pinn_burgers_pytorch.py

1 PDE, domain, and conditions

1D viscous Burgers equation

$$u_t + u\,u_x - \nu u_{xx} = 0, \qquad (x,t) \in \Omega := (-1,1) \times (0,1], \tag{1}$$

with viscosity

$$\nu = \frac{0.01}{\pi}. \tag{2}$$

2 What Is the Burgers Equation?

The Burgers equation is a canonical nonlinear PDE used as a minimal model for transport with dissipation. In one space dimension,

$$u_t + u\,u_x = \nu u_{xx}. \tag{3}$$

Its terms have a clear interpretation:

- u_t: time evolution of the field $u(x,t)$.
- $u\,u_x$: nonlinear self-advection (the state transports itself).
- νu_{xx}: diffusion/viscosity that smooths steep gradients.

For $\nu = 0$ (inviscid Burgers), the equation reduces to a nonlinear conservation law that can form shocks from smooth initial data. For $\nu > 0$ (viscous Burgers, used here), diffusion regularizes the solution and smooths shock-like structures. Because it combines nonlinearity and diffusion in a simple form, Burgers is a standard benchmark for numerical PDE methods and PINNs.

Boundary and initial conditions are

$$u(-1,t) = 0, \qquad t \in [0,1], \tag{4}$$

$$u(1,t) = 0, \qquad t \in [0,1], \tag{5}$$

$$u(x,0) = -\sin(\pi x), \qquad x \in [-1,1]. \tag{6}$$