Solving Continuous Games With Oracle Algorithms

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Domain

- Adversarial games
- for two players
- zero sum
- action spaces are continuums

Nash Equilibrium

- a stable state
- none of the players have a reason to deviate
- (from the adopted strategy)

Solving NE

- solved for games with finite action spaces
- create matrix M as utilities of every pair of actions
- Linear program:

$maximize\ x_0$

s.t. $M^T\boldsymbol{x} - \mathbf{1}x_0 \geq \mathbf{0}$

$\sum_{j \in A} x_j = 1$

$\boldsymbol{x} \geq 0$

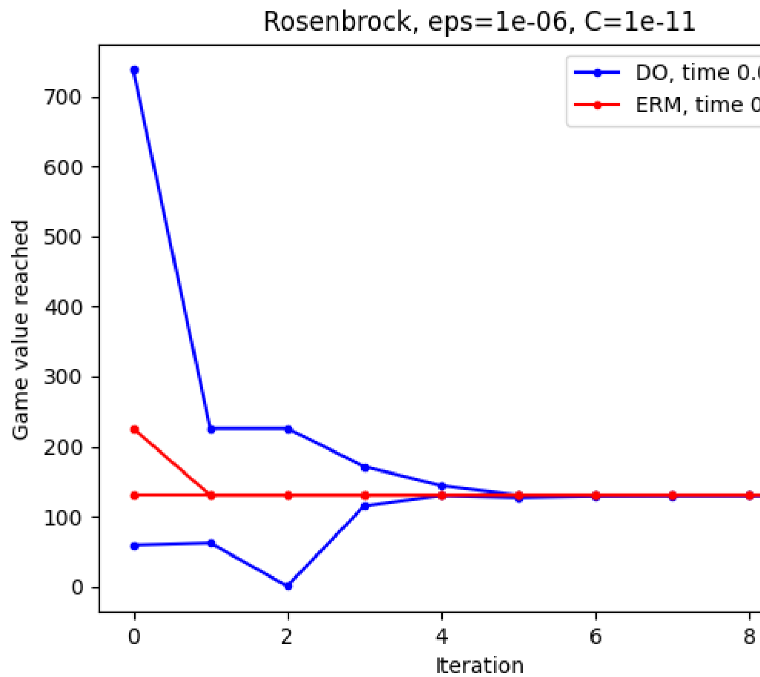
Infinite Games

- LP no longer applicable (cannot enumerate actions)
- $\rightarrow$ What else, then?
- settle for $\epsilon-$NE instead
- iterative algorithms based on *oracles*
- *oracles*:
  - *bestResponse oracle*: picks from the whole continuum
  - *value oracle*: the LP from previous slide, solves a subgame with finite action spaces

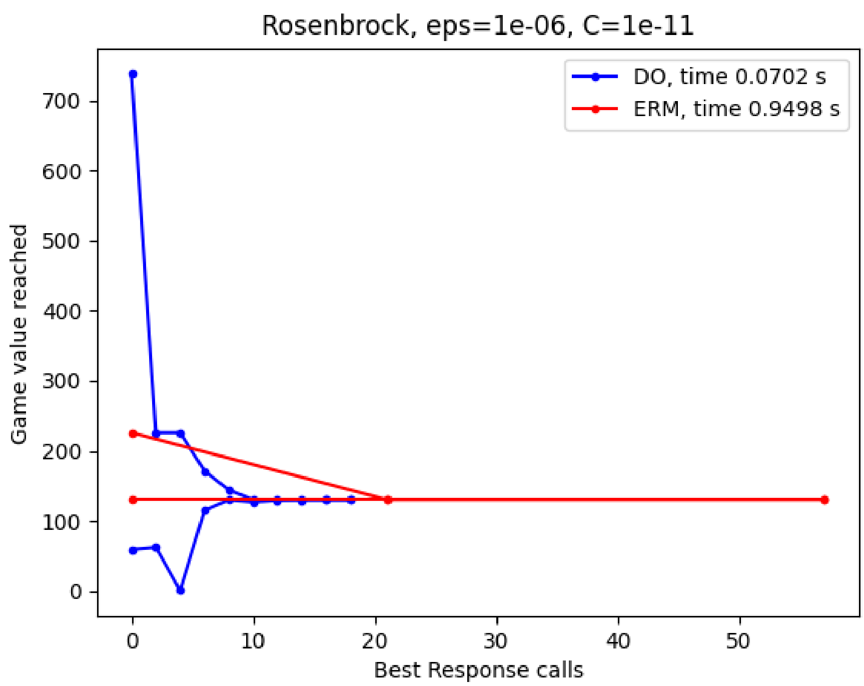
Algorithms

- **Fictitious Play**: the original one. Play *bestResponse* against the average of opponnent' history
- **Double Oracle**: Proven by FEL ČVUT (2021) to converge to ϵ-NE (much faster than FP)
- **Expected Regret Minimization**: Generalization of online learning into two-player setting, proposed 2023

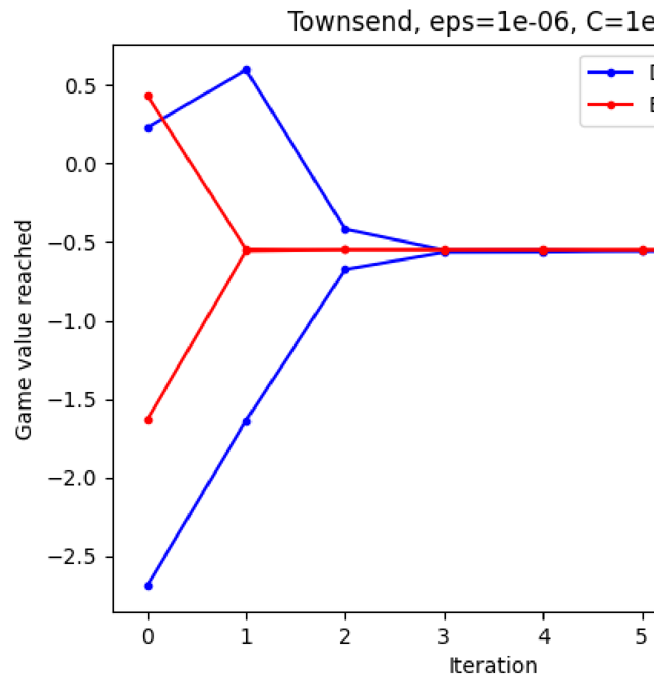
Convergence Comparison



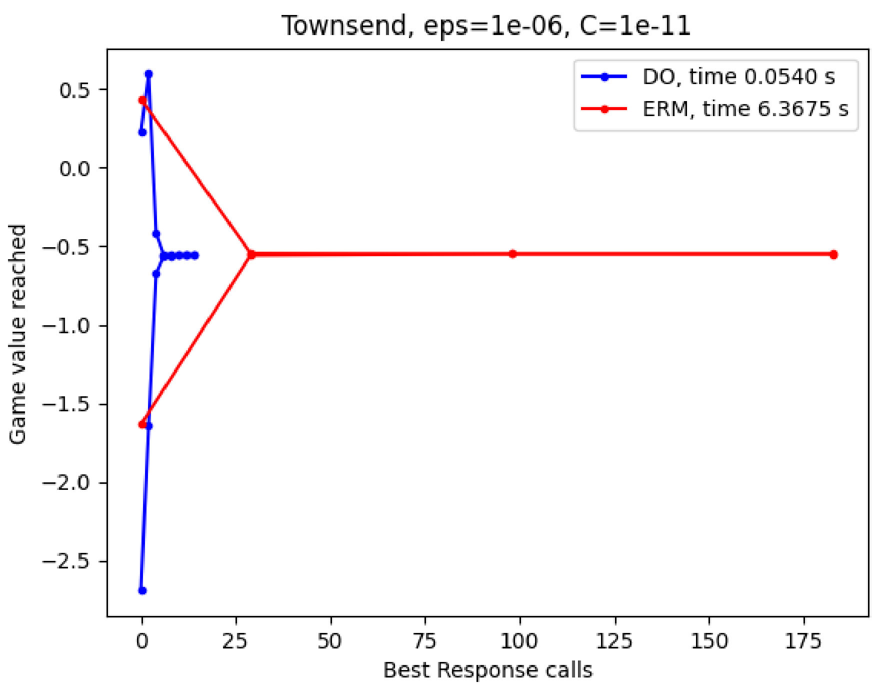
- Rosenbrock: $u(x, y) = (1-x)^2+100(y-x^2)^2$



Convergence Comparison

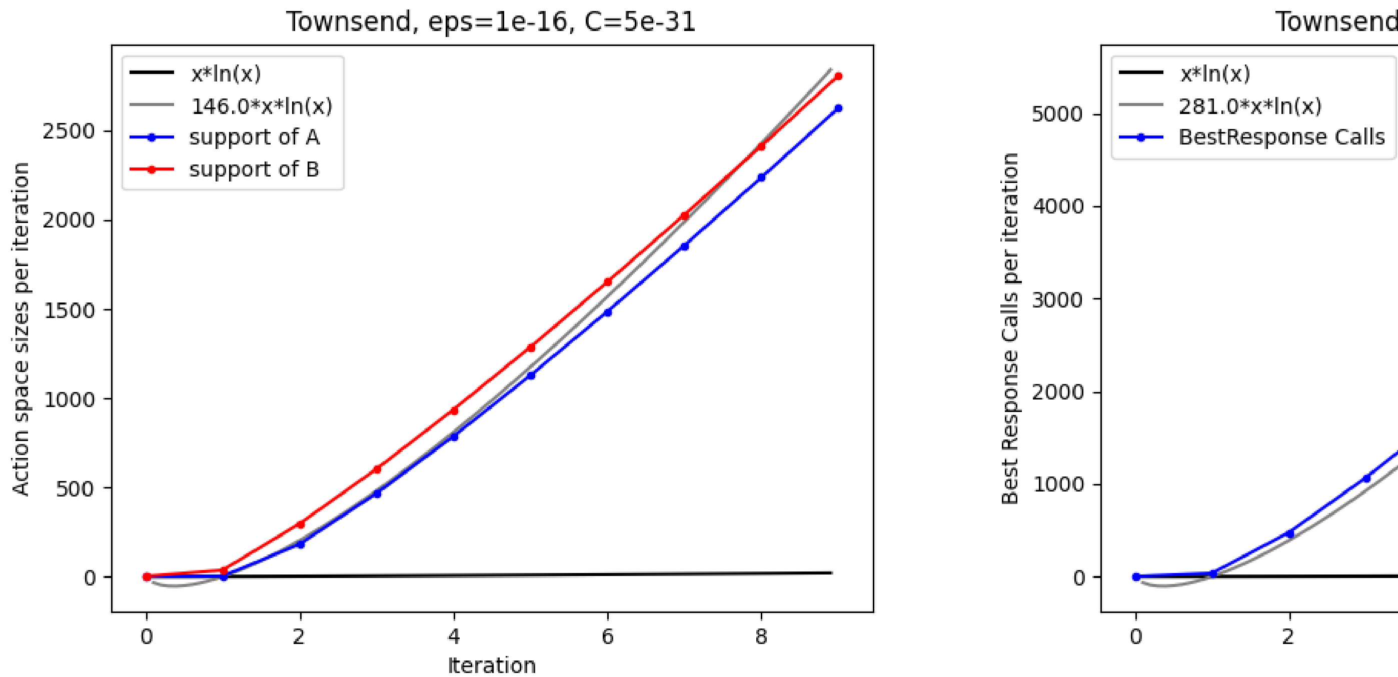


- Townsend: $u(x, y) = -[\cos((x-0.1)y)]^2 - x\sin(3x+y)$



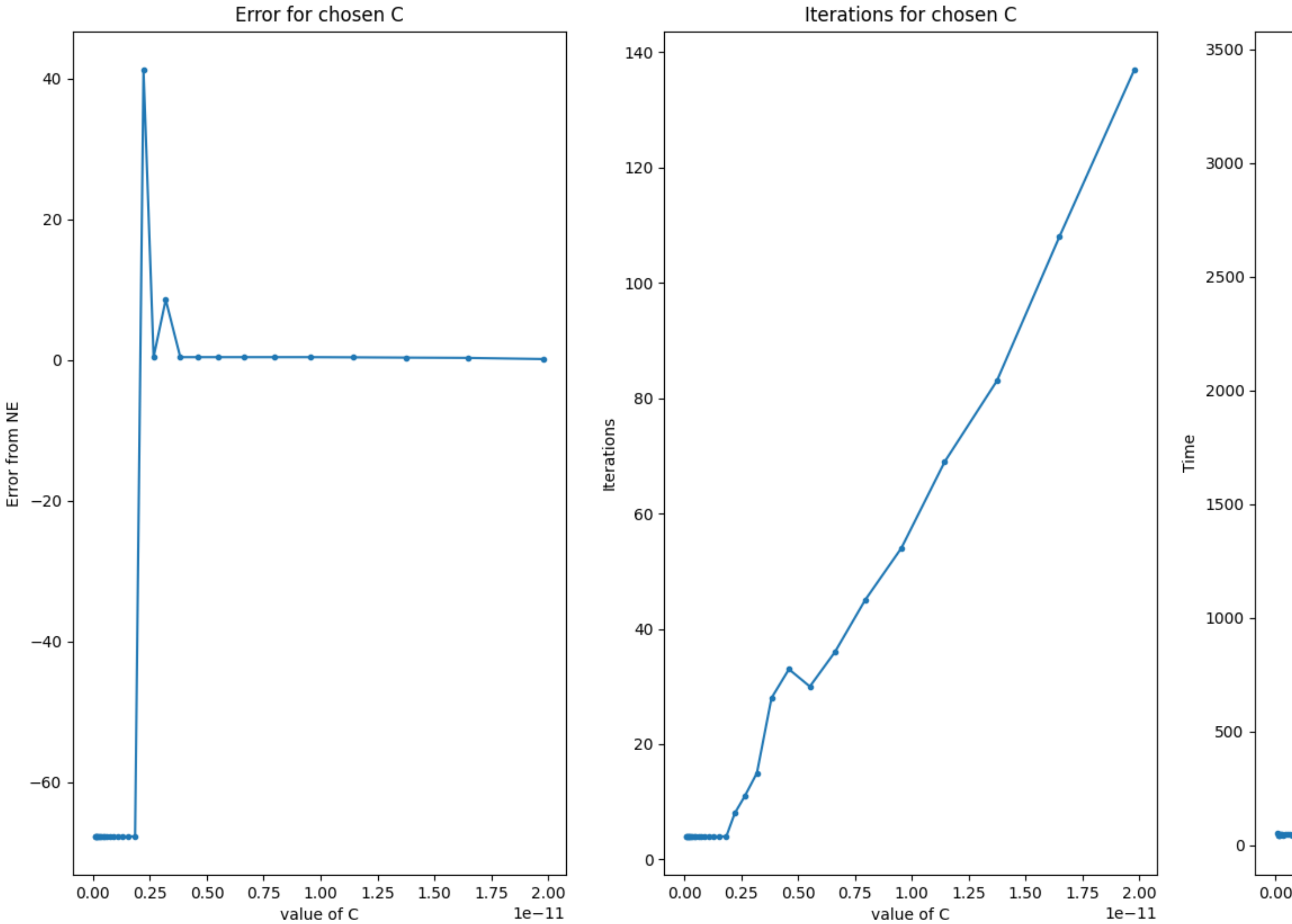
Testing Claims for ERM

Assume that ERM Algorithm runs for T iterations. Then, the number of oracle calls is bounded by $O(T/\epsilon^2 \cdot log(T/\epsilon^2))$.



Problem with the magic constant C

- Tested on the the Rosenbrock function



Error for chosen C
Error from NE
value of C
1e−11
Iterations for chosen C
Iterations
value of C
1e−11
Time