# Griffin Documentation

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**Abstract**

Griffin MMM (Media Mix Modeling) is a cutting-edge Bayesian tool aimed at helping marketers maximise their return on investment by understanding the effectiveness of different media channels. This document serves as a guide to Griffin MMM, covering everything from installation and configuration to data preprocessing, core modelling, and advanced customisations.

The Griffin MMM framework uses advanced statistical techniques, such as adstock transformations to capture delayed marketing effects and saturation transformations to model diminishing returns, providing a realistic representation of how marketing campaigns impact target metrics over time. Integration with Facebook's Prophet enables the inclusion of seasonality and holiday effects, offering a nuanced analysis that takes into account temporal fluctuations in consumer behaviour.

This guide walks users through the process of setting up Griffin MMM, configuring the model to reflect their specific business needs, and using the various visualisation tools available for interpreting results. Through features like lift testing and cross-validation, users are empowered to validate the effectiveness of their marketing strategies with robust statistical backing.

Griffin MMM provides intuitive visual outputs such as saturation curves, adstock effects, and channel contribution analyses to make it easier for stakeholders to understand the insights derived from the model. The Bayesian inference process offers posterior distributions for key metrics, helping marketers gauge the level of uncertainty and make well-informed decisions.

With its advanced reporting capabilities, underpinned by a rigorous statistical framework, Griffin MMM is well-suited for technically-minded marketers, data scientists, and analysts who are looking to derive actionable insights and optimise marketing performance effectively. This executive summary outlines the key aspects of Griffin MMM and underscores its value as a powerful tool for strategic marketing decision-making.

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## 1. Introduction

Griffin MMM (Media Mix Modelling) is a Bayesian analytical tool designed to enable marketers to accurately evaluate and optimise the performance of various marketing channels. In today's increasingly complex and multi-channel marketing environment, understanding how different channels contribute to overall marketing objectives is crucial. Griffin MMM offers a sophisticated solution for quantifying the contribution of each channel, helping businesses allocate their marketing budget effectively and maximise return on investment (ROI).

Griffin MMM stands out due to its advanced Bayesian modelling approach, which provides a robust framework for dealing with uncertainties and delivering more reliable insights. Traditional media mix models often struggle to incorporate uncertainty in their estimates, whereas Griffin MMM provides full posterior distributions for each parameter, giving a more complete picture of the underlying data and the potential outcomes. By applying Bayesian methods, Griffin MMM also allows users to leverage prior information, thereby improving model performance when data is sparse or incomplete.

This documentation serves as a complete guide to understanding how to set up, use, and customise Griffin MMM for your specific business needs. Whether you are new to media mix modelling or an experienced analyst, this guide will provide you with the necessary instructions to utilise Griffin MMM to its fullest potential. Key features of Griffin MMM include advanced adstock and saturation functions, intuitive configuration options, automated data preprocessing, and integrated lift testing capabilitiesâĂŤall of which help facilitate a deeper understanding of marketing impact.

The tool is particularly suited for businesses looking to improve their decision-making processes around marketing spend, providing insights into how different marketing channels interact and drive sales or conversions. With Griffin MMM, marketers can understand the incremental impact of each channel, uncover hidden synergies between channels, and simulate different budget scenarios to determine the optimal allocation of resources. The result is a data-driven approach that moves beyond simple heuristics and into sophisticated, evidence-based marketing strategies.

Griffin MMM also supports visualisation and reporting features, allowing users to easily interpret and communicate model outcomes. These visual insights, such as saturation curves and lift analyses, provide a clear representation of how marketing activities translate into results, which can be shared with stakeholders to justify marketing investments. The model's flexibility, scalability, and practical features make it suitable for both small agencies and large enterprises seeking to harness the power of data-driven decision making in their marketing efforts.

## 2. Getting Started

### 2.1. Installation

To get started with Griffin MMM, you need to follow standard steps to clone the repository and install dependencies. The tool is provided as an automated package, and the installation process should be straightforward. To install Griffin MMM, perform the following steps:

1. **Clone the Repository**: Begin by cloning the repository to your local machine. This can be done using Git:

```
git clone https://github.com/griffin-analytics/griffin-mmm.git
```

2. **Navigate to the Directory**: Change into the project directory:

```
cd griffin-mmm
```

3. **Install Dependencies**: Griffin MMM relies on several Python packages for its operation, including statistical modelling, data manipulation, and visualisation libraries. Use the following command to install all necessary dependencies:

```
pip install -r requirements.txt
```

The `requirements.txt` file includes all necessary dependencies, such as NumPy, Pandas, PyMC, and Matplotlib, ensuring that the environment is properly configured for executing Griffin MMM.

4. **Set Up Environment**: It is recommended to use a virtual environment to isolate dependencies. You can create and activate a virtual environment as follows:

```
python -m venv griffin_env
source griffin_env/bin/activate  # On Windows, use griffin_env\Scripts\activate
```

This helps avoid potential conflicts between package versions.

### 2.2. Directory Structure

The recommended directory structure for your Griffin MMM project is as follows:

```
project_directory/
          config.yaml
          input_data.csv
          results/
          notebooks/
              griffin_demo.ipynb
```

This structure ensures that all files required for Griffin MMM are properly organised. Each component of the directory serves a specific purpose:

- `config.yaml`: This file contains all the configuration settings for the model, including hyperparameters, priors, and data specifications. It allows users to customise how the model processes input data and the specifics of the Bayesian modelling approach.

- `input_data.csv`: This CSV file is the main source of input data for the model. It should contain information such as dates, media channel spends, and target metrics. Ensure that all columns are named correctly, matching the specifications in exttconfig.yaml.

- `results/`: This directory will store the outputs of your modelling process, including diagnostic plots, posterior estimates, and model summaries. Organising these outputs ensures that results can be easily accessed and reviewed.

- `notebooks/griffin_demo.ipynb`: This Jupyter notebook provides an interactive demonstration of Griffin MMM. It can be used to experiment with the tool, understand its workflow, and test custom configurations.

### 2.3. *Basic Setup*

After installation, ensure that you have the following key files:

- `config.yaml` for model configuration.
- `input_data.csv` for input data.
- `results/` folder for output storage.

#### 2.3.1. *Configuration File Setup*

The `config.yaml` file is critical for defining how the model will behave. It includes details like:

- **Model Name**: Defines the name of the model run, which helps in organising multiple experiments.
- **Data Granularity**: Specifies whether the data is daily, weekly, or monthly, which informs the time resolution for modelling.
- **Train-Test Split**: Defines the proportion of data used for training versus testing. A common setting is a 90-10 split.
- **Column Mappings**: Specifies columns such as exttdate_col, extttarget_col, and exttmedia that indicate where to find dates, sales targets, and media spends, respectively, in the input CSV.
- **Priors and Hyperparameters**: Allows users to set priors for Bayesian estimation, including distribution types and their parameters. This feature is particularly useful when users have prior knowledge about certain effects, such as the expected ROI of a channel.

#### 2.3.2. *Data Requirements*

Griffin MMM requires well-prepared input data for optimal performance. The exttinput_data.csv should follow these guidelines:

- **Date Column**: The date column must be correctly formatted (e.g., `YYYY-MM-DD`) and specified in `config.yaml`.
- **Media Spend Columns**: Ensure that media spend columns are named as per the configuration file and contain numerical values without missing entries.
- **Target Column**: The target metric, such as sales or conversions, should be clearly defined and consistent throughout the dataset.

#### 2.3.3. *Running Initial Setup*

Once all files are in place, the initial setup can be run to verify that everything is properly configured. This involves running the demonstration notebook or executing the model script to ensure there are no configuration errors:

```
python driver.py --config config.yaml --data input_data.csv
```

This command will initiate the model run based on the provided configuration and input data. The outputs, including model diagnostics, parameter summaries, and visualisations, will be stored in the `results/` directory.

## 3. Configuration Guide

Griffin MMM uses a configuration file `config.yaml` to specify key settings such as model structure, data paths, and hyperparameters. The configuration file provides a highly customisable setup, allowing users to control how the model processes input data, applies Bayesian inference, and optimises the use of available marketing data.

### 3.1. *Configuration File Example*

Below is an example of a YAML configuration file. Users should customise this file based on their data and requirements:

```yaml
################################
# MMM options
################################

data_rows:
  total: 171
  start_date: 2019-07-28
  end_date: 2022-10-30

################################
# Data handling options
################################

# Frequency of the input data (required)
# Values: daily, weekly
raw_data_granularity: weekly

# Proportion of the dataset to use for training (in-sample), with the remainder
# used for an out-of-sample test. Defaults to 90\% train, 10\% test.
train_test_ratio: 1.0

################################
# Column definitions
################################

# Column names from the input to be ignored (optional).
# All columns in the input data should be listed somewhere in this file,
# so use this field when testing out which columns to include.
ignore_cols:
  - "price"
  - "media_imp_5"
  - "media_imp_6"
  - "media_cost_5"
  - "media_cost_6"
  - "other_events"

# Column name for the date index of each row (optional,
# case-sensitive; defaults to lowercase "date").
# Values in this column must be ISO 8601 format (YYYY-MM-DD).
date_col: "date"

# Column name for the target or output metric (e.g. number of leads, sales volume)
# that we want to increase with marketing spend (required).
target_col: "subscribers"
target_type : "conversion" #revenue or conversion
# "entitlements_existing_all"

# Column names for "extra features", i.e. factors external to
# marketing that influence the target metric (optional).
extra_features_cols:
  - "covid_index"
  - "competitor_spend"
```

```yaml
  - "promo_events"

extra_features_impact: {
   "competitor_spend": "negative"
}

# Required: each block under "Media Channel" represents an advertising channel,
# a column name for cost data,
# and impressions data -- if not available then enter cost eg
# display_name: "Media Channel 5"
# impressions_col: media_cost_5
# spend_col: media_cost_5

media:
  - display_name: "Media Channel 1"
    impressions_col: media_imp_1
    spend_col: media_cost_1

  - display_name: "Media Channel 2"
    impressions_col: media_imp_2
    spend_col: media_cost_2

  - display_name: "Media Channel 3"
    impressions_col: media_imp_3
    spend_col: media_cost_3

  - display_name: "Media Channel 4"
    impressions_col: media_imp_4
    spend_col: media_cost_4

################################
# Model parameters
################################

# These PyMC options determine how many samples to take from the
# Markov chain Monte Carlo process; higher values mean slower runtime.
tune: 2000
draws: 2000
chains: 4
ad_stock_max_lag: 8
target_accept : 0.95 #default values is 0.95
# Adding Prophet seasonality and holidays to the model
prophet:
  include_holidays: true
  holiday_country: 'US'
  yearly_seasonality: true
  trend: true
  weekly_seasonality: true

# Fixed seed for MCMC process
seed: 42
```

### 3.2. *Configuration Details*

The `config.yaml` file is designed to be both flexible and comprehensive, enabling you to define every detail of the modelling process. Below are detailed explanations of each component of the configuration file.

#### 3.2.1. *Data Handling*

- **raw_data_granularity**: Specifies the temporal resolution of the input data, such as `daily` or `weekly`. This granularity informs the model on how to interpret time-series relationships.

- **train_test_ratio**: This defines how much of the data is used for training and how much for testing. A value of `1.0` indicates all data will be used for training, which is helpful for forecasting without immediate validation.

*3.2.2. Column Definitions*

- **ignore_cols**: Lists columns that should be excluded from the analysis. These could be irrelevant features or potentially confounding factors not to be included in the model.

- **date_col**: Specifies the column that holds the date for each observation. This is a mandatory field for time series data and must be in `YYYY-MM-DD` format.

- **target_col** and **target_type**: The `target_col` specifies the dependent variable (e.g., `subscribers`). The `target_type` can be either `revenue` or `conversion`, which helps define the optimisation goal.

- **extra_features_cols** and **extra_features_impact**: These define additional variables that may impact the target but are not directly linked to media spending, such as economic indicators. The `extra_features_impact` allows users to specify whether the feature has a `positive` or `negative` impact on the target.

*3.2.3. Media Channels*

- Under the `media` section, each media channel is defined with its `display_name`, `impressions_col`, and `spend_col`. This structure allows the model to process and differentiate between multiple advertising channels. Each channel must include a spend column, and optionally an impressions column if data is available.

*3.2.4. Model Parameters*

- **tune** and **draws**: These parameters control the number of samples taken during the Markov Chain Monte Carlo (MCMC) process. Higher values typically yield better model convergence but require more computational resources.

- **ad_stock_max_lag**: Defines the maximum lag period for adstock, allowing the model to capture delayed effects of marketing efforts up to a certain number of time units.

- **prophet**: The `prophet` section includes settings for adding seasonality components, holiday effects, and trend adjustments. This helps the model account for regular fluctuations, making the forecasts more reliable.

*3.2.5. Custom Priors*

- The `custom_priors` section allows the user to apply prior distributions to different model parameters. This is particularly useful for incorporating domain knowledge or biases about how media channels are expected to perform.

- Examples include defining the prior for `intercept` using a `LogNormal` distribution, setting priors for `adstock_alpha` with a `Beta` distribution, or adjusting `saturation_beta` to control the diminishing returns behaviour of media spend.

*3.3. Updating Configuration*

Updating and modifying the configuration file allows Griffin MMM to adapt to new campaigns or data sources. Changes might include adding new media channels, modifying seasonality parameters, or adjusting priors based on newly acquired insights. For example, the `holiday_country` can be updated to reflect the country-specific public holidays relevant to a new market.

The YAML structure of `config.yaml` makes these adjustments straightforward,

## 4. Data Handling and Preprocessing

### *4.1. Loading Data*

Griffin MMM expects input data to be formatted as a CSV file. This input file forms the foundation of the model, providing all the necessary data points for media mix modelling. Ensure that the data includes the following columns:

- `date`: The date for each observation. This column must be in ISO 8601 format (`YYYY-MM-DD`) to ensure correct parsing by the model.
- `impressions`: Media impressions per channel. This column represents the exposure of advertisements across different media channels, helping quantify the potential reach of marketing activities.
- `spend`: Media spend per channel. This column is crucial for understanding how much budget has been allocated to each media channel over time.
- `target`: The target metric (e.g., sales or conversions) that the model aims to predict or optimise. It serves as the dependent variable in the regression model, capturing the outcome that results from marketing efforts.
- `extra features`: These are optional columns that can include external factors such as economic indicators, competitor activity, or promotional events that might influence the target metric. Including these features helps the model account for non-media factors that impact outcomes.

It is essential that these columns are properly named and formatted, as Griffin MMM will use the configuration file to map each column to its respective role within the model. Users should perform initial data checks to confirm that the dataset is complete, with no missing or misformatted date entries.

### *4.2. Data Transformations*

Griffin MMM applies a series of data transformations to prepare the dataset for modelling. These transformations are crucial for ensuring that the input data is properly conditioned, which improves the model's performance and accuracy. The data transformation steps include:

#### *4.2.1. Normalisation and Scaling*

Media spend and impressions data are often on different scales, which can negatively impact model convergence. Griffin MMM applies standard scaling techniques to ensure all features are on a similar scale:

- **Standard Scaling**: This method transforms the data to have a mean of zero and a standard deviation of one. It is particularly useful when dealing with data that contains outliers, as it prevents large values from disproportionately affecting the model.
- **Min-Max Scaling**: Depending on user preference, Min-Max scaling can also be applied to rescale the data into a specific range, typically between 0 and 1. This can be particularly effective for non-Gaussian data.

#### *4.2.2. Handling Missing Data*

Griffin MMM provides multiple strategies for handling missing data, as incomplete datasets can significantly reduce model effectiveness:

- **Mean/Median Imputation**: Missing values in numerical columns can be imputed using either the mean or median of the column. Mean imputation is suitable for normally distributed data, while median imputation is more robust when dealing with skewed distributions or outliers.
- **Dropping Missing Values**: In cases where missing values are minimal, Griffin MMM can be configured to drop rows with missing values. This ensures that only complete records are used in training, albeit at the cost of reduced sample size.

#### *4.2.3. Feature Engineering*

Feature engineering plays an important role in enhancing model performance. Griffin MMM allows users to include additional features that may influence the target metric:

- **Lag Features**: Lagged versions of key metrics, such as previous week's impressions or spend, can be generated to help capture delayed effects of marketing.
- **Interaction Terms**: Users can define interaction terms between media channels to capture synergies. For example, TV and digital spend interactions can highlight how these channels work together to drive outcomes.
- **External Variables**: Variables such as weather, competitor actions, or macroeconomic indicators can be included to provide context and help the model differentiate between marketing-driven effects and broader market movements.

#### *4.2.4. Seasonality Adjustments*

Seasonality is a significant factor in marketing effectiveness, as consumer behaviour often follows seasonal patterns:

- **Yearly Seasonality**: Griffin MMM can include yearly seasonal components, accounting for changes like holiday periods or annual sales events that may influence customer behaviour.
- **Weekly Seasonality**: Weekly seasonality adjustments can capture day-of-the-week effects, such as increased consumer activity during weekends or specific weekday trends relevant to the business.
- **Prophet Integration**: Griffin MMM integrates with Prophet to add more advanced seasonality and trend components, allowing for better decomposition of the time series into seasonal, trend, and residual components.

#### *4.2.5. Adstock Transformation*

Griffin MMM applies adstock transformations to media spend to account for the delayed effects of marketing activities. The adstock transformation involves applying a decay function to capture how the impact of a media channel persists over time. Users can adjust the `ad_stock_max_lag` parameter in the configuration file to define how many time periods the effect is allowed to persist. The decay rate is controlled through the `adstock_alpha` prior, which can be customised based on empirical knowledge or historical data.

#### *4.2.6. Saturation Effects*

The model also incorporates saturation effects, recognising that increasing spend does not linearly translate to increased impact after a certain point. Griffin MMM uses non-linear transformations such as logistic or exponential functions to capture diminishing returns. The `saturation_beta` and `saturation_lam` parameters in the configuration file help specify the shape and rate of saturation for each media channel, making it possible to model realistic scenarios where channels reach a saturation threshold.

### *4.3. Data Validation*

To ensure that the data is ready for modelling, Griffin MMM performs several validation checks:

- **Date Consistency**: Verifies that dates are continuous and formatted correctly, ensuring there are no gaps in the time series.
- **Value Checks**: Ensures that spend, impressions, and target values are non-negative, as negative values could indicate data entry errors and lead to incorrect model outputs.
- **Column Verification**: Confirms that all specified columns in `config.yaml` are present in the dataset, helping avoid runtime errors.

Data handling and preprocessing are foundational steps that can significantly impact the quality of the model output. By following these guidelines and using the built-in transformation and validation features of Griffin MMM, users can ensure that their data is well-prepared, resulting in more reliable and actionable insights.

## 5. Core Modeling

Griffin MMM offers core modeling capabilities, including sophisticated techniques to represent real-world marketing effects, such as saturation and adstock transformations. These modelling elements ensure that the nuances of how marketing spend impacts business outcomes are accurately represented, providing marketers with meaningful insights to optimise future budgets.

### *5.1. Adstock Transformations*

Adstock transformations are a fundamental feature of Griffin MMM, designed to simulate the delayed and accumulated effect of marketing activities. The concept of adstock originates from the idea that marketing spend does not yield instant, complete returns but instead influences consumer behaviour over a period of time. By applying adstock transformations, Griffin MMM captures the carryover effect, effectively modelling how exposure from advertising decays gradually rather than dropping off immediately after the campaign ends.

The adstock transformation is defined by two critical components:

- **Decay Rate (Adstock Alpha)**: The decay rate, typically denoted by `adstock_alpha`, represents how quickly the effect of the advertisement diminishes over time. The decay factor can range from 0 to 1, where a value closer to 1 indicates a slow decay, meaning the advertising impact is prolonged, whereas a value closer to 0 suggests that the advertising effect wears off quickly.
- **Maximum Lag Period (Adstock Lag)**: The `ad_stock_max_lag` parameter controls how many time units the effect of advertising can persist. For instance, setting `ad_stock_max_lag` to 8 means that the effect of advertising is considered for up to 8 weeks or time periods after the initial spend. This allows Griffin MMM to appropriately capture the long-term contribution of marketing activities.

Mathematically, the adstock effect at time $t$ is calculated using the following recursive formula:

$$A_t = X_t + \alpha A_{t-1} \tag{1}$$

where:

- $A_t$ is the adstock effect at time $t$.
- $X_t$ is the media spend at time $t$.
- $\alpha$ is the decay factor, determining the proportion of the previous period's effect carried forward.

The adstock effect is crucial for accurately representing the impact of campaigns like television ads, which often influence consumer behaviour for weeks after the initial exposure. The choice of $\alpha$ can significantly affect model results, and Griffin MMM provides flexibility to set this parameter based on empirical evidence or domain expertise.

### *5.2. Saturation Transformations*

Saturation transformations in Griffin MMM are used to model diminishing returns from increasing media spend. In real-world scenarios, doubling the marketing budget does not necessarily double the impact due to market saturation. Griffin MMM incorporates non-linear transformations to represent this effect, ensuring that the model reflects realistic spending dynamics.

#### 5.2.1. *Logistic Saturation Function*

One commonly used approach in Griffin MMM is the logistic function to represent saturation effects. The logistic function is well-suited for capturing diminishing returns because it asymptotically approaches a maximum value, reflecting the idea that beyond a certain point, additional spend yields minimal incremental gains.

The logistic saturation effect is defined by the following equation:

$$S(x) = \frac{\lambda}{1 + e^{-\beta(x-\theta)}} \tag{2}$$

where:

- $S(x)$ is the saturation-adjusted effectiveness.
- $\lambda$ is the upper limit of the saturation effect, representing the maximum potential impact.
- $\beta$ is the growth rate, controlling how quickly saturation is reached.
- $\theta$ is the midpoint, indicating the point of inflection where saturation begins to have a significant effect.

#### 5.2.2. *Implementation in Griffin MMM*

In Griffin MMM, each media channel can have different saturation parameters to accurately represent their unique behaviours. For example, digital channels like paid social media may reach saturation faster than traditional media channels like TV, depending on the audience size and the extent of exposure. The `saturation_beta` and `saturation_lam` parameters in the configuration file allow users to specify these characteristics for each media channel.

### 5.3. *Model Structure and Bayesian Inference*

Griffin MMM uses Bayesian inference to estimate the model parameters, allowing users to quantify uncertainty in their estimates. The Markov Chain Monte Carlo (MCMC) process is used to generate samples from the posterior distributions of the model parameters, which allows for a full understanding of the likely range of effects for each channel.

#### 5.3.1. *Markov Chain Monte Carlo (MCMC) Sampling*

MCMC sampling is a powerful tool for estimating the posterior distribution of model parameters. The `tune` and `draws` parameters in the configuration file control the number of iterations used in the MCMC process:

- **Tuning Phase** (`tune`): During the tuning phase, the MCMC algorithm adjusts itself to better explore the parameter space. The number of tuning iterations helps the sampler converge to a stable state, and higher values (e.g., `2000`) are often recommended for complex models.
- **Drawing Samples** (`draws`): Once tuning is complete, the sampler draws from the posterior distribution. These samples form the basis for the model's predictions and the estimated effects of media channels.

### 5.4. *Base Model*

The base model uses a Bayesian approach to estimate the effect of different channels. Users simply provide the 'config.yaml' and formatted data, and Griffin MMM will generate outputs including channel contributions, effectiveness estimates, and ROI analysis.

The combination of adstock, saturation, and Bayesian inference makes Griffin MMM a powerful tool for understanding the effectiveness of marketing spend across different channels, taking into account both immediate and long-term effects, as well as diminishing returns. This allows marketers to make well-informed decisions on where to allocate budgets to maximise overall impact.

## 6. Validation

Griffin MMM includes validation utilities to ensure data consistency, accuracy, and suitability for model training. Data validation is a critical part of the modelling process as it helps to catch errors early, preventing issues that could compromise the quality of model outputs. Proper validation ensures that the input data is clean, reliable, and structured in a way that maximises model performance.

### *6.1. Validation Functions*

The validation process in Griffin MMM comprises several automated checks designed to guarantee that the dataset is correctly formatted and devoid of common pitfalls that could lead to poor model performance. Key validation steps include:

#### *6.1.1. Target Variable Validation*

Ensuring the target variable is non-empty and free of missing values is essential for effective model training. Griffin MMM performs the following checks:

- **Non-Empty Target Column**: The model verifies that the target metric (e.g., sales, conversions) has valid entries for all time periods. Missing values in the target column are imputed or, if imputation is not feasible, the rows containing missing targets are flagged for removal.
- **Statistical Summary Checks**: A statistical summary of the target variable is generated to identify anomalies such as extreme outliers or zero-variance scenarios, which could skew model predictions.

#### *6.1.2. Date Column Validation*

Time series analysis requires a continuous and correctly formatted date column. Griffin MMM checks the following aspects:

- **Uniqueness of Dates**: Each date entry must be unique, with no duplicates. Duplicate dates can lead to ambiguity in time-series modelling, affecting adstock and seasonality calculations.
- **Date Format and Continuity**: The date column must follow the `YYYY-MM-DD` ISO format. Griffin MMM also validates the continuity of the date sequence, ensuring there are no gaps in the data, which is crucial for modelling consistent temporal patterns.
- **Handling Missing Dates**: If gaps in the date sequence are detected, Griffin MMM can either interpolate the missing entries or raise warnings so users can correct the data manually.

#### *6.1.3. Media Spend and Impressions Validation*

Media spend and impressions are core components of MMM. Validating these metrics ensures the integrity of the model's drivers:

- **Non-Negative Values**: All media spend and impressions data must be non-negative. Negative values are typically indicative of data entry errors and could lead to erroneous conclusions about the effectiveness of campaigns.
- **Range and Scaling Checks**: Griffin MMM evaluates whether the media spend values fall within a reasonable range. For instance, if media spend appears unusually high or low for a particular channel, the system will flag this for further inspection. This helps avoid scenarios where unrealistic values disproportionately influence model parameters.
- **Missing Data Handling**: Missing values in spend or impressions columns can significantly affect adstock and saturation calculations. Griffin MMM offers strategies such as forward-filling or interpolation to handle missing data, ensuring the temporal consistency required for effective marketing mix modelling.

*6.1.4. Extra Features Validation*

Extra features, such as economic indicators or promotional events, can play an important role in the model. Griffin MMM performs the following checks on extra features:

- **Impact Definition Validation**: For each extra feature, users must specify the expected impact direction (e.g., positive or negative). Griffin MMM validates that all specified features are aligned with the assumptions made in the model configuration.
- **Correlation Analysis**: The model computes correlations between extra features and the target variable to ensure that these features provide meaningful information. Features that exhibit little to no correlation may be excluded to improve model parsimony and reduce the risk of overfitting.

*6.1.5. Cross-Validation Setup*

Griffin MMM supports cross-validation to evaluate the robustness of the model. The validation setup includes:

- **Train-Test Splitting**: The `train_test_ratio` specified in the configuration determines how data is split between training and testing. The model checks that the training set is sufficiently large to enable effective parameter estimation.
- **Temporal Consistency**: When splitting time series data, it is important to maintain temporal consistency by ensuring that training data precedes testing data. Griffin MMM enforces this rule to prevent information leakage that could bias model evaluation.

*6.1.6. Priors and Hyperparameter Validation*

Griffin MMM allows for the definition of custom priors and hyperparameters. Proper validation of these settings is critical:

- **Priors Consistency Check**: The system ensures that all specified priors are valid for the distributions chosen. For example, if a `Beta` distribution is used, the parameters must be positive, and Griffin MMM will validate these conditions before running the model.
- **Hyperparameter Bounds**: The model checks that hyperparameters, such as `adstock_alpha` and `saturation_beta`, fall within acceptable bounds to prevent unrealistic or unstable behaviour during training.

*6.1.7. Visual Validation Reports*

Griffin MMM generates visual reports summarising key validation checks. These visualisations include:

- **Missing Data Heatmaps**: These heatmaps help users quickly identify any patterns in missing data across all features, allowing them to address inconsistencies prior to model training.
- **Distribution Plots**: Distribution plots for target, spend, and impressions are generated to highlight outliers or unusual distributions that may need further investigation.

These validation steps help ensure that the data provided is suitable for modeling, reducing the chance of errors during model training and ultimately leading to more accurate and reliable model outputs. By incorporating rigorous data validation, Griffin MMM enables users to trust the insights derived from the marketing mix model, ensuring that resource allocation decisions are based on well-validated, high-quality data.

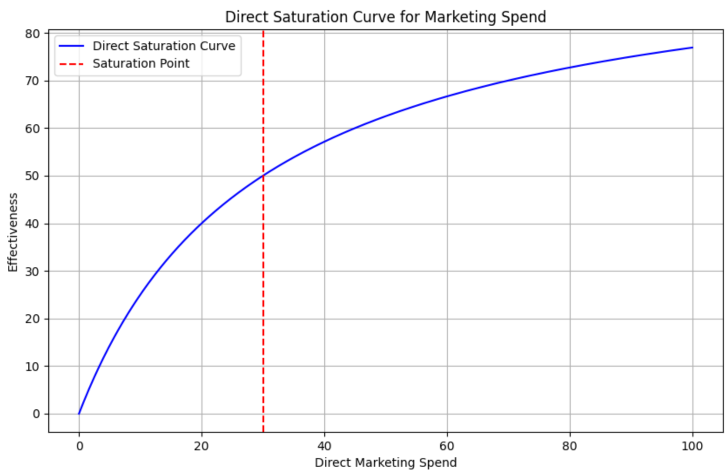
## 7. Visualization

Visualizing the model results is essential for interpreting and communicating the insights derived from Griffin MMM. Proper visual representation allows marketers to better understand the relationships between media spend and business outcomes, thereby informing strategic decision-making regarding budget allocation and channel effectiveness.

### *7.1. Plotting Saturation Curves*

Saturation curves are a core visual output of Griffin MMM, providing insight into the concept of diminishing returns on marketing investment. Griffin MMM generates these curves for each media channel, illustrating how the effectiveness of marketing efforts changes as spend increases. This visualisation allows users to identify the optimal point of spend – the point where further increases yield little to no additional impact, signalling saturation.

Figure 1: Example Saturation Curve Showing Diminishing Returns




The plot in Figure 1 shows a typical saturation effect where effectiveness grows rapidly at lower levels of spend but begins to plateau as the channel becomes saturated. The curve typically follows an S-shape (sigmoid) or a logarithmic form, depending on the channel dynamics and market conditions. The point at which the curve flattens indicates diminishing returns, meaning that additional spend no longer provides proportional gains in output.

#### *7.1.1. Technical Details of Saturation Curve Calculation*

Griffin MMM uses a combination of non-linear transformations, including logistic or exponential functions, to represent saturation. The calculation takes into account:

- **Saturation Parameters**: The `saturation_beta` and `saturation_lam` parameters in the configuration file control the rate and shape of the saturation effect for each media channel. These parameters are estimated as part of the Bayesian inference process, allowing the model to capture how quickly each channel reaches its maximum effectiveness.
- **Interaction with Spend**: The saturation curve is derived by applying these transformations to the adstocked spend data. This means that the plotted curves incorporate both the delayed effect (through adstock) and the non-linear returns (through saturation), giving a holistic view of the channel's performance.

The flexibility of saturation modelling in Griffin MMM allows users to adapt the shape of the curve based on empirical insights or to allow the model to learn the saturation behaviour directly from the data.

*7.2. Plotting Adstock Effects*

Visualising adstock effects helps illustrate the temporal influence of marketing spend. Griffin MMM generates adstock plots that show how the impact of a media spend decays over time, capturing the lingering effect of advertising even after the initial campaign ends.

Figure 2: Example Adstock Effect Showing Delayed Impact of Media Spend

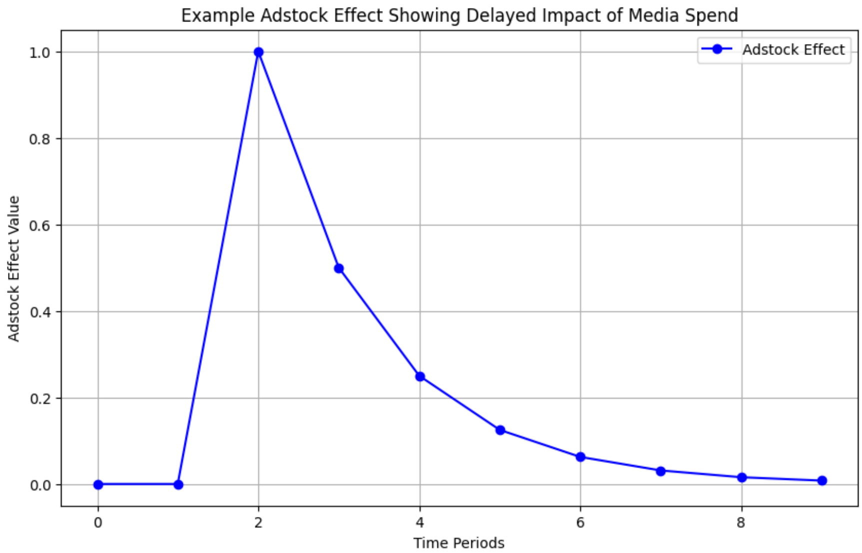



Figure 2 represents the adstock effect for a particular media channel. The curve illustrates how the initial impact of a campaign diminishes gradually rather than dropping off instantly, reflecting real-world consumer response behaviours. By understanding these lagged effects, users can better time their media investments, especially for channels where delayed responses are prominent (e.g., TV and radio).

*7.3. Lift Test Analysis Plots*

Lift test analysis plots are used to evaluate the incremental impact of specific marketing campaigns or interventions. These plots provide a visual representation of the treatment versus control performance, highlighting the effectiveness of a given marketing effort.

Figure 3 shows an example of a lift test analysis. The difference between the control and treatment lines represents the incremental gain achieved due to the intervention. Shaded areas around the lines indicate confidence intervals, providing a sense of the uncertainty around the estimates.

*7.4. Channel Contribution Visualisation*

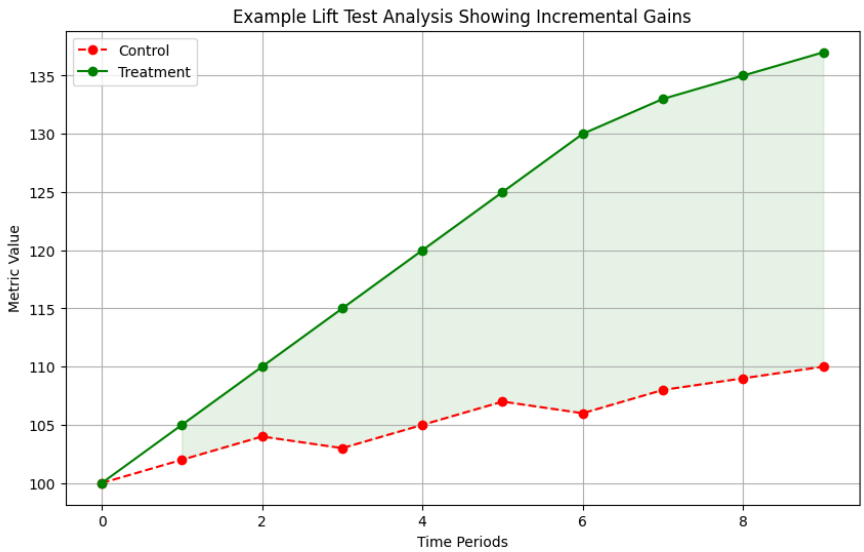
Griffin MMM also provides visualisations that display the contribution of each media channel to the overall target metric (e.g., total sales). These stacked bar charts or line graphs help break down the aggregate impact of all channels into individual contributions, making it easier for marketers to understand which channels are driving performance.

*7.4.1. Stacked Bar Chart of Channel Contributions*

The stacked bar chart visualises how much each media channel contributes to the target metric over time. This chart is particularly useful for seeing how different channels perform during specific periods, such as product launches or promotional campaigns.

Figure 4 demonstrates how contributions from various channels evolve over time. The visualisation helps pinpoint which channels are providing the most value and when their impact peaks, enabling marketers to optimise spend allocation.

Figure 3: Example Lift Test Analysis Showing Incremental Gains




*7.5. Uncertainty and Posterior Distributions*

One of the advantages of using a Bayesian framework is the ability to visualise uncertainty in the model's predictions. Griffin MMM generates plots that show the posterior distributions of key parameters, such as the return on investment (ROI) for each channel.

*7.5.1. Posterior Distribution Plots*

Posterior distribution plots help users understand the range and uncertainty associated with each estimated parameter:

- **ROI Distributions**: These plots show the distribution of ROI estimates for each media channel, providing insights into the variability and reliability of the returns. Channels with narrow distributions indicate more certainty around their effectiveness, while broader distributions suggest more variability.
- **Parameter Uncertainty**: Posterior plots for parameters like `adstock_alpha` or `saturation_beta` allow users to see the range of likely values for these parameters, which is crucial for understanding the potential impact of different modelling assumptions.

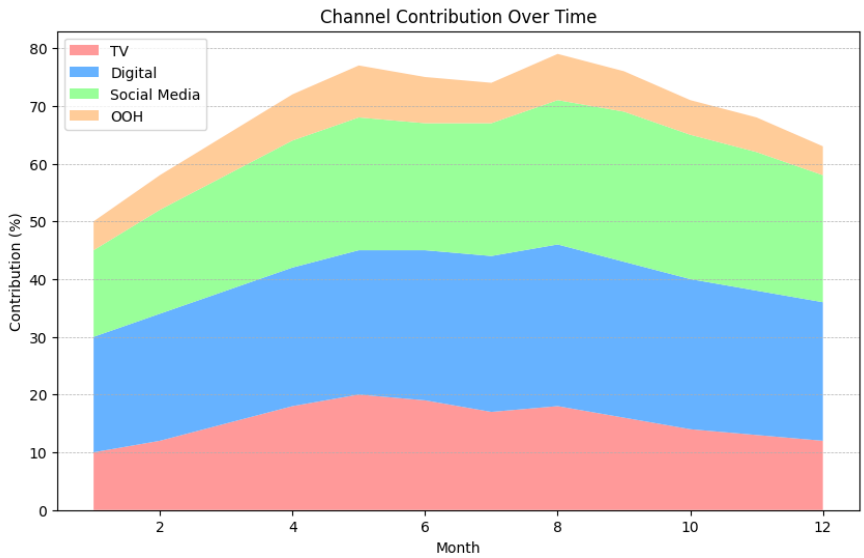
Figure 5 provides an example of a posterior distribution for media channel ROI. The distribution captures the full range of possible values, allowing marketers to make more informed decisions that take uncertainty into account.

*7.6. Practical Uses of Visualizations*

Visualisations generated by Griffin MMM serve several practical purposes:

- **Budget Allocation**: By understanding saturation points and channel contributions, marketers can make informed decisions about where to allocate their marketing budgets to maximise ROI.
- **Stakeholder Communication**: Complex modelling outputs are translated into intuitive visual formats, making it easier to communicate results and recommendations to stakeholders who may not have a technical background.

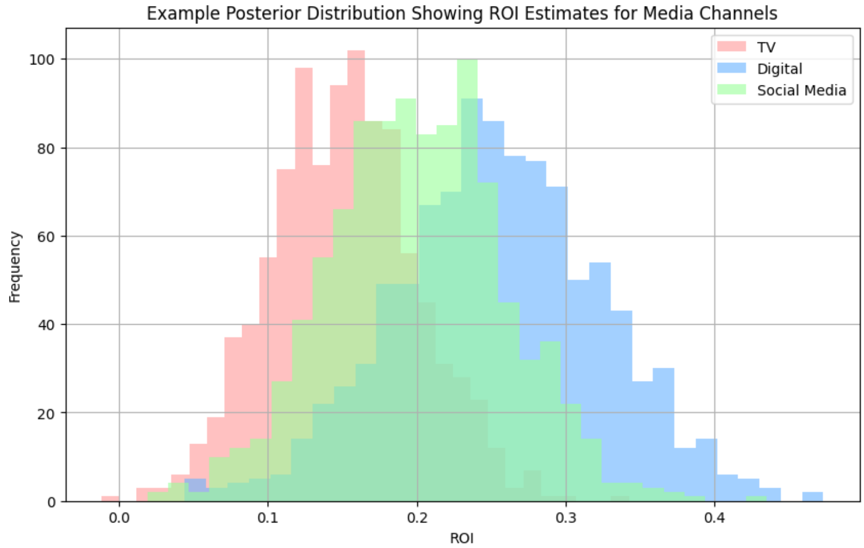
Figure 4: Example Stacked Bar Chart of Media Channel Contributions




- **Campaign Planning**: Adstock and lift visualisations help determine the timing and duration of future campaigns based on observed lag effects and measured incremental gains.

Effective visualisation is crucial to extracting and communicating actionable insights from the Griffin MMM model. By providing a wide range of plots, Griffin MMM ensures that both technical and non-technical stakeholders can derive value from the results, ultimately leading to better marketing decisions and improved business outcomes.

Figure 5: Example Posterior Distribution Showing ROI Estimates for Media Channels




## 8. Advanced Topics

### *8.1. Prophet Integration*

Griffin MMM integrates with Facebook's Prophet, a powerful forecasting tool designed to handle time series data that includes seasonal effects and holiday influences. Prophet is particularly well-suited for business data that exhibits strong seasonal trends and periodic changes, such as weekly and yearly variations in consumer behaviour. By leveraging Prophet, Griffin MMM is able to provide more accurate and realistic estimates by accounting for temporal patterns that impact marketing effectiveness.

The integration with Prophet allows users to:

- **Incorporate Seasonality**: Adding yearly, weekly, and even daily seasonal components helps the model to better capture predictable patterns that occur at regular intervals, thus improving the overall predictive accuracy.
- **Adjust for Holidays and Special Events**: Many industries see significant variations in customer activity during holidays or special events. Griffin MMM, with Prophet's holiday integration, allows users to model these effects, accounting for spikes or drops in consumer behaviour during these periods.
- **Detect Trends**: Prophet helps capture underlying trends beyond seasonal fluctuations, such as gradual increases or decreases in consumer activity, which may be due to market growth, changing consumer preferences, or other macroeconomic factors.

Prophet's flexibility allows Griffin MMM to dynamically adjust seasonal components, resulting in a model that adapts to changing business conditions and provides a more nuanced understanding of marketing performance across different time horizons.

### *8.2. Adding Seasonality Components*

Griffin MMM users can add seasonality components like yearly and weekly trends through the `config.yaml` file. This enables a more fine-tuned adjustment for recurring patterns that might significantly affect marketing performance. Below, we explore the specifics of configuring and using these seasonality components.

#### 8.2.1. *Yearly and Weekly Seasonality*

Adding yearly and weekly seasonality helps Griffin MMM account for repeating patterns observed over these periods. The `prophet` section in the configuration file allows users to include seasonality components:

```
prophet:
  include_holidays: true
  holiday_country: 'US'
  yearly_seasonality: true
  trend: true
  weekly_seasonality: true
```

- **Yearly Seasonality**: Yearly seasonality accounts for annual patterns, such as increased sales during holidays or reduced marketing efficiency during off-peak seasons. These seasonal components help explain variance in the target metric that aligns with recurring annual events, such as Christmas, summer holidays, or fiscal year-end boosts in corporate spending.
- **Weekly Seasonality**: Weekly seasonality helps capture consumer behaviours that vary by day of the week, such as increased activity on weekends or higher conversion rates during weekdays. Including this component is especially helpful for businesses where customer engagement is known to be cyclical within a week.

These seasonal adjustments are applied during the preprocessing phase, and their effects are incorporated into the model's baseline trends. Including such components helps isolate the effects of marketing spend from predictable temporal variations, thereby providing a more accurate estimate of marketing-driven uplift.

#### 8.2.2. *Holiday Effects*

Holiday effects can have a major impact on marketing performance, often causing spikes or drops in consumer engagement that must be accounted for to avoid skewed results:

- **Holiday Integration**: The `include_holidays` option allows Griffin MMM to automatically adjust for major holidays in the specified `holiday_country`. This is especially useful for global brands running campaigns across different markets, each with unique holiday schedules.
- **Custom Holiday Definitions**: In addition to built-in holidays, users can also define custom events that are significant for their business (e.g., product launches, promotional events). These custom holidays can be added to the Prophet configuration, allowing for bespoke seasonal adjustments that reflect unique business dynamics.

Including holiday effects ensures that the spikes or troughs in the time series caused by external factors do not wrongly influence the estimated effectiveness of marketing activities. This improves the reliability of the model by distinguishing between seasonality-driven and spend-driven variations in the target metric.

### 8.3. *Advanced Trend Modelling with Prophet*

In addition to seasonality, Prophet provides advanced trend modelling capabilities that can be leveraged by Griffin MMM to capture non-linear trends and structural changes in the target metric:

#### 8.3.1. *Additive and Multiplicative Seasonality*

Griffin MMM allows users to choose between additive and multiplicative models of seasonality, depending on how they expect the seasonal effects to interact with the overall level of the target metric:

- **Additive Seasonality**: This is suitable when the magnitude of the seasonal effect does not depend on the level of the target metric. For example, a consistent seasonal boost of 10 units during a particular month regardless of the base level of the metric.

- **Multiplicative Seasonality**: This is used when the seasonal impact scales with the level of the target metric. For instance, a 10% increase in sales during a holiday season that scales with the overall baseline.

Choosing the right type of seasonality is crucial for accurately capturing the nature of recurring trends in the data.

*8.3.2. Changepoint Detection*

Prophet has built-in changepoint detection that is integrated into Griffin MMM to help identify moments in the time series when the trend shifts significantly. These changepoints can be caused by a variety of factors, such as market shifts, regulatory changes, or significant marketing interventions:

- **Automatic Changepoint Detection**: Prophet automatically identifies potential changepoints where the time series trajectory alters significantly. These changepoints are included in the model, allowing it to better fit the data around periods of structural change.
- **Manual Changepoint Specification**: Users can also manually specify changepoints if they have prior knowledge of significant events. This feature is particularly useful for incorporating known disruptions, such as competitor launches or major internal changes, directly into the model.

Changepoint detection helps Griffin MMM adapt to significant shifts in the underlying trend, providing a more flexible and accurate representation of how marketing activities are influencing the target metric over time.

*8.4. Practical Applications of Prophet Integration*

The integration of Prophet with Griffin MMM offers several practical benefits for marketers:

- **Forecasting with Seasonality and Trend Components**: The combined power of MMM and Prophet allows marketers to not only measure historical effectiveness but also forecast future performance by projecting seasonal and trend components forward in time.
- **Campaign Planning**: By understanding the expected seasonal and trend fluctuations, marketers can better plan their campaigns to take advantage of periods with high baseline activity or avoid times when customer interest is predicted to be low.
- **Scenario Analysis**: Users can simulate the impact of upcoming holidays or expected seasonal peaks and troughs to determine how best to allocate marketing spend in the future, ultimately helping to optimise campaign timing and effectiveness.

Prophet integration provides Griffin MMM with a robust framework to adjust for temporal complexities, ensuring that seasonality and other recurring patterns are correctly accounted for. This results in a more accurate attribution of marketing impact, supporting data-driven decision-making for budget allocation and campaign optimisation.

## 9. Example Workflow

To run a complete Griffin MMM model, follow these steps:

1. **Prepare Your Data**: Load your data into the correct CSV format, ensuring it matches the expected structure. The dataset should include columns for `date`, media spend per channel, impressions (if available), extra features (such as economic indicators or promotional events), and the target metric (e.g., sales or conversions). Each column must be clearly mapped in the `config.yaml` file to ensure proper preprocessing and model input.
2. **Configure the Model**: Set parameters in the `config.yaml` file, including channels, target variables, and time granularity. Make sure to:
   - Define `media` channels with their respective `spend_col` and `impressions_col`.
   - Set `adstock_alpha` and `saturation_beta` for each channel to control the decay and saturation behaviours.
   - Include seasonality and holiday components under the `prophet` section to improve accuracy.
   - Specify custom priors to reflect domain knowledge or adjust hyperparameters for specific campaigns.
3. **Preprocess the Data**: Run the preprocessing utilities provided by Griffin MMM to handle missing values, scale data appropriately, and add any lagged or interaction terms. Proper preprocessing ensures that the model receives data in a format that maximises the efficiency of Bayesian inference.
4. **Fit the Model**: Execute the model run using the provided tools. Griffin MMM will utilise the configured parameters and input data to perform Bayesian analysis of marketing effectiveness. The MCMC sampling process will estimate the impact of each channel while also quantifying uncertainty. During this phase, the model uses the specified `tune` and `draws` parameters to ensure robust parameter convergence.
5. **Evaluate and Visualize Results**: Utilize the visual outputs generated by Griffin MMM to gain insights into channel performance and effectiveness:
   - **Saturation Curves**: Plot saturation curves for each channel to understand diminishing returns and identify optimal spend levels.
   - **Adstock Effects**: Visualise adstock curves to assess the delayed impact of marketing activities.
   - **Lift Analysis**: Perform lift testing to validate the incremental impact of specific marketing interventions.
   - **Channel Contribution**: Review channel contribution graphs to see how different channels contribute to overall target metric performance over time.
6. **Adjust and Iterate**: Based on the evaluation, adjust the model configuration and repeat the workflow as necessary. Iteration is key to refining the accuracy of the modelâĂŤtweak hyperparameters, adjust priors, or modify seasonality settings to achieve better alignment with observed results.
7. **Report Insights**: Once the model is satisfactorily tuned and validated, generate a report summarising the key insights. Include visualisations, ROI estimates, and performance comparisons across channels. This report can then be shared with stakeholders to inform strategic decisions regarding future marketing investments.

The example workflow provided above is iterative, meaning that marketers and analysts should expect to refine and re-run the model multiple times to optimise performance. Each iteration allows for improved parameter tuning, better data handling, and ultimately a more precise understanding of how marketing investments impact business outcomes.

## 10. API

### *10.1. API Overview*

Griffin MMM provides a set of methods and tools internally to facilitate the transformation, modelling, and visualisation of marketing data. Advanced users who wish to extend the functionality of Griffin MMM or integrate it into other workflows can access the API for greater control over each modelling step. Below is an overview of key components:

#### *10.1.1. Core API Methods*

Griffin MMM exposes several core classes and functions that advanced users can interact with directly:

- **Data Handling API**: Methods for loading and validating data are part of the `InputData` class. Functions include `validate_data()`, `transform_input_generic()`, and `load_csv()`. These functions help ensure data readiness for modelling.
- **Model Configuration and Setup**: The `MMM` class is used for initiating model fitting. It accepts user-provided configuration parameters from `config.yaml` and is responsible for setting up model priors, hyperparameters, and constraints.
- **Custom Transformations and Features**: Users can apply custom transformations on input data using the `transformers` module. This module includes predefined adstock, decay, and interaction transformations that can be applied to media data to enrich the feature set.
- **Bayesian Inference Methods**: The `fit()` method leverages PyMC's MCMC capabilities to estimate model parameters. This method includes options for specifying tuning iterations, draw counts, and chains to achieve convergence.
- **Visualisation Tools**: The `plot()` functions from the `plot.py` module generate key visual insights, including saturation curves, adstock effects, and channel contributions, making it easier for advanced users to incorporate these visuals into custom reporting.

For advanced users, detailed API documentation can be requested, which includes information on how to extend existing classes, customise priors, and modify the internal behaviour of the MMM model.

### *10.2. Common Issues and Troubleshooting*

Griffin MMM is designed to handle a wide variety of input data and configurations, but users may still encounter issues during setup and modelling. This section provides a guide to addressing common problems:

#### *10.2.1. Data-Related Issues*

- **Missing Date Column**: Ensure that the `date_col` specified in `config.yaml` matches the actual date column in your dataset. Dates should be formatted in ISO 8601 (`YYYY-MM-DD`) to ensure compatibility with Prophet and time series processing.
- **Negative Channel Values**: Verify that all media spend and impressions values are non-negative. Negative values can indicate data entry errors and lead to inaccurate or non-convergent models. Use the preprocessing utilities in `transform.py` to clean and standardise such data before model fitting.
- **Duplicate Dates**: The date column must contain unique values for each time period. Duplicate entries will cause inconsistencies in time series processing and result in errors during the model run. Griffin MMM provides a validation utility that automatically checks for duplicate date entries.
- **Handling Missing Values**: Missing data in key columns can negatively affect model performance. Griffin MMM offers several imputation methods, such as forward-filling or interpolation, to handle these missing values. Users can specify imputation strategies in the `config.yaml` file or preprocess the data manually.

#### 10.2.2. *Configuration Errors*

- **Incorrect Media Channel Configuration**: Each media channel must be properly defined with `spend_col` and `impressions_col` in `config.yaml`. Omitting either of these definitions can lead to errors during the model's adstock and saturation transformations. Double-check the configuration file for missing or incorrectly named attributes.

- **Priors and Hyperparameters**: Incorrect specification of priors or unrealistic hyperparameter values can result in poor model convergence or extremely high/low posterior estimates. It is advisable to start with default priors and gradually adjust them based on the data. If the model fails to converge, consider simplifying the priors by using distributions with broader support, such as switching from `Beta` to `Uniform` distributions.

- **Prophet Integration Issues**: If Prophet integration fails, check that the `prophet` options in `config.yaml` are correctly configured. Ensure that `include_holidays` is set to a valid country code, and that no conflicting seasonality options are specified (e.g., using both yearly and custom seasonalities incorrectly).

#### 10.2.3. *Model Fitting Challenges*

- **Non-Convergence of MCMC Chains**: Bayesian modelling relies on MCMC sampling, which may sometimes fail to converge. This can be due to improper prior specifications, a small number of `tune` or `draws`, or highly collinear features. Increase the number of `tune` steps and use diagnostic tools like trace plots to verify that all chains converge to the same posterior distribution.

- **Runtime Performance**: Model fitting may be slow, especially for datasets with high granularity (e.g., daily data with hundreds of features). Consider reducing the number of features by excluding less relevant ones or aggregating data to a weekly level. Optimising hyperparameters such as `chains` and `draws` can also help balance accuracy with runtime performance.

### 10.3. *Useful Tips for Efficient Modelling*

- **Start with a Simple Model**: Begin with a limited number of channels and features to ensure that the model runs correctly. Once validated, incrementally add more complexity.

- **Use Visual Diagnostics**: Make use of the posterior distribution plots and diagnostic plots (e.g., trace plots) provided by Griffin MMM. These visualisations can help identify problems such as non-convergence, poor mixing of chains, or extreme parameter values.

- **Leverage Cross-Validation**: Use the built-in cross-validation tools to validate the model's predictive performance on out-of-sample data. This is particularly important for ensuring that the model generalises well beyond the training dataset.

- **Custom Priors**: Tailor the priors to fit your specific use case. For example, if you know that a certain channel has historically had a minimal impact, use a tighter prior distribution to reflect this domain knowledge.

By following these guidelines, users can ensure smoother implementation and achieve more reliable marketing insights.

## 11. Marketing Mix Modelling: Concepts and Methodology

Marketing serves as a vital catalyst for company growth, often representing significant financial commitments. Consequently, assessing the efficiency and fine-tuning the distribution of marketing funds is crucial for those in the field. Marketing Mix Modelling (MMM) has long been a key strategic instrument used by marketers to meet these objectives. Recent shifts towards consumer privacy, such as Apple's IDFA changes in iOS 14, highlight the critical need for marketers to future-proof their measurement strategies using tools like MMM.

While experimental designs and causal analytical methods are frequently applied to infer causality, they may be impractical or prohibitively expensive in certain contexts. MMM provides an alternative by utilizing aggregate time-series data alongside regression analysis to delineate the impact of marketing on sales. Additionally, MMM can be customized to account for variables like seasonal patterns, trends, and other external factors, and to incorporate geographic hierarchies. Importantly, the primary goal of MMM is often to assess the incremental impact of various marketing activities, not just sales forecasting.

Several complexities must be navigated when developing MMM. The rapid evolution of advertising media demands continuous adaptation to new marketing channels, presenting modelers with the "large p, small n" challenge. Moreover, aiming for detailed data granularity for actionable insights can lead to sparse data and anomalies. Modelers are tasked with balancing the availability of reliable historical data against the need for detailed data analysis. The sequential nature of data can also lead to correlated errors, undermining the foundational assumptions of ordinary least squares modelling. Additionally, the frequent practice of basing marketing budgets on projected revenues introduces endogeneity and multicollinearity issues, exacerbating difficulties with channel spend correlation. Notably, the tendency of demand-focused channels to exhibit self-selection bias necessitates careful treatment to avoid overstated results. Furthermore, the complexity of MMM involves considerable investment and diverse stakeholder engagement, raising the stakes for model clarity and interpretability. Lastly, traditional machine learning techniques like cross-validation are not always applicable for MMM parameter tuning and model selection due to data limitations and the potential non-representativeness of holdout samples for forecasting.

MMM models are instrumental in providing insight for future allocations of budgets. The influence of advertising is subject to fluctuations based on seasonality and various other factors, such as competitive, shape, carryover, and lag effects. Among these, the shape effect is particularly crucial for marketers. It illustrates how sales react to varying levels of advertising intensity, a phenomenon also known as the saturation effect. This effect hypothesises that consumer responses tend to plateau when advertising investments are ramped up. A widely accepted notion in this context is that brands typically see minimal responsiveness to low advertising investments, peak responsiveness at moderate investment levels, and diminishing returns at higher levels.

Most current models suggest an S-curve pattern in sales response to increased advertising. Additionally, the competitive effect is key, gauging how a brand's advertising efficiency stacks up against its market competitors. The carryover effect represents the residual impact of advertising beyond its immediate deployment, whereas the lag effect denotes the time delay in consumer response to specific advertising efforts. These effects have been extensively researched for three decades. However, the emergence of online media channels has strengthened our comprehension of consumer interactions with advertisements.

MMM is grounded in well-established economic and statistical theories that provide a framework for understanding how marketing activities influence consumer behaviour and business outcomes. At its core, MMM applies econometric modelling to estimate the relationship between marketing inputs and a desired business metric, such as sales, revenue, or brand awareness. This section delves into the theoretical aspects that underpin MMM, including the foundational principles, types of transformations used, and methodological considerations.

### *11.1. Econometric Foundations*

MMM is based on econometric principles that model the relationship between independent variables (e.g., media spend across various channels) and a dependent variable (e.g., sales). The core econometric approach used is often a form of multiple linear regression, adjusted for complexities such as temporal dependencies, seasonality, and interactions among channels. The Bayesian framework, frequently applied in MMM, allows for the incorporation of prior beliefs and uncertainties, making the model robust to variations in marketing activities and external influences.

The equation for a typical MMM model can be represented as follows:

$$y_t = \alpha + \sum_{m=1}^{M} \beta_m x_{t,m} + \sum_{c=1}^{C} \gamma_c z_{t,c} + \epsilon_t,$$

where:

- $y_t$: Dependent variable at time $t$, representing the KPI (e.g., sales).
- $\beta_m$: Coefficient representing the effect of marketing channel $m$.
- $x_{t,m}$: Marketing spend for channel $m$ at time $t$.
- $\gamma_c$: Coefficient for control variable $c$.
- $z_{t,c}$: Control variables (e.g., price, promotions, external economic indicators).
- $\epsilon_t$: Error term capturing residual variance not explained by the model.

*11.2. Adstock and Lag Effects*

A critical concept in MMM is the delayed impact of advertising, which is modelled using adstock transformations. Adstock helps represent the cumulative effect of past advertising efforts on the current performance, recognising that marketing impact is not always immediate but can decay over time. This decay is captured using an adstock rate $\theta$, which determines how quickly the effect of advertising diminishes. Mathematically, the adstocked value $x'_{t,m}$ for media channel $m$ is defined recursively as:

$$x'_{t,m} = x_{t,m} + \theta x'_{t-1,m},$$

where $0 \leqslant \theta \leqslant 1$ controls the decay rate.

*11.3. Saturation and Diminishing Returns*

Another key theoretical aspect of MMM is the concept of diminishing returns, often represented using saturation functions. The relationship between marketing spend and its effect on sales is typically nonlinear; higher levels of spend often yield smaller incremental gains, which is referred to as saturation. The Hill function is commonly used to model this saturation effect:

$$\text{Hill}(x; K, S) = \frac{1}{1 + (x/K)^{-S}},$$

where $K$ is the half-saturation point and $S$ controls the steepness of the curve. This function provides a realistic depiction of how the response to advertising saturates at high levels of spend.

*11.4. Hierarchical Bayesian Framework*

MMM often employs a hierarchical Bayesian framework to account for the variability across different products, regions, or time periods. This framework allows for the modelling of parameters that vary by group, while also sharing information across groups to improve parameter estimation. The hierarchical structure is represented as follows:

$$\beta_{m,g} \sim \mathcal{N}(\mu_{\beta_m}, \tau^2),$$

where $\mu_{\beta_m}$ represents the average effect of media channel $m$ across all groups, and $\tau$ controls the variability of this effect across groups $g$.

*11.5. Endogeneity and Control Variables*

Endogeneity, often caused by the correlation between media spend and unobserved factors influencing sales, is a significant challenge in MMM. For example, media spend might increase during periods when sales are expected to be high (e.g., holidays), creating a bias in the estimated effect of advertising. To mitigate this, MMM includes control variables such as pricing, promotions, and macroeconomic indicators. Including these factors helps isolate the true impact of marketing activities on the target metric.

*11.6. Model Calibration and Validation*

The robustness of an MMM model is ensured through calibration and validation processes. Cross-validation is often used to assess the model's ability to generalise to unseen data, while out-of-sample testing helps validate the model's predictive performance. Due to the temporal nature of the data, MMM models often use time-series cross-validation, which respects the sequential structure of the data.

Calibration of parameters like adstock rate $\theta$ and saturation parameters $K$ and $S$ is performed by iteratively adjusting them to minimise the difference between predicted and actual values. Bayesian calibration, which uses prior distributions for these parameters, is particularly effective in scenarios with limited historical data.

## 12. Mathematical Foundations and Key Concepts

### *12.1. Response Functions and Shape Effects*

The shape effect in Media Mix Modelling characterises the non-linear relationship between marketing input and consumer response. We formalise this through multiple response functions:

#### *12.1.1. Hill-Saturation Response*

The Hill function, adapted from biochemistry, provides a flexible S-shaped response curve:

$$f(x) = \frac{\sigma}{1 + e^{-\beta(x-\lambda)}} \tag{3}$$

where:

- $\sigma$ represents the maximum attainable response
- $\beta$ controls the steepness of the response curve
- $\lambda$ determines the inflexion point

#### *12.1.2. Michaelis-Menten Kinetics*

For diminishing returns scenarios, we employ the Michaelis-Menten function:

$$f(x) = \alpha \cdot \frac{x}{\lambda + x} \tag{4}$$

where:

- $\alpha$ represents the asymptotic maximum effect
- $\lambda$ is the half-saturation constant

### *12.2. Saturation Effects and Adstock Transformations*

#### *12.2.1. Geometric Adstock*

The temporal carryover effect is modelled through geometric adstock:

$$A_t = \sum_{l=0}^{L} \theta^l x_{t-l} \tag{5}$$

where:

- $A_t$ is the adstock value at time $t$
- $\theta$ is the decay parameter $(0 \leqslant \theta \leqslant 1)$
- $L$ is the maximum lag considered
- $x_{t-l}$ represents the marketing input at lag $l$

*12.2.2. Weibull Adstock*

For more flexible decay patterns, we implement the Weibull adstock:

$$w(t) = \frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1} e^{-(t/\lambda)^k} \tag{6}$$

where:

- $k$ is the shape parameter
- $\lambda$ is the scale parameter

*12.3. Competitive Effects and Market Dynamics*

*12.3.1. Cross-Channel Elasticity*

We quantify competitive effects through cross-channel elasticity matrices:

$$\eta_{ij} = \frac{\partial \log(y_i)}{\partial \log(x_j)} \tag{7}$$

where:

- $\eta_{ij}$ represents the elasticity of brand $i$'s response to brand $j$'s activity
- $y_i$ is brand $i$'s performance metric
- $x_j$ is brand $j$'s marketing input

*12.3.2. Market Share Dynamics*

The competitive landscape is modelled through a market share attraction model:

$$s_i = \frac{A_i}{\sum_{j=1}^{n} A_j} \tag{8}$$

where:

- $s_i$ is brand $i$'s market share
- $A_i$ is brand $i$'s market attraction
- $n$ is the number of competitors

*12.4. Bayesian Implementation*

*12.4.1. Prior Specifications*

We employ informative priors for key parameters:

$$\beta \sim \text{LogNormal}(\mu_\beta, \sigma_\beta) \tag{9}$$
$$\lambda \sim \text{Gamma}(\alpha_\lambda, \beta_\lambda) \tag{10}$$
$$\theta \sim \text{Beta}(\alpha_\theta, \beta_\theta) \tag{11}$$

#### 12.4.2. *Hierarchical Structure*

The model implements a hierarchical structure for channel-specific parameters:

$$\beta_j \sim \text{LogNormal}(\mu_j, \sigma_j) \tag{12}$$
$$\mu_j \sim \text{Normal}(\mu_0, \tau_0) \tag{13}$$
$$\sigma_j \sim \text{HalfNormal}(\nu_0) \tag{14}$$

where j indexes the marketing channels.

### 12.5. *Model Diagnostics*

#### 12.5.1. *Convergence Assessment*

We employ multiple diagnostics:

- Gelman-Rubin statistic ($\hat{R}$)
- Effective sample size (ESS)
- Monte Carlo standard error (MCSE)

#### 12.5.2. *Predictive Checks*

Model validation includes:

$$\text{ELPD} = \sum_{i=1}^{n} \log \left( \frac{1}{S} \sum_{s=1}^{S} p(y_i|\theta^s) \right) \tag{15}$$

where:

- ELPD is the expected log pointwise predictive density
- $S$ is the number of posterior samples
- $\theta^s$ represents the parameter values in sample $s$