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1. $\begin{cases} x_1 + 3x_2 - x_3 = -1 \\ x_1 - x_2 + 3x_3 = 11 \\ 2x_1 + x_2 + x_3 = 7 \end{cases}$

а)

$$D = \begin{vmatrix} 1 & 3 & -1 \\ 1 & -1 & 3 \\ 2 & 1 & 1 \end{vmatrix} = -1 - 1 + 18 - 2 - 3 - 3 = 8$$

$$D_1 = \begin{vmatrix} -1 & 3 & -1 \\ 11 & -1 & 3 \\ 7 & 1 & 1 \end{vmatrix} = 1 - 11 + 63 - 7 + 3 - 33 = 16$$

$$D_2 = \begin{vmatrix} 1 & -1 & -1 \\ 1 & 11 & 3 \\ 2 & 7 & 1 \end{vmatrix} = 11 - 7 - 6 + 22 - 21 + 1 = 0$$

$$D_3 = \begin{vmatrix} 1 & 3 & -1 \\ 1 & -1 & 11 \\ 2 & 1 & 7 \end{vmatrix} = -7 - 1 + 66 - 2 - 11 - 21 = 24$$

Ответ:
$$x_1 = \frac{D_1}{D} = \frac{16}{8} = 2; \quad x_2 = \frac{0}{8} = 0; \quad x_3 = \frac{24}{8} = 3.$$

б)
$$A = \begin{pmatrix} 1 & 3 & -1 \\ 1 & -1 & 3 \\ 2 & 1 & 1 \end{pmatrix}; \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}; \quad B = \begin{pmatrix} -1 \\ 11 \\ 7 \end{pmatrix}.$$

$$X = A^{-1} \cdot B; \quad A^{-1} = \frac{1}{|A|} \cdot C^T.$$

$$|A| = 8$$

$$A^T = \begin{pmatrix} 1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 1 \end{pmatrix}; \quad C^T = \begin{pmatrix} -4 & -4 & 8 \\ 5 & 3 & -4 \\ 3 & 5 & -4 \end{pmatrix},$$

$a_{11} = (-1)^2 \cdot \begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} = -4$

$a_{12} = (-1)^3 \cdot \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = -4$

$a_{13} = (-1)^4 \cdot \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} = 8$

$a_{21} = (-1)^3 \cdot \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 5$

$a_{22} = (-1)^4 \cdot \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = 3$

$a_{23} = (-1)^5 \cdot \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} = -4$

$a_{31} = (-1)^4 \cdot \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = 3$

$a_{32} = (-1)^5 \cdot \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 5$

$a_{33} = (-1)^6 \cdot \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} = -4$

$$A^{-1} = \frac{1}{8} \cdot \begin{pmatrix} -4 & -4 & 8 \\ 5 & 3 & -4 \\ 3 & 5 & -4 \end{pmatrix} = $$

$$= \begin{pmatrix} -1/2 & -1/2 & 1 \\ 5/8 & 3/8 & -1/2 \\ 3/8 & 5/8 & -1/2 \end{pmatrix}$$

Ответ:
$$X = \begin{pmatrix} -1/2 & -1/2 & 1 \\ 5/8 & 3/8 & -1/2 \\ 3/8 & 5/8 & -1/2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 11 \\ 7 \end{pmatrix} =$$

$$= \begin{pmatrix} 1/2 - 11/2 + 7 \\ -5/8 + 33/8 - 7/2 \\ -3/8 + 55/8 - 7/2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}.$$

Проверка:
$\begin{cases} 2 + 0 - 3 = -1 \\ 2 - 0 + 9 = 11 \\ 4 + 0 + 3 = 7 \end{cases}$

2. $\begin{cases} x_1 + 2x_2 - 3x_3 + x_4 = 2 \\ 2x_1 - x_2 - x_3 - 3x_4 = -1 \\ 3x_1 - x_2 - 2x_3 - 4x_4 = -1 \end{cases}$

$$A|B = \left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 2 \\ 2 & -1 & -1 & -3 & -1 \\ 3 & -1 & -2 & -4 & -1 \end{array}\right) \begin{matrix} (-2)+ \\ (-3)+ \end{matrix} \sim \left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 2 \\ 0 & -5 & 5 & -5 & -5 \\ 0 & -7 & 7 & -7 & -7 \end{array}\right) \begin{matrix} \cdot(-\frac{1}{5}) \\ (-\frac{7}{5})+ \end{matrix} \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right) (-2)+ \sim \left(\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 1 & 1 \end{array}\right), \quad r_A = 2;\ r_{A|B} = 2$$

совместная

$r < n \Rightarrow$ беск. мн-во реш.

$\begin{cases} x_1 - x_3 - x_4 = 0 \\ x_2 - x_3 + x_4 = 1 \end{cases}$, x_1, x_2 – базисные; x_3, x_4 – своб.

Ответ:

$x_1 = x_3 + x_4$

$x_2 = x_3 - x_4 + 1$

$x_3 \in R$

$x_4 \in R$

Проверка: $x_1 = 1 + 2 = 3$

$x_2 = 1 - 2 + 1 = 0$

$\begin{cases} 3 - 1 - 2 = 0 \\ 0 - 1 + 2 = 1 \end{cases}$

3. $f(A) = A^2 + 2A^{-1} + E$; $A = \begin{pmatrix} 4 & 3 \\ 9 & 7 \end{pmatrix}$

$$A^2 = \begin{pmatrix} 4 & 3 \\ 9 & 7 \end{pmatrix} \cdot \begin{pmatrix} 4 & 3 \\ 9 & 7 \end{pmatrix} = \begin{pmatrix} 16+27 & 12+21 \\ 36+63 & 27+49 \end{pmatrix} = \begin{pmatrix} 43 & 33 \\ 99 & 76 \end{pmatrix}.$$

$|A| = 28 - 27 = 1$

$A^T = \begin{pmatrix} 4 & 9 \\ 3 & 7 \end{pmatrix}$; $C^T = \begin{pmatrix} 7 & -3 \\ -9 & 4 \end{pmatrix}$,

$a_{11} = (-1)^2 \cdot 7 = 7$

$a_{12} = (-1)^3 \cdot 3 = -3$

$a_{21} = (-1)^3 \cdot 9 = -9$

$a_{22} = (-1)^4 \cdot 4 = 4$

$$A^{-1} = \frac{1}{1} \cdot \begin{pmatrix} 7 & -3 \\ -9 & 4 \end{pmatrix} = \begin{pmatrix} 7 & -3 \\ -9 & 4 \end{pmatrix}.$$

$$2A^{-1} = \begin{pmatrix} 14 & -6 \\ -18 & 8 \end{pmatrix}.$$

Ответ:

$$f(A) = \begin{pmatrix} 43 & 33 \\ 99 & 76 \end{pmatrix} + \begin{pmatrix} 14 & -6 \\ -18 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 58 & 27 \\ 81 & 85 \end{pmatrix}.$$