Boban 12/5

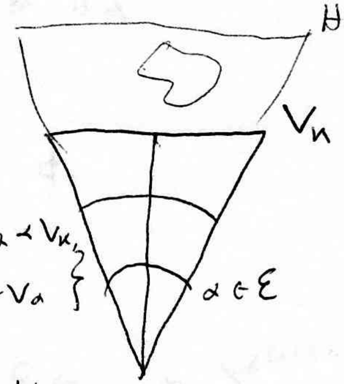
Recall

ϕ infinitary propositional formula in CNF

$\mathbb{H}(\phi)$ p.o. consisting of finite partial assignments

nonempty

Fix inaccessible κ w/ $\phi \in V_\kappa$.



$\mathcal{E} = \{\alpha : V_\alpha \prec V_\kappa,\ \phi \in V_\alpha\}$

$\lambda \in \mathcal{E} \cup \{\kappa\}$

$\mathbb{P}^*_\lambda$ poset of preconditions

$p = (\mathcal{M}_p, w_p)$

$\{M_0 \in M_1 \in \dots \in M_{n-1}\}$

$\lambda_{M_0} < \lambda_{M_1} < \dots < \lambda_{M_{n-1}}$

$C_\alpha = \{M : \text{ctbl}$

$M \models ZFC;\ \alpha \in M \prec \hat{M} \prec V_\alpha\}$

$\text{Hull}(M, V_\alpha) \xrightarrow{\pi} \text{Hull}(M, V_\alpha)$

$\pi[M]$

$M \in C_{\lambda_M}$

$w_p \in \mathbb{H}(\phi)$ is finite partial assignment

$q \leq p$ iff $w_p \subseteq w_q$ and $\forall M \in \mathcal{M}_p\ \exists N \in \mathcal{M}_q$

$\lambda_M = \lambda_N,\ S_M = S_N,\ M \prec N$

$M \cap \omega_1$

$\mathbb{P}_\lambda$. $p \in \mathbb{P}^*_\lambda$

Game $G_\lambda(p)$:

I	Q_n
II	

$p_0 = p$

$\geq$

p_1

$\geq$

p_2

$\vdots$

I. Q_n: $C \in \phi$

II. $p_{n+1} \leq p_n$ ~~s.t.~~ $\exists\, l \in C$ $l \in p_{n+1}$

OR

I Q_n: (M, D) $M \in \mathcal{M}_{p_n}$, $D \in M$ dense in $\mathbb{P}_{\lambda_M}$

II produces $q \in D$ $\text{Hull}(M, \{q\}) \cap \omega_1 = M \cap \omega_1$ $p_{n+1} \leq p_n, q$.

$\mathbb{P}_\lambda = \{p \in \mathbb{P}_\lambda^* : \text{II has w.s. in } G_\lambda(p)\}$

Rmk

$\forall c \in \Phi$ $\{p \in \mathbb{P}_\lambda : \exists \ell \in C, \ell \in u_p\}$ is dense in $\mathbb{P}_\lambda$

(assuming $H(\Phi) \neq \emptyset$)

—

Forcing w/ $\mathbb{P}_\lambda$ adds a satisfying assignment for ϕ.

(Kasum) **AS-consistency**

Given ϕ, ϕ is AS-consistent iff $\forall$ stat $S \subseteq \omega_1$, in $V^{Col(\omega, \omega_1)}$:

$\forall \mu \models \phi$ $\exists j : V \to W$, CRT $j = \omega_1^V \in j(S)$

$\mu \in V^{Col(\omega, \omega_1)}$ $\exists \hat{\mu} \models j(\phi)$ s.t. $j[\mu] \subseteq \hat{\mu}$.

Thm

ϕ AS-consistent $\Rightarrow$ $\mathbb{P}_\kappa$ is SSP.

Def

$\mathbb{P} \in M \prec H_\theta$ (ctbl). $\mathbb{P}$ is semiproper for M if $\forall p \in M$ $\exists q \leq p$ q is $(M, \mathbb{P})$-semigeneric

Fact

$\mathbb{P}$ is SSP iff $\{M \prec H_\theta : \mathbb{P}$ is semiproper for $M\}$ is proj. stat.

i.e. for every $S \subseteq \omega_1$ stat, $\{M \prec : M \cap \omega_1 \in S\}$ is stat.

Def

Suppose $\theta >> \kappa$ regular, $M \prec H_\theta$ ctbl, $\Phi, \kappa, \phi \in M$

M is good if for every $p \in \mathbb{P}_\kappa \cap M$

$\exists q \in \mathbb{P}_\kappa$ $q \leq p$ $\exists \lambda \in E$ $M \restriction \lambda \in M_q$.

$\left(\pi(\kappa) = \lambda, \ \pi : M \to M \restriction \lambda, \ \mathrm{Hull}(M, V_\lambda) \cap V_\kappa = V_\lambda \right)$

Prop

If M is good, then $\mathbb{P}_\kappa$ is semiproper for M.

Rmk

"self-generic" instead of "good"

Sps $\overset{\omega\,\mathbb{P}_\kappa}{r \le q}$ $\exists N \in M_r$ $M\restriction\lambda \subseteq_{\omega_1} N$, $\delta_N = \delta_{M\restriction\lambda}$

Given $D \in M$ and $s \in D$, $\mathrm{Hull}(M, \{s\}) \cap \omega_1 = M \cap \omega_1$

$\exists u \le r, s$

$\pi : M \to M\restriction\lambda$, $\pi(\kappa) = \lambda$, $\pi(D) \in M\restriction\lambda$

I	... $(N, \pi(D))$
II	

$\mathcal{Y} = \{M \prec H_\theta : M \text{ is good}\}$ is a <u>local club</u> in

$X \prec H_\theta$, $|X| = \omega_1$, $\omega_1 \subseteq X$

<u>Claim</u> $\{M : M \prec X \text{ ctbl, not good}\}$ is not stationary coll of T

Sps ow.

X is union of M_ξ, $\xi < \omega_1$ $\nearrow$, $M_\xi \cap \omega_1 = \xi$.

Fix $T \subseteq [X]^\omega$ st $p \in X$ and

$\forall M \in T$ $\nexists q \in \mathbb{P}_\kappa$ $q \le p$ ... $M\restriction\lambda \in M_q$ ...

$\nexists \lambda$

Fix $\lambda \in E$ $\mathrm{Hull}^{H_\theta}(V_\lambda) \cap V_\kappa = V_\lambda$.

Let h be V-generic / $\mathrm{Col}(\omega, \kappa)$.

In $V[h]$, fix g V-generic / $\mathbb{P}_{\dot{x}}$, $p \in g$.

Let $\mu = \mu_g$ be the generic assignment from g.

By AS consistency, $\exists j : V \to W$ (in $V[h]$) w/ $\mathrm{crit}\, j = \omega_1^V$

$\omega_1^V \in j(T)$,

$\exists \hat{\mu} \models j(\phi)$ $\hat{\mu} \supseteq j[\mu]$.

$N = \mathrm{Hull}^{j(H_\theta)}(\omega_1^V \cup \{j(p)\})$

$\prec$

$j(H_\theta)$

$N \in W$, $j(p) \in N$, $N \cap \omega_1^W = \omega_1^V$

$q = (j(\mu_p) \cup \{N \mapsto j(\lambda)\}, j(r_p))$