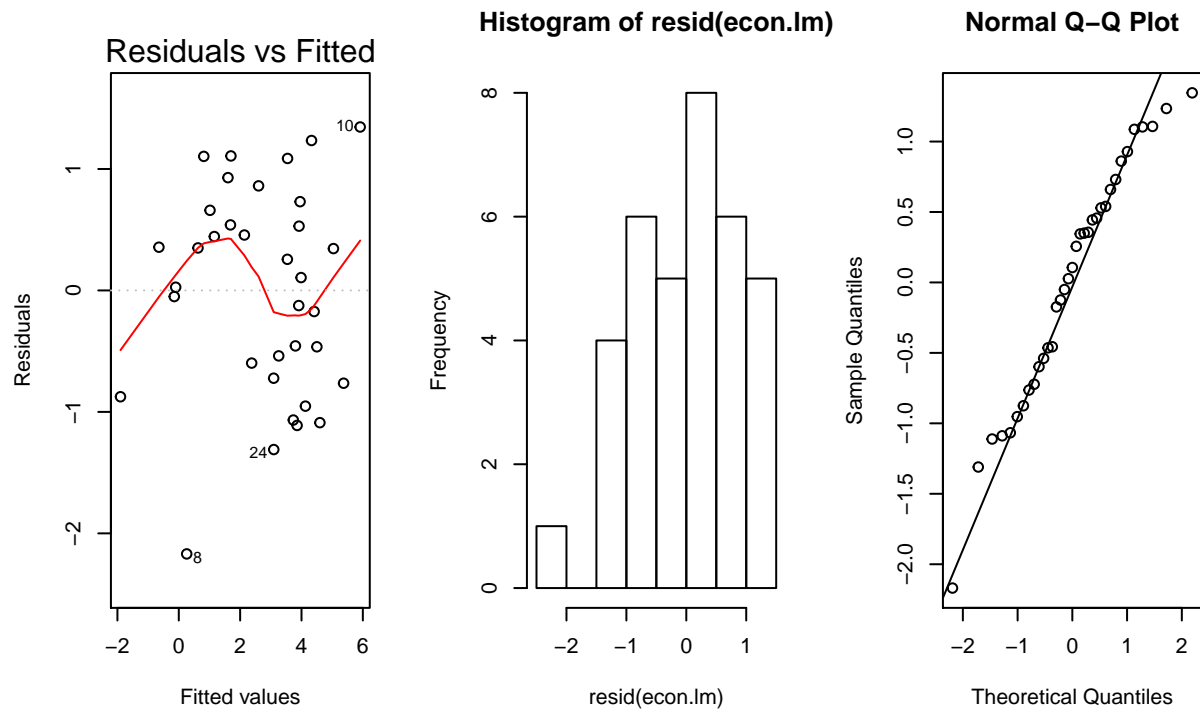
```
##   YEAR GDP_Rate Inflation IntrRate PrimEducRate SecEducRate TerEducRate
## 8 1982 -1.910891     6.16    8.15         100.68         92.75         56.77
##   MCapRate Population Unemployment Imports Exports S.P500 Poverty
## 8 43.55364    231.664          9.69 3.03183e+11 2.83209e+11 0.2042     15
##   President Senate House TradeRatio
## 8         0         0         1    1.070527
```

## Assumptions

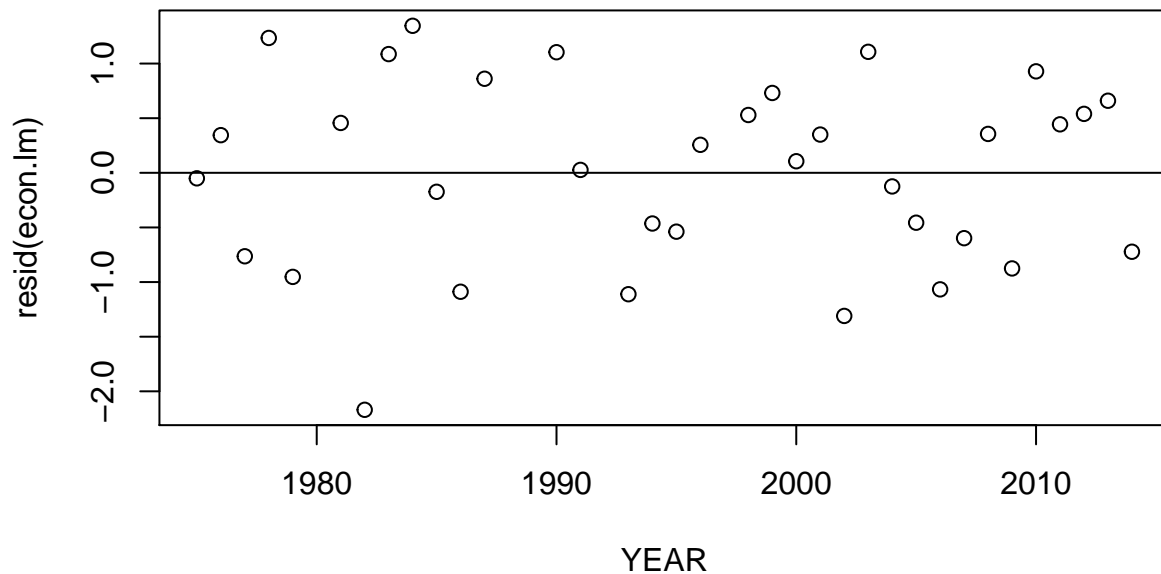
Before we can proceed with backwards selection, we must check the assumptions of our model. The QQ plot has some skewedness in both tails. However, the distribution of the residuals seems to be pretty normal. There is a small amount of left skewedness but it is not too bad. The plots of residuals vs. year, however, support the idea that our model satisfies the independence assumption. It doesn't really appear nearby years are clustered close to each other, possibly as a result of using growth rates instead of raw values. Hence, the independence assumption appears to hold.

```
par(mfrow=c(1,3))
plot(econ.lm, which=1)
```

```
hist(resid(econ.lm))
qqnorm(resid(econ.lm))
qqline(resid(econ.lm))
```



```
par(mfrow=c(1,1))
plot(resid(econ.lm)~YEAR, data=econ1)
abline(h=0)
```



## Backwards Selection

Not surprisingly, we notice that our categorical political categorical variables are insignificant. Due to the presence of some collinearity, we conduct an ANOVA F test on all three to determine if any are worth including in our model. With a p-value of .5791, we fail to reject the null hypothesis that the reduced model is adequate at a 5% level of significance. As such, we exclude all the political categorical variables.

```
summary(econ.lm)
```

```
##
## Call:
## lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
##      SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
##      Unemployment + S.P500 + Poverty + President + Senate + House,
##      data = econ1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.1691 -0.6604  0.1059  0.5997  1.3457
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  108.72889    21.33741     5.096 5.52e-05 ***
## Inflation     -0.69789     0.23486    -2.971 0.007544 **
## IntRate        0.49458     0.21635     2.286 0.033300 *
## PrimEducRate  -0.80467     0.16485    -4.881 9.03e-05 ***
## SecEducRate   -0.24819     0.18544    -1.338 0.195774
## TerEducRate    0.24413     0.07685     3.177 0.004740 **
## MCapRate       0.07054     0.02573     2.742 0.012573 *
```

```
## Population      -0.16521    0.04257   -3.881 0.000929 ***
## TradeRatio      3.48477    2.83813    1.228 0.233762
## Unemployment    -1.20213    0.34747   -3.460 0.002476 **
## S.P500          -6.57091    1.96663   -3.341 0.003253 **
## Poverty         1.70100    0.54882    3.099 0.005653 **
## President       -0.20704    0.71548   -0.289 0.775280
## Senate          -0.95317    0.77570   -1.229 0.233411
## House           0.91721    1.03723    0.884 0.387047
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.114 on 20 degrees of freedom
## Multiple R-squared:  0.8289, Adjusted R-squared:  0.7091
## F-statistic: 6.919 on 14 and 20 DF, p-value: 6.287e-05
```

```
vif(econ1.lm)
```

```
##      Inflation      IntrRate PrimEducRate  SecEducRate  TerEducRate
##      9.924518      8.226730   14.335413   16.543879   30.914312
##      MCapRate      Population  TradeRatio Unemployment      S.P500
##      30.724187     55.210966    6.016254    8.630038    2.969426
##      Poverty      President      Senate      House
##      13.124616     3.580850    4.153552    7.277964
```

```
econ1.lm<-lm(GDP_Rate ~ Inflation + IntrRate + PrimEducRate +SecEducRate + TerEducRate + MCapRate+ Popul.
anova(econ1.lm,econ1.lm)
```

```
## Analysis of Variance Table
```

```
##
```

```
## Model 1: GDP_Rate ~ Inflation + IntrRate + PrimEducRate + SecEducRate +
##      TerEducRate + MCapRate + Population + TradeRatio + Unemployment +
##      S.P500 + Poverty
```

```
## Model 2: GDP_Rate ~ Inflation + IntrRate + PrimEducRate + SecEducRate +
##      TerEducRate + MCapRate + Population + TradeRatio + Unemployment +
##      S.P500 + Poverty + President + Senate + House
```

```
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
```

```
## 1      23 27.338
```

```
## 2      20 24.834  3    2.5041 0.6722 0.5791
```

We know that high collinearity suggests that the standard errors for coefficients may be larger than usual. Thus, the high VIF value ( $>5$ ) for SecEducRate may explain its insignificance. Despite all the very large VIF values, every coefficient/explanatory variable is otherwise significant - i.e. the standard errors are relatively small despite the high levels of collinearity. To check for the significance of SecEducRate, we theorize that only PrimEducRate significantly impacts GDP. However, the ANOVA F-test shows that this theory is very wrong, as the p-value of excluding both TerEducRate and SecEducRate is .00159. Thus, we reject the null hypothesis at a 5% level of significance that the reduced model is adequate. With no theoretical reason to remove SecEducRate, we seek to include it in our model despite the high t-statistic of 3.812. We attribute this high t-statistic value to the corresponding high VIF value. Thus, this is our final model.

```
summary(econ1.lm)
```

```
##
```

```
## Call:
```

```
## lm(formula = GDP_Rate ~ Inflation + IntrRate + PrimEducRate +
##      SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
##      Unemployment + S.P500 + Poverty, data = econ1)
```

```
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.3958 -0.6067  0.0982  0.6568  1.3301
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  100.53594   14.31160    7.025 3.70e-07 ***
## Inflation    -0.58182    0.17360   -3.352 0.002764 **
## IntRate       0.60274    0.17612    3.422 0.002329 **
## PrimEducRate -0.78982    0.12964   -6.092 3.25e-06 ***
## SecEducRate  -0.15591    0.15516   -1.005 0.325440
## TerEducRate   0.26747    0.07017    3.812 0.000896 ***
## MCapRate      0.06811    0.01907    3.572 0.001617 **
## Population   -0.18218    0.03838   -4.746 8.76e-05 ***
## TradeRatio    4.80290    2.13529    2.249 0.034367 *
## Unemployment -0.97990    0.26751   -3.663 0.001294 **
## S.P500        -5.56625    1.69989   -3.274 0.003328 **
## Poverty       1.47626    0.48650    3.034 0.005893 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.09 on 23 degrees of freedom
## Multiple R-squared:  0.8116, Adjusted R-squared:  0.7215
## F-statistic: 9.007 on 11 and 23 DF, p-value: 5.728e-06
```

```
vif(econ1.lm)
```

```
##      Inflation      IntRate PrimEducRate  SecEducRate  TerEducRate
##      5.664226      5.695029      9.262676      12.099923      26.923969
##      MCapRate  Population  TradeRatio Unemployment      S.P500
##      17.627397      46.896672      3.557571      5.343653      2.317630
##      Poverty
##      10.773957
```

```
econ2.lm<-lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate + MCapRate+ Population + TradeRatio + Unempl
anova(econ1.lm,econ2.lm)
```

```
## Analysis of Variance Table
##
## Model 1: GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate +
##      TerEducRate + MCapRate + Population + TradeRatio + Unemployment +
##      S.P500 + Poverty
## Model 2: GDP_Rate ~ Inflation + IntRate + PrimEducRate + MCapRate + Population +
##      TradeRatio + Unemployment + S.P500 + Poverty
##   Res.Df    RSS Df Sum of Sq    F Pr(>F)
## 1      23 27.338
## 2      25 47.876 -2   -20.538 8.6395 0.00159 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

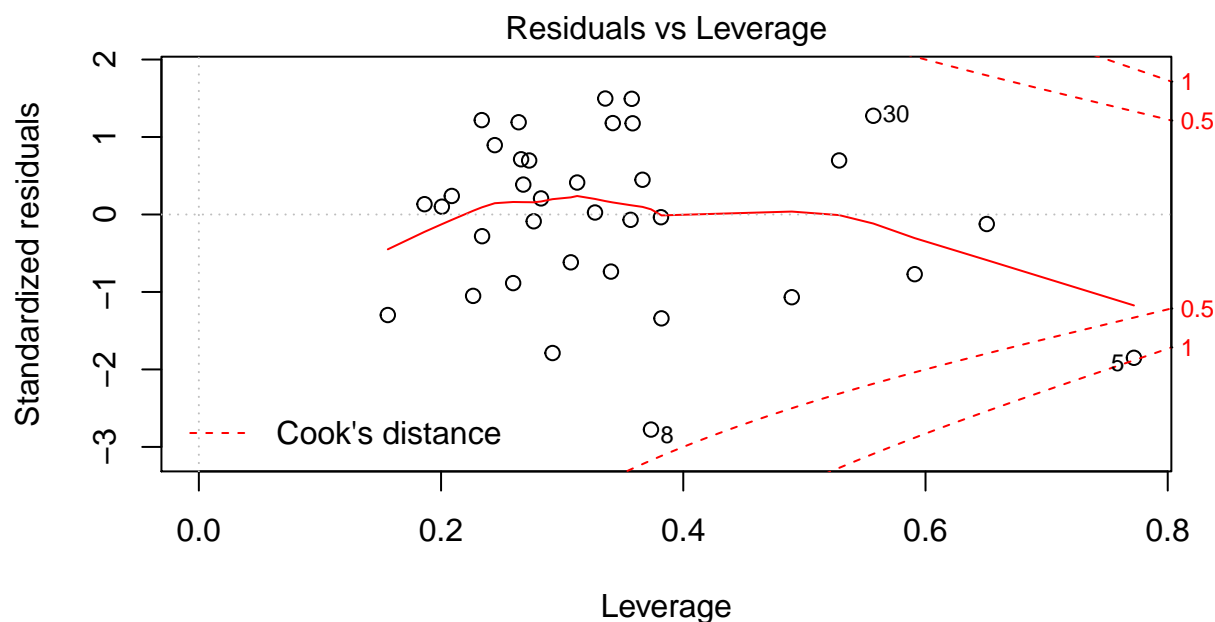
## Case Influence Stats

Due to a studentized residual of about -2 and a Cook's distance of nearly 1, we exclude case 5 or the year 1979 from the data set and refit. We feel comfortable excluding this data point due to high inflation that year. There are no other significant cases and we move forward this data set and model. Additionally, with



the exclusion of this data point we now find SecEducRate is significant.

```
plot(econ1.lm,which=5)
```



lm(GDP\_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRate)

```
cooks.distance(econ1.lm)
```

```
##      1      2      3      4      5
## 2.342698e-03 9.678346e-03 2.335933e-02 3.767469e-02 9.661790e-01
##      7      8      9     10     11
## 4.548597e-02 3.825515e-01 1.033420e-01 9.428882e-02 2.464054e-04
##     12     13     14     15     16
## 2.290960e-02 1.255797e-03 6.019361e-02 6.326758e-05 9.258033e-02
##     17     18     19     20     21
## 1.409505e-02 2.200247e-04 3.378423e-04 1.534912e-02 2.158366e-02
##     22     23     24     25     26
## 6.442513e-03 2.690603e-05 1.099303e-01 4.237163e-02 1.410776e-03
##     27     28     29     30     31
## 2.000723e-03 2.693562e-02 2.594091e-02 1.700269e-01 9.116704e-02
##     32     33     34     35     36
## 6.442722e-02 1.517257e-02 2.123367e-04 4.540390e-03 7.152057e-02
```

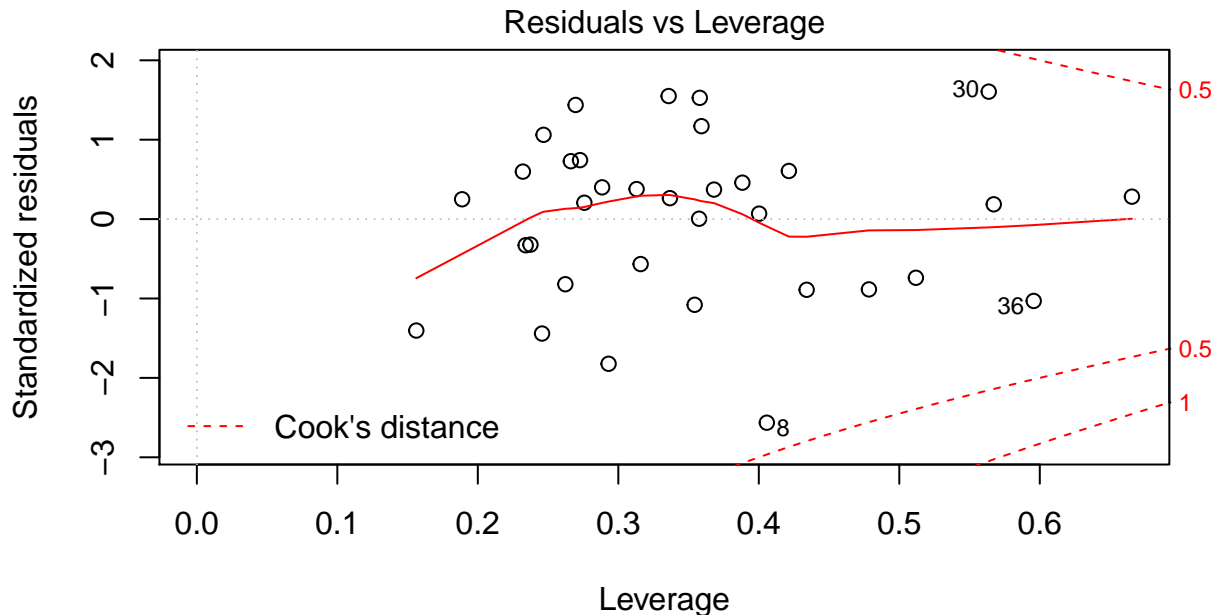
```
econ1[5,]
```

```
##   YEAR GDP_Rate Inflation IntRate PrimEducRate SecEducRate TerEducRate
## 5 1979 3.175691    11.27    4.07         90.45        78.57         52.5
##   MCapRate Population Unemployment Imports Exports S.P500 Poverty
## 5 37.74836    225.055         5.85 2.52675e+11 2.30129e+11 0.1852    11.7
##   President Senate House TradeRatio
## 5         1         1         1 1.097971
```

```
econ2<-econ1[-5,]
```

```
econ1.lm<-lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate +SecEducRate + TerEducRate + MCapRate+ Popul
```

```
plot(econ1.lm,which=5)
```



lm(GDP\_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate + TerEducRate .

```
summary(econ1.lm)
```

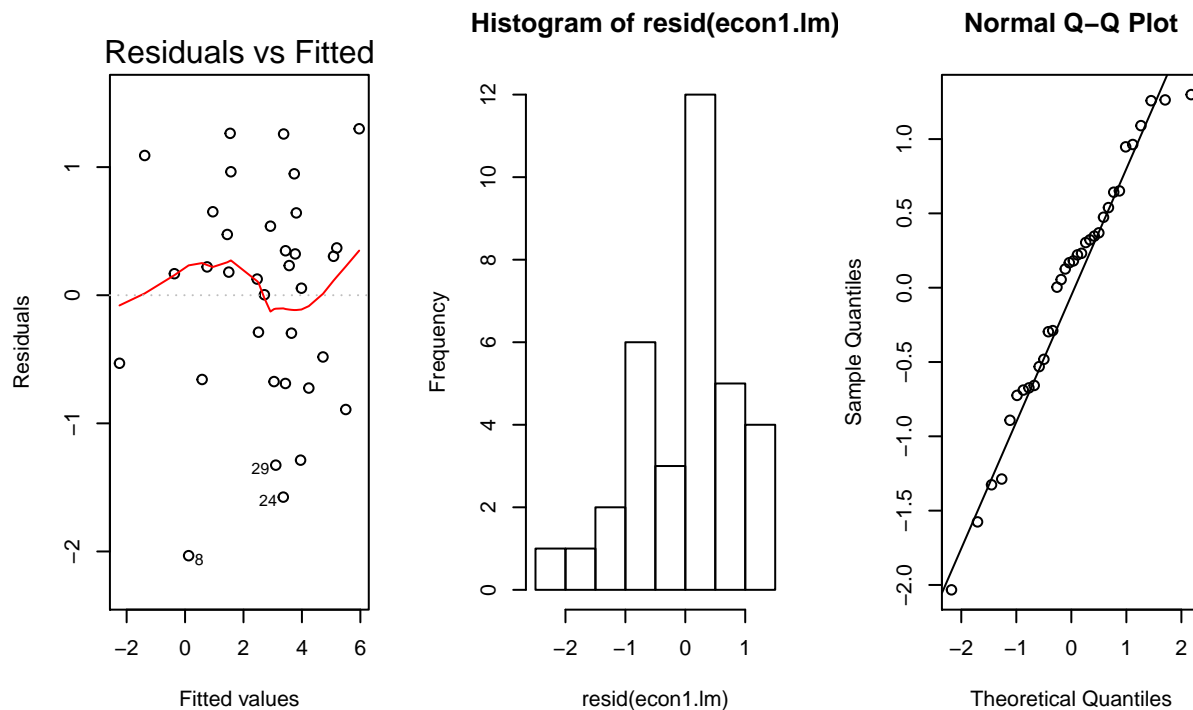
```
##
## Call:
## lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
##     SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
##     Unemployment + S.P500 + Poverty, data = econ2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.0329 -0.6260  0.1742  0.5227  1.2992
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  119.04515   16.47302   7.227 3.06e-07 ***
## Inflation     -0.63586    0.16606  -3.829 0.000914 ***
## IntRate        0.67946    0.17069   3.981 0.000632 ***
## PrimEducRate  -0.80852    0.12267  -6.591 1.25e-06 ***
## SecEducRate   -0.47958    0.22061  -2.174 0.040750 *
## TerEducRate    0.23841    0.06783   3.515 0.001953 **
## MCapRate       0.06047    0.01840   3.286 0.003375 **
## Population    -0.15190    0.03936  -3.859 0.000850 ***
## TradeRatio     6.82267    2.26237   3.016 0.006359 **
## Unemployment  -1.33420    0.31036  -4.299 0.000291 ***
## S.P500        -4.64101    1.67156  -2.776 0.011008 *
## Poverty        2.06225    0.54765   3.766 0.001066 **
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.028 on 22 degrees of freedom
## Multiple R-squared:  0.8394, Adjusted R-squared:  0.7591
## F-statistic: 10.45 on 11 and 22 DF,  p-value: 2.229e-06
```

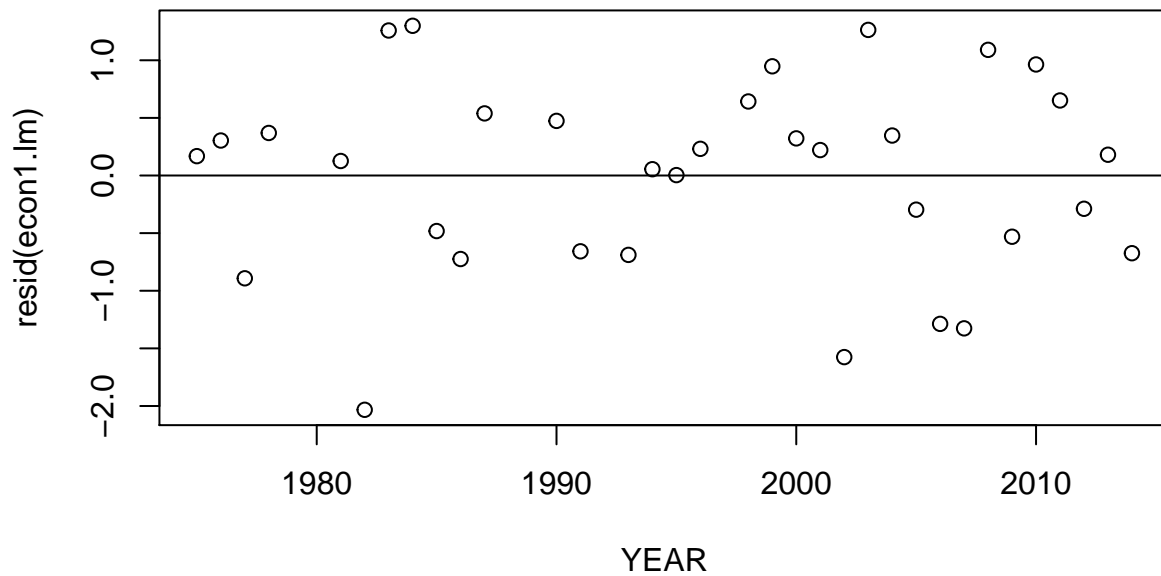
## Assumptions

The distribution of the residuals does seem to be relatively normal. After removing the insignificant terms and influential cases, normality seems to be violated on the basis of the qqplot. On the basis of the residual plot, normality and constant variance seems to be satisfied. Once again, our data shows no obvious signs of clustering in groups of nearby years. Hence, independence also seems to be satisfied.

```
par(mfrow=c(1,3))
plot(econ1.lm, which=1)
hist(resid(econ1.lm))
qqnorm(resid(econ1.lm))
qqline(resid(econ1.lm))
```



```
par(mfrow=c(1,1))
plot(resid(econ1.lm)~YEAR, data=econ2)
abline(h=0)
```



## Final model

After removing the influential cases, we have found our final model and can confirm this with an ANOVA F-test.

*#Full model:*

```
econ2.lm<-lm(GDP_Rate ~ Inflation + IntRate + PrimEducRate +SecEducRate + TerEducRate + MCapRate+ Popul
summary(econ2.lm)
```

```
##
## Call:
## lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
##     SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
##     Unemployment + S.P500 + Poverty + President + House + Senate,
##     data = econ2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.7956 -0.6252  0.1586  0.5363  1.3629
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  122.60327    21.14719   5.798 1.38e-05 ***
## Inflation     -0.69577     0.21953  -3.169 0.005048 **
## IntRate        0.56366     0.20523   2.746 0.012833 *
## PrimEducRate  -0.79852     0.15411  -5.182 5.31e-05 ***
## SecEducRate   -0.55480     0.23279  -2.383 0.027752 *
## TerEducRate    0.20853     0.07406   2.816 0.011041 *
## MCapRate       0.06432     0.02425   2.652 0.015741 *
```

```
## Population      -0.13621    0.04241   -3.212 0.004594 **
## TradeRatio      6.32590    3.01839    2.096 0.049728 *
## Unemployment    -1.51640    0.36173   -4.192 0.000494 ***
## S.P500          -5.33703    1.94165   -2.749 0.012771 *
## Poverty         2.25010    0.58360    3.856 0.001066 **
## President       0.04190    0.68055    0.062 0.951545
## House          1.00381    0.97047    1.034 0.313961
## Senate         -0.80791    0.72876   -1.109 0.281434
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.042 on 19 degrees of freedom
## Multiple R-squared:  0.8577, Adjusted R-squared:  0.7529
## F-statistic: 8.183 on 14 and 19 DF,  p-value: 2.511e-05
```

```
anova(econ1.lm,econ2.lm)
```

```
## Analysis of Variance Table
##
## Model 1: GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate +
##      TerEducRate + MCapRate + Population + TradeRatio + Unemployment +
##      S.P500 + Poverty
## Model 2: GDP_Rate ~ Inflation + IntRate + PrimEducRate + SecEducRate +
##      TerEducRate + MCapRate + Population + TradeRatio + Unemployment +
##      S.P500 + Poverty + President + House + Senate
##   Res.Df    RSS Df Sum of Sq    F Pr(>F)
## 1      22 23.271
## 2      19 20.611  3    2.6594 0.8172 0.5003
```

```
summary(econ1.lm)
```

```
##
## Call:
## lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
##      SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
##      Unemployment + S.P500 + Poverty, data = econ2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.0329 -0.6260  0.1742  0.5227  1.2992
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  119.04515   16.47302    7.227 3.06e-07 ***
## Inflation    -0.63586    0.16606   -3.829 0.000914 ***
## IntRate       0.67946    0.17069    3.981 0.000632 ***
## PrimEducRate -0.80852    0.12267   -6.591 1.25e-06 ***
## SecEducRate  -0.47958    0.22061   -2.174 0.040750 *
## TerEducRate   0.23841    0.06783    3.515 0.001953 **
## MCapRate      0.06047    0.01840    3.286 0.003375 **
## Population   -0.15190    0.03936   -3.859 0.000850 ***
## TradeRatio    6.82267    2.26237    3.016 0.006359 **
## Unemployment -1.33420    0.31036   -4.299 0.000291 ***
## S.P500       -4.64101    1.67156   -2.776 0.011008 *
## Poverty       2.06225    0.54765    3.766 0.001066 **
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.028 on 22 degrees of freedom
## Multiple R-squared:  0.8394, Adjusted R-squared:  0.7591
## F-statistic: 10.45 on 11 and 22 DF,  p-value: 2.229e-06
```

## Tables and Graphs for Paper

Model summary:

```
library(stargazer)
```

```
##
## Please cite as:
## Hlavac, Marek (2015). stargazer: Well-Formatted Regression and Summary Statistics Tables.
## R package version 5.2. http://CRAN.R-project.org/package=stargazer
```

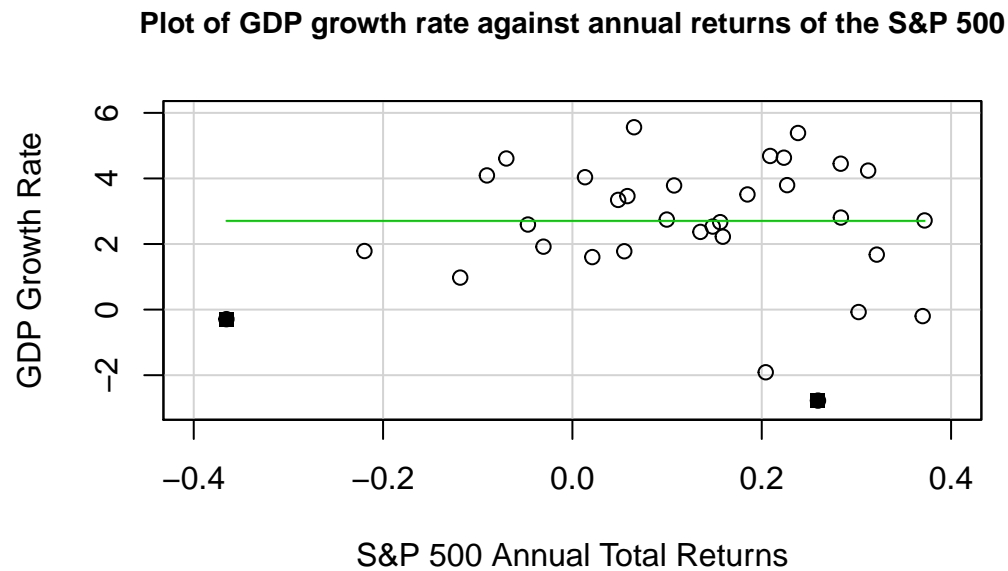
```
stargazer(econ1.lm,single.row=TRUE)
```

```
##
## % Table created by stargazer v.5.2 by Marek Hlavac, Harvard University. E-mail: hlavac at fas.harvard.edu
## % Date and time: Sun, Jun 04, 2017 - 19:46:51
## \begin{table}[!htbp] \centering
##   \caption{}
##   \label{}
##   \begin{tabular}{@{\extracolsep{5pt}}lc}
##     \hline
##     \hline \hline
##     & \multicolumn{1}{c}{\textit{Dependent variable:}} & \\
##     \cline{2-2}
##     \hline & GDP\_Rate & \\
##     \hline \hline
##     Inflation &  $-\$0.636^{***}$  & (0.166) \\
##     IntRate &  $0.679^{***}$  & (0.171) \\
##     PrimEducRate &  $-\$0.809^{***}$  & (0.123) \\
##     SecEducRate &  $-\$0.480^{**}$  & (0.221) \\
##     TerEducRate &  $0.238^{***}$  & (0.068) \\
##     MCapRate &  $0.060^{***}$  & (0.018) \\
##     Population &  $-\$0.152^{***}$  & (0.039) \\
##     TradeRatio &  $6.823^{***}$  & (2.262) \\
##     Unemployment &  $-\$1.334^{***}$  & (0.310) \\
##     S.P500 &  $-\$4.641^{**}$  & (1.672) \\
##     Poverty &  $2.062^{***}$  & (0.548) \\
##     Constant &  $119.045^{***}$  & (16.473) \\
##     \hline \hline
##     Observations & 34 & \\
##     R2 & 0.839 & \\
##     Adjusted R2 & 0.759 & \\
##     Residual Std. Error & 1.028 (df = 22) & \\
##     F Statistic &  $10.453^{***}$  (df = 11; 22) & \\
##     \hline
##     \hline \hline
##     \textit{Note:} & \multicolumn{1}{r}{ $^{*}p < 0.1$ ;  $^{**}p < 0.05$ ;  $^{***}p < 0.01$ } & \end{table}
```

```
## \end{tabular}
## \end{table}
```

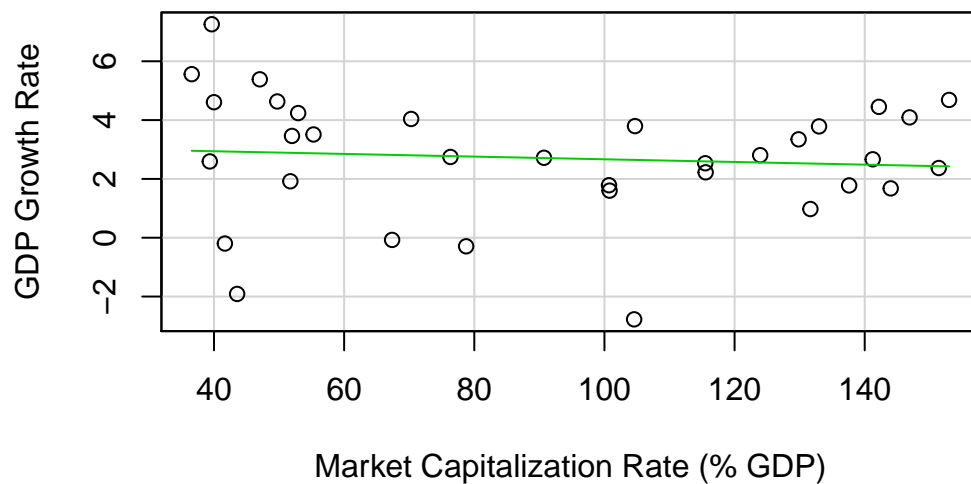
Plot of GDP growth vs. S&P 500 returns. Financial crisis years (2008, 2009) are squares

```
econ2Nocrisis <- econ[-c(5,6,30,31),]
econcrisis <- econ[c(30,31),]
scatterplot(GDP_Rate ~ S.P500 , data=econ2, boxplots = F, smoother=F, xlab = "S&P 500 Annual Total Return")
points(econcrisis$S.P500, econcrisis$GDP_Rate, pch =15)
```



```
scatterplot(GDP_Rate ~ MCapRate , data=econ2, boxplots = F, smoother=F, xlab = "Market Capitalization Rate")
points(econcrisis$S.P500, econcrisis$GDP_Rate, pch =15)
```

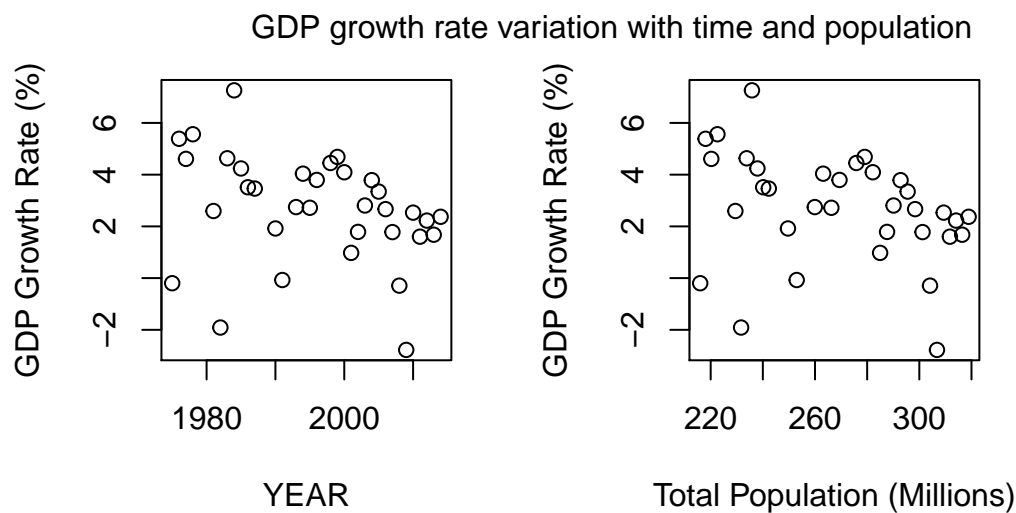
Plot of GDP growth rate against the market capitalization rate in percent (



```
par(mfrow = c(1,2) ,oma = c(0, 0, 1, 0))
plot(GDP_Rate ~ YEAR , data=econ2, xlab = "YEAR", ylab = "GDP Growth Rate (%)", cex.main = 0.85)

plot(GDP_Rate ~ Population , data=econ2, xlab = "Total Population (Millions)", ylab = "GDP Growth Rate (%)", cex.main = 0.85)
par(mfrow = c(1,1))

mtext("GDP growth rate variation with time and population", outer = F, cex = 1, side = 3, adj = 0.48, p
```

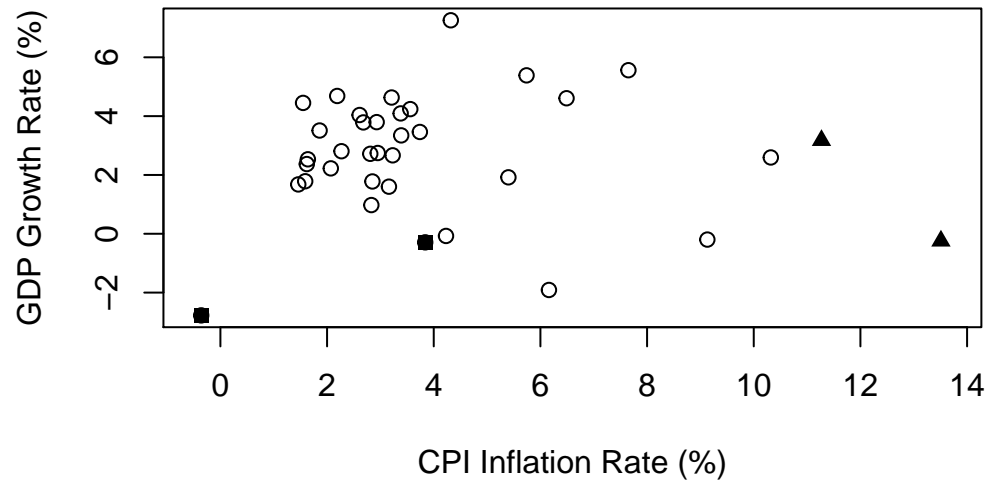


Plot to show inflation vs. GDP rate (w/ 1979, 1980 shown).



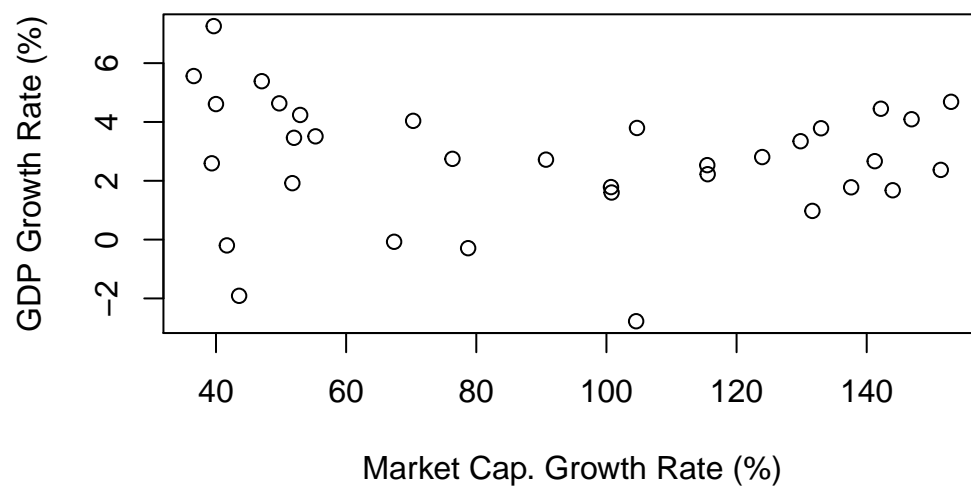
```
econcut <- econ[c(5,6),]
plot(GDP_Rate ~ Inflation , data=econ2, xlab = "CPI Inflation Rate (%)", ylab = "GDP Growth Rate (%)",
points(econcut$Inflation, econcut$GDP_Rate, pch = 17)
points(econcrisis$Inflation, econcrisis$GDP_Rate, pch = 15)
```

**GDP growth rate against Consumer Price Index (CPI) inflation rate**



```
plot(GDP_Rate ~ MCapRate , data=econ2, xlab = "Market Cap. Growth Rate (%)", ylab = "GDP Growth Rate (%)")
```

**Plot of annual GDP growth rate against total market capitalization growth**



```
summary(econ1.lm)
```

```
##
```

```

## Call:
## lm(formula = GDP_Rate ~ Inflation + IntRate + PrimEducRate +
##      SecEducRate + TerEducRate + MCapRate + Population + TradeRatio +
##      Unemployment + S.P500 + Poverty, data = econ2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.0329 -0.6260  0.1742  0.5227  1.2992
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  119.04515    16.47302   7.227 3.06e-07 ***
## Inflation     -0.63586     0.16606  -3.829 0.000914 ***
## IntRate        0.67946     0.17069   3.981 0.000632 ***
## PrimEducRate  -0.80852     0.12267  -6.591 1.25e-06 ***
## SecEducRate   -0.47958     0.22061  -2.174 0.040750 *
## TerEducRate    0.23841     0.06783   3.515 0.001953 **
## MCapRate       0.06047     0.01840   3.286 0.003375 **
## Population    -0.15190     0.03936  -3.859 0.000850 ***
## TradeRatio     6.82267     2.26237   3.016 0.006359 **
## Unemployment  -1.33420     0.31036  -4.299 0.000291 ***
## S.P500        -4.64101     1.67156  -2.776 0.011008 *
## Poverty        2.06225     0.54765   3.766 0.001066 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.028 on 22 degrees of freedom
## Multiple R-squared:  0.8394, Adjusted R-squared:  0.7591
## F-statistic: 10.45 on 11 and 22 DF, p-value: 2.229e-06

# Market Cap 95% CI
0.060+c(-1,1)*qt(.975,df=22)*0.01840

## [1] 0.02184074 0.09815926

# S.P500 95% CI
-4.64101+c(-1,1)*qt(.975,df=22)*1.67156

## [1] -8.107613 -1.174407

# Pop 95% CI
-0.15190+c(-1,1)*qt(.975,df=22)*0.03936

## [1] -0.23352764 -0.07027236

predict(econ1.lm,newdata = econ2[28,], interval="prediction")

##              fit              lwr              upr
## 30 -1.382178 -4.04932  1.284963

```