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MSDS 460 Decision Analytics

Linear Programming Problem – Diet Example

Standard Form

Decision Variables:

Let x_1, x_2, x_3, x_4, and x_5, represent the number of servings of Vanilla Protein Shake, Honey Bunches of Oats Cereal, Barilla Protein+ Pasta, Horizon Organic Whole Milk, and Smucker's Chunky Natural Peanut Butter respectively to be consumed over a week's time.

Objective Function:

Minimize the total cost of the food items for a week.
Minimize $C = 3.44x_1 + 0.47x_2 + 0.51x_3 + 0.70x_4 + 0.27x_5$

Nutritional Constraints:

1. Sodium Constraint: $290x_1 + 180x_2 + 0x_3 + 135x_4 + 95x_5 \leq 5000$
2. Energy Constraint: $370x_1 + 170x_2 + 190x_3 + 160x_4 + 190x_5 \geq 2000$
3. Protein Constraint: $20x_1 + 3x_2 + 10x_3 + 8x_4 + 8x_5 \geq 50$
4. Vitamin D Constraint: $3x_1 + 2x_2 + 0x_3 + 2.5x_4 + 0x_5 \geq 20$
5. Calcium Constraint: $390x_1 + 10x_2 + 14x_3 + 300x_4 + 18x_5 \geq 1300$
6. Iron Constraint: $1.5x_1 + 16.2x_2 + 2x_3 + 0x_4 + 1x_5 \geq 18$
7. Potassium Constraint: $650x_1 + 80x_2 + 245x_3 + 410x_4 + 201x_5 \geq 4700$

Non-negativity Constraint:

$x_1, x_2, x_3, x_4, x_5 \geq 0$

Description:

The goal of the model is to find the optimal number of servings of each of the five food items over the course of a week that meets the daily nutritional requirements at the minimum cost. The decision variables represent the servings of each food, and the objective is to minimize the total cost. There is a minimum constraint regarding energy, protein, vitamin d, calcium, iron, and potassium and a maximum constraint regarding sodium. The non-negativity constraint ensures that the number of each food item is not less than 0.

Nutritional Constraint Layout:

Nutritional Component	Max/Min	Daily Amount & Measure	Weekly Amount & Measure
Sodium	Maximum	5000 mg	35000 mg
Energy	Minimum	2000 cal	14000 cal
Protein	Minimum	50 g	350 g
Vitamin D	Minimum	20 mcg	140 mcg
Calcium	Minimum	1300 mg	9100 mg
Iron	Minimum	18 mg	126 mg
Potassium	Minimum	4700 mg	32900 mg

Food Item Nutritional and Cost Breakdown per serving:

	Vanilla Gatorade Nutrition Shake	Honey Bunches of Oats Cereal	Barilla rotini protein+ pasta	Horizon Organic Whole Milk	Smucker's Chunky Natural Peanut Butter
Sodium (mg)	290	180	0	135	95
Energy (cal)	370	170	190	160	190
Protein (g)	20	3	10	8	8
Vitamin D (mcg)	3	2	0	2.5	0
Calcium (mg)	390	10	14	300	18
Iron (mg)	1.5	16.2	2	0	1
Potassium (mg)	650	80	245	410	201
Cost per 1 serving (USD)	3.44	0.47	0.51	0.7	0.27

Solution Description:

The linear programming problem has been solved with the following optimal solution: (week)

Barilla Rotini: 0 servings
Honey Bunches of Oats: 5 servings
Horizon Organic Milk: 52 servings
Smucker's Peanut Butter: 56 servings
Vanilla Gatorade: 0 servings

This solution minimizes the weekly cost to **$53.87**. To meet the nutritional requirements, I would need to consume the specified servings of each food item. Here's a breakdown of the nutritional values for the optimal solution: (week)

Sodium: 8460 mg
Energy: 19690 cal
Protein: 983 g
Vitamin D: 150 mcg
Calcium: 17578 mg
Iron: 185 mg
Potassium: 36100 mg

This solution represents the most cost-effective way to meet the specified nutritional requirements while considering the cost of each food item. The minimum cost solution indicates that purchasing 5 servings of Honey Bunches of Oats, 52 servings of Horizon Organic Milk, and 56 servings of Smucker's Peanut Butter, with no servings of Barilla Rotini and Vanilla Gatorade, would satisfy the constraints at the lowest cost. The total expenditure on food each week to achieve this optimal solution is $53.87.

To comply with the additional request that each food item must be consumed at least 1 time, I must change the lower bound in the linear programming code to 1 for each item. This completely changes the results of the program.

Here's a brief explanation of the modifications:

Optimal Servings:

Barilla Rotini Servings: Increased from 0 to 2.
Honey Bunches of Oats Servings: Unchanged at 5.
Horizon Organic Milk Servings: Decreased from 52 to 51.

Smucker's Peanut Butter Servings: Decreased from 56 to 52.
Vanilla Gatorade Servings: Increased from 0 to 1.

Optimal Weekly Cost:

Increased from **$53.87** to **$56.55.**

Measurement of Each Nutritional Value Under Optimal Solution:

Sodium: Increased from 8460.0 mg to 8595.0 mg per week.
Energy: Increased from 19690.0 cal to 19790.0 cal per week.
Protein: Increased from 983.0 g to 989.0 g per week.
Vitamin D: Decreased from 150.0 mcg to 146.0 mcg per week.
Calcium: Decreased from 17578.0 mg to 17162.0 mg per week.
Iron: Increased from 185.0 mg to 187.0 mg per week.
Potassium: Decreased from 36100.0 mg to 35716.0 mg per week.

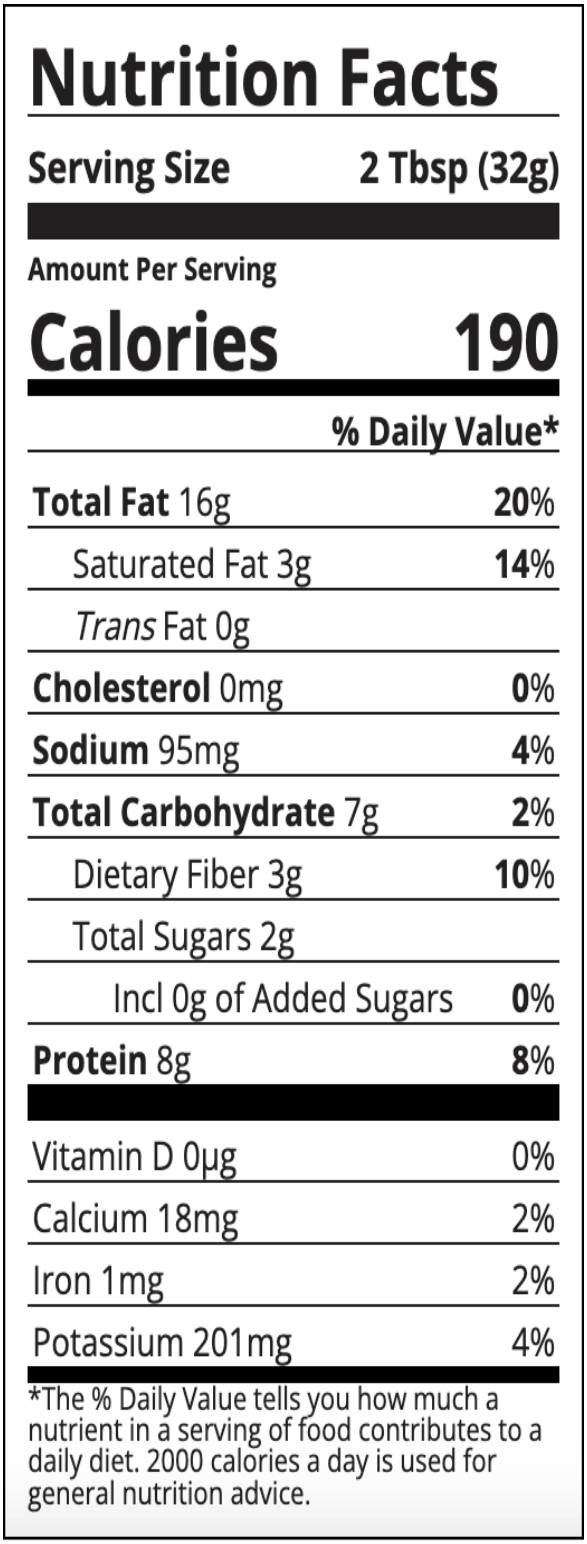
Overall, the changes in the optimal solution indicate adjustments in the servings of each food item to meet the nutritional constraints while minimizing the cost. The increase in the weekly cost suggests a trade-off in the quantities of food items consumed to achieve a less cost-effective solution.

Appendix A

Serving size	1 Cup (240mL)
Amount per serving	
Calories	**160**
	% Daily Value*
Total Fat 8g	**10%**
Saturated Fat 5g	25%
Trans Fat 0g	
Polyunsaturated Fat 0g	
Monounsaturated Fat 2.5g	
Cholestrerol 35mg	**12%**
Sodium 135mg	**6%**
Total Carbohydrate 13g	**5%**
Dietary Fiber 0g	0%
Total Sugars 12g	
Includes 0g Added Sugars	0%
Protein 8g	**16%**
Vitamin D 2.5mcg	**15%**
Calcium 300mg	**25%**
Iron 0mg	**0%**
Potassium 410mg	**8%**
Vitamin A 90mcg	**10%**
Riboflavin 0.4mg	**30%**
Vitamin B12 1.2mcg	**50%**
Phosphorus 240mg	**20%**

($5.58 / 8 servings = .70$ per serving)

Appendix B



Nutrition Facts

Serving Size	**2 Tbsp (32g)**
Amount Per Serving	
Calories	**190**
	% Daily Value*
Total Fat 16g	**20%**
Saturated Fat 3g	**14%**
Trans Fat 0g	
Cholesterol 0mg	**0%**
Sodium 95mg	**4%**
Total Carbohydrate 7g	**2%**
Dietary Fiber 3g	**10%**
Total Sugars 2g	
Incl 0g of Added Sugars	**0%**
Protein 8g	**8%**
Vitamin D 0µg	0%
Calcium 18mg	2%
Iron 1mg	2%
Potassium 201mg	4%

*The % Daily Value tells you how much a nutrient in a serving of food contributes to a daily diet. 2000 calories a day is used for general nutrition advice.

($4.29 / 16 servings = 0.27$ per serving)

Appendix C

Nutrition Facts

About X servings per container

Serving size **1 cup (42g)**

Amount per serving

Calories **170**

	% Daily Value*
Total Fat 3g	**4 %**
Saturated Fat 0g	**0 %**
Trans Fat 0g	
Polyunsaturated Fat 0.5g	
Monounsaturated Fat 1.5g	
Cholesterol 0mg	**0 %**
Sodium 180mg	**8 %**
Total Carbohydrate 34g	**12 %**
Dietary Fiber 2g	**9 %**
Total Sugars 9g	
Incl. 8g Added Sugars	**15 %**
Protein 3g	
Vitamin D 2mcg	10 %
Calcium 10mg	0 %
Iron 16.2mg	90 %
Potassium 80mg	0 %
Vitamin A	40 %
Thiamin	40 %
Riboflavin	35 %
Niacin	40 %
Vitamin B_6	40 %
Folate 400mcg DFE (240mcg folic acid)	100 %
Vitamin B_{12}	100 %
Phosphorus	6 %
Magnesium	6 %
Zinc	10 %

* The % Daily Value (DV) tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.

($8.99 / 19 servings = .47$ per serving)

Appendix D

Nutrition Facts

1 serving per container
Serving Size 11.16 fl oz (330 mL)

Amount Per Serving
Calories 370

	% Daily Value*
Total Fat 8g	**11%**
Saturated Fat 1.5g	**8%**
Trans Fat 0g	
Cholesterol 10mg	**4%**
Sodium 290mg	**13%**
Total Carbohydrate 55g	**20%**
Dietary Fiber <1g	**3%**
Total Sugars 29g	
Includes 28g Added Sugars	**57%**
Protein 20g	**40%**
Vitamin D 3mcg	15%
Calcium 390mg	30%
Iron 1.5mg	8%
Potassium 650mg	15%
Vitamin A 320mcg	35%
Vitamin C 135mg	150%
Vitamin E 6mg	40%
Riboflavin 0.59mg	45%
Vitamin B6 0.34mg	20%
Vitamin B12 2.88mcg	120%
Biotin 63mcg	210%
Pantothenic Acid 1mg	20%
Phosphorus 560mg	45%
Iodine 53mcg	35%
Magnesium 105mg	25%
Zinc 7.7mg	70%
Manganese 0.58mg	25%

*The % Daily Value (DV) tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.

(3.44$ per bottle)

Appendix E

NUTRITION FACTS

NON GMO Project VERIFIED
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Serving size: 2oz. (56g)
Servings per container: about 7

Amount / Serving		(% Daily Value*)
Calories	**190**	
Total Fat	**1g**	(1%)
Saturated Fat	**0g**	(0%)
Trans fat	**0g**	
Cholesterol	**0mg**	(0%)
Sodium	**0mg**	(0%)
Total Carb.	**39g**	(14%)
Dietary Fiber	**4g**	(17%)
Soluble Fiber	**2g**	
Insoluble Fiber	**2g**	
Total Sugars	**2g**	
Protein	**10g**	

Vitamin D 0mcg	0%
Calcium 14mg	2%
Iron 2mg	10%
Potassium 245mg	6%
Thiamin 0.5mg	40%
Riboflavin 0.2mg	15%
Niacin 5mg	30%
Folate 199mcg DFE (112mcg folic acid)	50%

Not a signficant source of added sugars.

*The % Daily Value tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice

($3.57 / 7 servings = .51$ per serving)

Appendices