MATH560 HW3

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Question 1

Describe the null hypotheses to which the p-values given in Table 3.4 correspond. Explain what conclusions you can draw based on these p-values. Your explanation should be phrased in terms of sales, TV, radio, and newspaper, rather than in terms of the coefficients of the linear model.

Answer: The p-values in table 3.4 correspond to the null hypotheses that each variable is equal to zero, or has no effect on the response. Based on the p-values we see for TV and radio, we can conclude that there is statistical evidence at the 0.0001 level of significance that these two variables have an effect on the response variable, and we reject the null hypothesis. For the newspaper variable, we conclude that there is not evidence at any standardly used significance level that this variable has an effect on the response variable and so we fail to reject the null hypothesis. That is, increased expenditure on TV and radio advertising have an effect on total sales, while expenditure on newspaper advertising does not have an effect on total sales.

Question 7

It is claimed in the text that in the case of simple linear regression of Y onto X, the R^2 statistic (3.17) is equal to the square of the correlation between X and Y (3.18). Prove that this is the case. For simplicity, you may assume that $\bar{x} = \bar{y} = 0$.

Answer:
We want to show that in the case of simple linear regression with the above

assumptions,

$$R^2 = r^2 = \frac{\sum (x_i y_i)^2}{\sum x_i^2 \sum y_i^2}.$$

For simple linear regression of Y onto X, setting $\bar{x} = \bar{y} = 0$, we have

$$\hat{y_i} = \hat{\beta}_1 x_i + \hat{\beta}_0.$$

Notice that since $\hat{\beta}_0 = \bar{y}\hat{\beta}_1\bar{x}$, $\hat{\beta}_0 = 0$. We also have

$$\hat{\beta}_1 = \frac{\sum (x_i y_i)}{\sum x_i^2},$$

which follows that

$$\hat{y_i} = \hat{\beta}_1 x_i = \frac{\sum (x_i y_i)}{\sum x_i}.$$

Our assumptions also lead to

$$R^2 = \frac{\sum \hat{}(y_i)^2}{\sum y^2}.$$

Now we plug in the result obtained earlier for y_i into R^2 to obtain

$$\begin{aligned}
R^2 &= \frac{\sum \hat{}(y_i)^2}{\sum y^2} \\
&= \frac{\sum (\hat{\beta}_1 x_i)^2}{\sum y^2} \\
&= \hat{\beta}_1{}^2 \frac{\sum (x_i)^2}{\sum y^2} \\
&= \left(\frac{\sum (x_i y_i)}{\sum x_i^2}\right)^2 \frac{\sum (x_i)^2}{\sum y^2} \\
&= \frac{\sum (x_i y_i)^2}{\sum x_i^2 \sum y_i^2},
\end{aligned}$$

which equals r^2 and satisfies the result we were trying to prove.