

Homework 3

Alberc Ej Salcedo

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1 Knowledge Representations

P is a Knight and Q is a Knave

Let's say that there are four cases

1. $p = \text{knight}$ and $q = \text{knight}$
2. $p = \text{knave}$ and $q = \text{knight}$
3. $p = \text{knight}$ and $q = \text{knave}$
4. $p = \text{knave}$ and $q = \text{knave}$

Knave always tell a lie.

Knight always tells the Truth.

If p is a knight then what p says has to be true since knights always tell truth. Therefore, case 1 is impossible. Although Case 3 can occur.

If p is a knave then what he says will always be false. making case 2 and 4 impossible since knave always lies.

Thus making case 3 the only possible case.

A is Knave and B is a Knight

1. $a = \text{knight}$ and $b = \text{knight}$
2. $a = \text{knave}$ and $b = \text{knight}$
3. $a = \text{knight}$ and $b = \text{knave}$
4. $a = \text{knave}$ and $b = \text{knave}$

Knave always tell a lie.

Knight always tells the Truth.

If A is a knight then what he says is true, making case 1 and 3 impossible

If B is a knave then what he says is false, making case 4 impossible

Thus, making case 2 the only possible case.

2 Logical Identities

1. $\neg(p \rightarrow (q \rightarrow p))$
 $\neg p \rightarrow \neg(q \rightarrow p)$
 $\neg p \rightarrow (\neg q \rightarrow \neg p)$
2. $\neg((p \wedge q) \rightarrow (q \vee p))$
 $\neg(p \wedge q) \rightarrow \neg(q \vee p)$
 $(\neg p \vee \neg q) \rightarrow (\neg q \wedge \neg p)$

3 Logical Equivalences

p	q	r	$p \rightarrow (q \rightarrow r)$	$(p \wedge q) \rightarrow r$	$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	1	1	1	1

The propositions are equivalent since the final column of the truth table receives a value of one for all cases, making $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$ is always true. Therefore, the pair is equivalent.

p	q	r	$p \rightarrow (q \rightarrow r)$	$(p \rightarrow q) \rightarrow r$	$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \rightarrow q) \rightarrow r)$
0	0	0	1	0	0
0	0	1	1	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	1	1	1	1

The propositions are not equivalent since the final column of the truth table, $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \rightarrow q) \rightarrow r)$, does not receive a value of one for all cases.

4 Logical Consequence

1. This is valid since we have no way of knowing if the conclusion is false or not.
2. This is valid since Puerto Rico is surrounded by water and since all islands are surrounded by water, then Puerto Rico has to be an island.