Томчук Г. 4326 / Вариант 20

① $\int \frac{dx}{9x^2-2} = \frac{1}{9}\int \frac{dx}{x^2-\left(\sqrt{\frac{2}{9}}\right)^2} = \frac{1}{9}\cdot\frac{1}{2\sqrt{\frac{2}{9}}}\cdot \ln\left(\left|\frac{x-\sqrt{\frac{2}{9}}}{x+\sqrt{\frac{2}{9}}}\right|\right) =$

$= \frac{1}{6\sqrt{2}}\cdot \ln\left(\left|\frac{x-\sqrt{\frac{2}{9}}}{x+\sqrt{\frac{2}{9}}}\right|\right) = \frac{\sqrt{2}}{12}\cdot \ln\left(\left|\frac{x-\frac{\sqrt{2}}{3}}{x+\frac{\sqrt{2}}{3}}\right|\right) =$

$= \frac{\sqrt{2}}{12}\cdot \ln\left(\left|\frac{3x-\sqrt{2}}{3x+\sqrt{2}}\right|\right) + C.$

② $\int \frac{tg^2 x}{\cos^2 x}\,dx = \int tg^2 x \cdot d(tg x) = \frac{tg^3 x}{3} + C.$

③ $\int \arccos 2x \cdot dx = \frac{1}{2}\int \arccos 2x \cdot d(2x) = \left(\begin{matrix} u = \arccos 2x \\ du = -\frac{2}{\sqrt{1-4x^2}}dx \end{matrix} ; \right.$

$\left. \begin{matrix} dv = d(2x) \\ v = 2x \end{matrix} \right) = \frac{1}{2}\left(\arccos 2x \cdot 2x - \int \frac{-4x}{\sqrt{1-4x^2}}dx\right) =$

$= \frac{1}{2}\left(\arccos 2x \cdot 2x - \frac{1}{2}\int \frac{d(1-4x^2)}{\sqrt{1-4x^2}}\right) = \frac{1}{2}\left(\arccos 2x \cdot 2x - \right.$

$\left. - \frac{1}{\cancel{2}}\cdot \cancel{2}\sqrt{1-4x^2}\right) = \arccos 2x \cdot x - \frac{1}{2}\sqrt{1-4x^2} + C.$

④ $\int \frac{2x-1}{\sqrt[3]{(x-1)^2}}dx = \left(\begin{matrix} x-1 = t \\ x = t+1 \\ dx = dt \end{matrix} \right) = \int \frac{2t+1}{t^{\frac{2}{3}}}dt =$

$= \int \frac{2t^{\cancel{x}\,\frac{1}{3}}}{\cancel{t^{\frac{2}{3}}}}dt + \int \frac{dt}{t^{\frac{2}{3}}} = \frac{\cancel{2}t^{\frac{4}{3}}\cdot 3}{\cancel{4}_2} + 3t^{\frac{1}{3}} =$

$= \frac{3\sqrt[3]{(x-1)^4}}{2} + 3\sqrt[3]{x-1} + C.$

⑤ $\int \frac{x}{x^3-1^3}dx = \int \frac{x}{(x-1)(x^2+x+1)}dx = \left(\frac{x}{(x-1)(x^2+x+1)} = \right.$

$= \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1};\quad x = (x^2+x+1)A + (x-1)(Bx+C);$

$$x = Ax^2 + Ax + A + Bx^2 + Cx - Bx - C;$$

$$x = (A+B)x^2 + (A+C-B)x + (A-C);$$

$$\begin{cases} A+B=0 \\ A+C-B=1 \\ A-C=0 \end{cases}; \quad \begin{cases} -2C+B=-1 \\ -2B+C=1 \end{cases}; \quad \left(\begin{cases} B=-\frac{1}{3} \\ C=\frac{1}{3} \\ A=\frac{1}{3} \end{cases}\right) =$$

$$= \int\left(\frac{\frac{1}{3}}{x-1} + \frac{-\frac{1}{3}x+\frac{1}{3}}{x^2+x+1}\right)dx = \int\left(\frac{1}{3(x-1)} + \frac{-x+1}{3(x^2+x+1)}\right)dx =$$

$$= \frac{1}{3}\int\frac{d(x-1)}{x-1} + \frac{1}{3}\int\frac{-\frac{1}{2}(2x+1)+\frac{3}{2}}{x^2+x+1}dx =$$

$$= \frac{1}{3}\int\frac{d(x-1)}{x-1} + \frac{1}{3}\left(-\frac{1}{2}\int\frac{d(x^2+x+1)}{x^2+x+1} + \frac{3}{2}\int\frac{dx}{x^2+x+1}\right) =$$

$$= \cancel{\frac{1}{3}\int\frac{d(x-1)}{}} \frac{1}{3}\ln|x-1| + \frac{1}{3}\left(-\frac{1}{2}\ln(x^2+x+1) + \sqrt{3}\,\cdot\right.$$

$$\left.\cdot\, \operatorname{arctg}\left(\frac{2\sqrt{3}x+\sqrt{3}}{3}\right)\right) = \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln(x^2+x+1) +$$

$$+ \frac{\sqrt{3}}{3}\cdot\operatorname{arctg}\left(\frac{2\sqrt{3}x+\sqrt{3}}{3}\right) + C.$$

(7) $$\int\cos 3x\cdot\cos 5x\,dx = \int\frac{1}{2}\left(\cos(2x)+\cos(8x)\right)dx =$$

$$= \frac{1}{2}\left(\int\cos 2x\,dx + \int\cos 8x\,dx\right) = \frac{1}{2}\left(\frac{1}{2}\cdot\sin 2x +\right.$$

$$\left.+ \frac{1}{8}\sin 8x\right) = \frac{\sin 2x}{4} + \frac{\sin 8x}{16} + C.$$

(6) $$\int\frac{dx}{\sin^2 x-3} = \int\frac{1}{\cos^2 x}\left(-\frac{1}{2\operatorname{tg}^2 x+3}\right)dx =$$

$$= -\int\frac{1}{2\operatorname{tg}^2 x+3}\,d(\operatorname{tg}x) = -\frac{1}{2}\int\frac{d(\operatorname{tg}x)}{\operatorname{tg}^2 x+\frac{3}{2}} =$$

$$= -\frac{1}{2}\cdot\frac{1}{\sqrt{\frac{3}{2}}}\operatorname{arctg}\left(\frac{\operatorname{tg}x}{\sqrt{\frac{3}{2}}}\right) = -\frac{1}{\sqrt{6}}\cdot\operatorname{arctg}\left(\sqrt{\frac{2}{3}}\operatorname{tg}x\right)+C.$$