

HW8

Question 1:

$$(p \wedge q) \rightarrow (q \rightarrow r)$$

$$= \sim(p \wedge q) \vee (q \rightarrow r) \text{ implication elimination}$$

$$= (\sim p \vee \sim q) \vee (\sim q \vee r) \text{ de Morgan}$$

$$= \sim p \vee \sim q \vee \sim q \vee r = \sim p \vee \sim q \vee r$$

$$\text{Given: } p \rightarrow (q \rightarrow r)$$

$$= \sim p \vee (q \rightarrow r) \text{ implication elimination}$$

$$= \sim p \vee \sim q \vee r. \text{ implication elimination}$$

Negate the query $(p \vee q \vee \sim r)$, then resolve it with $\sim p \vee \sim q \vee r = \text{empty}$

Since we have reached empty, and there is a contradiction. Hence, the query is correct with the given statement.

Questions 2:

(1).

$$1. \forall x \text{ child}(x) \rightarrow \text{love}(x, \text{Santa})$$

$$2. \forall x \text{ love}(x, \text{Santa}) \rightarrow \forall y (\text{reindeer}(y) \rightarrow \text{love}(x, y))$$

$$3. \text{reindeer}(\text{Rudolph}) \wedge \text{red nose}(\text{Rudolph})$$

$$4. \forall x \text{ Red nose}(x) \rightarrow (\text{weird}(x) \vee \text{clown}(x))$$

5. $\forall x \text{ reindeer}(x) \rightarrow \sim \text{clown}(x)$

6. $\forall x \text{ weird}(x) \rightarrow \sim \text{love}(\text{scrooge}, x)$

7. $\sim \text{child}(\text{scrooge})$

(2).

1. $\forall x (\text{Austinite}(x) \wedge \sim \text{conservative}(x)) \rightarrow \exists y (\text{Armadillo}(y) \wedge \text{love}(x, y))$

2. $\forall x \text{ wear}(x, \text{marron-and-white shirts}) \rightarrow \text{Aggie}(x)$

3. $\forall x \forall y \text{ Aggie}(x) \rightarrow (\text{dog}(y) \wedge \text{love}(x, y))$

4. $\sim \exists x \forall y \sim \exists z (\text{dog}(y) \rightarrow \text{love}(x, y)) \wedge (\text{armadillo}(z) \wedge \text{love}(x, z))$

5. $\text{Austinite}(\text{Clem}) \wedge \text{wear}(\text{Clem}, \text{maroon-and-white shirts})$

6. $\exists x \text{ Austinite}(x) \wedge \text{conservative}(x)$

Question 3:

1.

Alice: $\sim \text{murderer}(\text{Alice}) \rightarrow (\text{friends}(\text{Barney}, \text{Victor}) \wedge \sim \text{friends}(\text{Caddy}, \text{Victor}))$

Barney: $\sim \text{murderer}(\text{Barney}) \rightarrow \sim \text{friends}(\text{Barney}, \text{Victor})$

Caddy: $\sim \text{murderer}(\text{Caddy}) \rightarrow \text{friends}(\text{Barney}, \text{Victor})$

2.

$\forall x \forall y \text{ friends}(x, y) \rightarrow (\sim \text{murder}(x, y) \wedge \sim \text{murder}(y, x))$

$\exists x \exists y \text{ murderer}(x) \wedge \text{murderer}(y) \wedge (x = y)$

$\forall x \sim \text{murderer}(x) \rightarrow \sim \text{lie}(x)$

3.

1. $(\text{murderer}(\text{Alice}) \wedge \text{friends}(\text{Barney}, \text{Victor})) \wedge (\text{murderer}(\text{Alice}) \vee \sim \text{friends}(\text{Caddy}, \text{Victor}))$

2. $\text{murderer}(\text{Barney})$

3. $\sim \text{friends}(\text{Barney}, \text{Victor})$

3. $\text{murderer}(\text{Caddy})$

4. $\text{friends}(\text{Barney}, \text{Victor})$

5. $(\sim \text{friends}(x, y) \vee \sim \text{murder}(x, y)) \wedge (\sim \text{friends}(x, y) \vee \sim \text{murder}(y, x))$

6. $\text{murderer}(a) \wedge \text{murderer}(b) \wedge (a = b)$

7. $\text{murderer}(x)$

8. $\sim \text{lie}(x)$

4.

$\exists x \text{murderer}(x) \wedge \text{murder}(\text{Victor}, x)$

5.

Caddy: $\text{Friend}(\text{Caddy}, \text{Victor})$

6.

Goal: $\text{murderer}(\text{Alice})$

Negate the Goal: $\sim \text{murderer}(\text{Alice})$.

Process:

Step 1: combine statement 1 and negated goal will get

$\text{friends}(\text{Barney}, \text{Victor}) \wedge \sim \text{friends}(\text{Caddy}, \text{Victor})$

Step 2: combine Caddy's statement that has been added in part 5

and result of step 1 will get

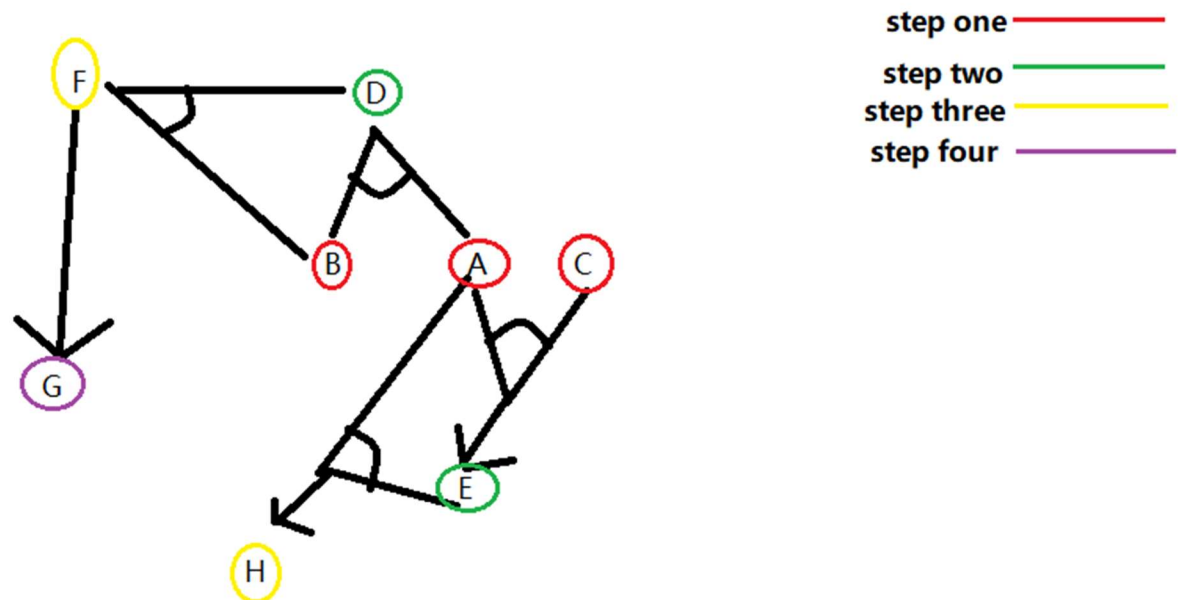
$\text{Friends}(\text{Barney}, \text{Victor})$

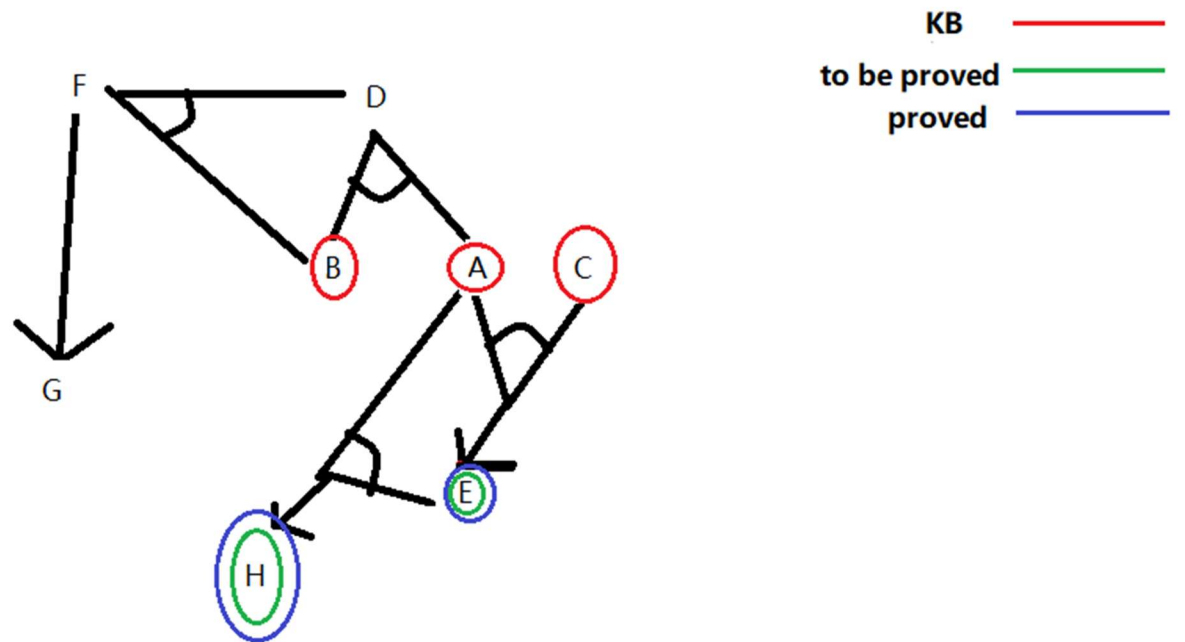
Step 3: combine statement 3 and result of step 2 will get

Empty

Thus, there is a contradiction, which means the goal is unsatisfiable.P

Question 4:

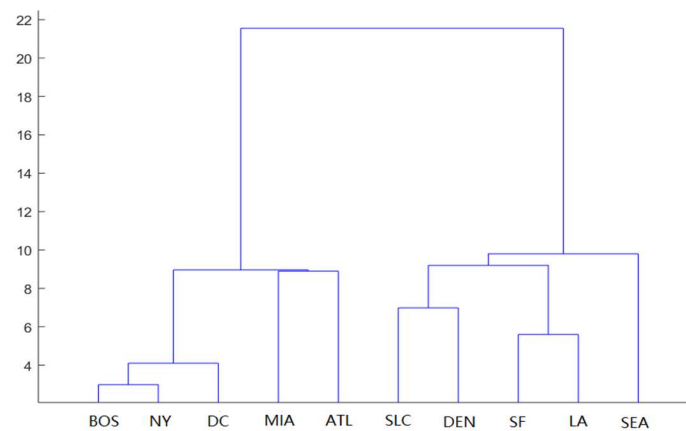




2.

Question 5:

1.



a.

b. Group 1: [BOS, NY, DC, MIA, ATL]

Group 2: [SLC, DEN, SF, LA]

Group 3: [SEA]

2.

a. Group A: [BOS, NY, DC]

Group B: [MIA, SLC, SEA, SF, LA, DEN, ATL]

b. Group A new center coordinate: $c1 = (41, 74.0333)$

Group B new center coordinate: $c2 = (37.07143, 106.3286)$

c. Group A: [BOS, NY, DC, MIA, ATL]

Group B: [SLC, SEA, SF, LA, DEN]