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# ANALYSIS AND SIMULATION OF A QPSK SYSTEM
# EQ2310 Digital Communications

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| number of authors | | ≤ 2 ☐ | > 2 ☐ | |
| author name(s) and personal id number(s) stated | | yes ☐ | no ☐ | |
| the text is easy to follow and the language correct | yes ☐ | mostly ☐ | sometimes ☐ | no ☐ |
| results and conclusions are clearly stated | yes ☐ | most of them ☐ | only a few ☐ | no ☐ |
| results are correct | yes ☐ | most of them ☐ | only a few ☐ | no ☐ |
| the extent of the work is satisfactory | yes ☐ | almost ☐ | no ☐ | |
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| the mathematics/theory is correct | yes ☐ | mostly ☐ | no ☐ | hard to tell ☐ |
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# EQ2310 Digital Communication
# Analysis and Simulation of a QPSK System

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## 1 Abstract

In this report, the performance parameters of the QPSK system in the channel are studied, including bit error probability, synchronization, carrier phase estimation, eye diagram, spectrum characteristics, etc. The main task completed in this paper is to complete the four MATLAB functions to realize the simulation of the system. In the Problem Areas section, this paper studies the comparison of theoretical and simulated BER performance, and selects four other issues for discussion: signal constellation, power spectral density, eye diagram, and the effect of training sequence length.

## 2 Background and Problem Formulation

The description for the system is as follows: The QPSK system considered in this report include bit source, transmitter, channel, receiver, and a bit sink. The bit source generates a stream of information bits to be sent by a transmitter. Typically, random bit generator is used as bit sources in simulations. This is also the case with the article. Transmitter converts bits to QPSK sign and applies optional pulse shaping and upconversion. The output of transmitter is fed through a channel, which in its simplest form is an AWGN channel. But in some specific examples, multipath channels will also be used, introducing inter-symbol interference (ISI). The receiver block takes the output from the channel and sequentially passes through bandpass filtering, downconversion and sampling, matched filtering, synchronization, sampling and phase estimation on the signal received by the channel. After the receiver estimates the timing and phase offset, it demodulates the received QPSK symbols into information bits, which are fed to the bit receiver. Typically, in a simulation environment, the bit receiver simply counts the number of errors that occur to gather statistics that can be used to investigate the performance of the system.

The first task we need to solve is to write the 4 missing matlab functions:

- d = qpsk(b) A QPSK modulator, mapping pairs of bits $0, 1$ into complex-valued symbols.
- tsamp = sync(mf, b train, Q, t start, t end) Determines when to sample the matched filter outputs.
- phihat = phase estimation(r, b train) Phase estimator operating on the received baseband signal.

- bhat = detect(r) Determines the received bits given a received sequence of (phase-corrected) QPSK symbols. Gray coding of the individual bits is assumed.

The second task we completed was writing MATLAB code that answered the following 5 questions in Problem Area:

- BER performance: Determine the received bits given a received sequence of QPSK symbols.
- Signal constellation: Study the signal constellation in the receiver for various values of $E_b/N_0$. What are the implications from an error in the phase estimate?
- Power density spectrum: Plot the power density spectrum of the transmitted signal. How is it affected if a root raised cosine pulse is chosen instead of a rectangular pulse shape?
- Eye diagram: Plot the eye diagram and study it for both AWGN and two-path ISI channel. Study the ISI's impact on BER.
- Training sequence length: Investigate the performance of phase estimation and synchronization for different lengths of the training sequence.

# 3 Methodology

The general solution to these five problems is as follows.

- BER: Plot and compare the expression we have derived with the estimated BER from simulation.
- Signal constellation: Change the size of $E_b/N_0$ and observe the changes in the constellation diagram.
- Power density spectrum: Because the generated QPSK symbols need to pass through an optional pulse shaping filter, the root raised cosine pulse and the rectangular pulse are selected respectively for comparison.
- Eye diagram: It is an oscilloscope display that uses for showing the effects of channel noise. Compare the eye diagrams of AWGN channel and two path ISI channel when SNR is 10dB.
- Training sequence length: Change the length of the training bits and study how the synchronization error and phase estimation error change accordingly.

# 4 Results

In order to investigate the properties of the QPSK system, some missing functions need to be filled in. Besides, some additional functions have to be completed, such as plotting the BER, plotting the eye diagram, etc. We have completed these tasks before solving the problem. The specific code can be found in appendix.

## 4.1 BER Performance

The exact expression for the BER of the QPSK system is

$$BER = Q(\sqrt{\frac{2E_b}{N_0}}) \tag{1}$$

The data which pulse shape is rectangular are transmitted through an AWGN channel, and then phase estimation and synchronization are used. Finally we obtain the data bits at the receiver and calculate the difference between the original data and the received data as BER of simulation. We use Equation 1 to calculate the BER in theory and compare it with the BER in simulation when SNR from 0 dB to 10 dB. The results are shown in Figure 1. It can be seen that the curve of BER in simulation and in theory match very well. The BER will decrease as SNR increases. This trend means that the increased SNR helps to reduce errors and stabilize the system.

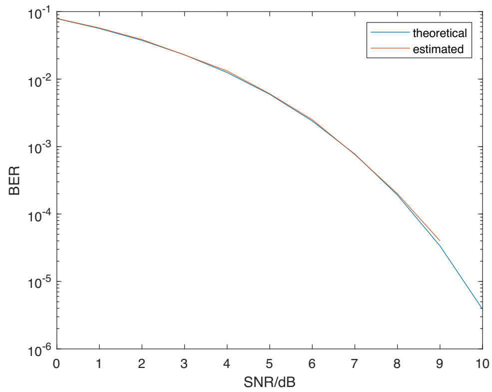



Figure 1: Simulated and Theoretical Result of BER Performance

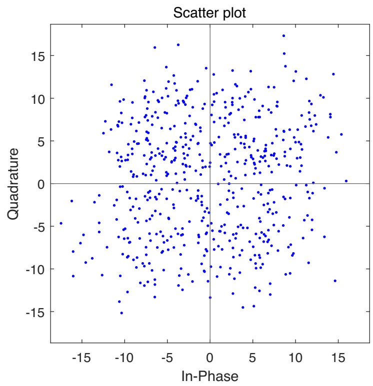
## 4.2 Signal Constellation

The data at different SNR are mapped into QPSK constellation plots. The results are shown in Figure 2. We select 4 constellation plots at different SNR so that the effect of SNR can be seen more clearly. Ideally, the QPSK constellation will be concentrated in the specific four points. However, noise will cause these constellation to deviate from their center. The lower the SNR, the more scattered the constellation will be. In the plot with SNR = 0, it can be seen that the noise causes the constellations to be very scattered. There are many points near the boundaries of the distinguishing symbols, which are very easy to cause detection errors.

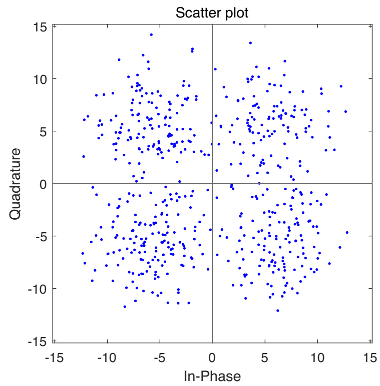
However, noise is only one of the reasons of detection errors. The channel and various filter in receiver will cause unknown phase offset which will also increase the probability of detection errors. But an error in phase estimation can lead to the same consequences. Therefore, it is necessary to estimate the offset correctly in order to demodulate the signal successfully.

## 4.3 Power Density Spectrum

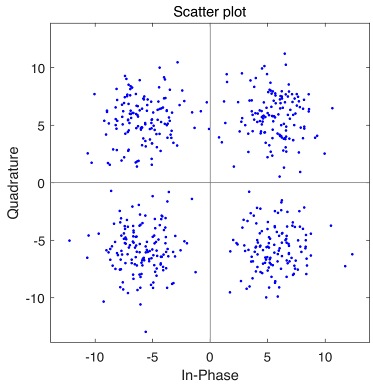
Using the MATLAB function **pwelch**, the power density spectrum with rectangular and root raise cosine (RRC) pulse shape are shown in Figure 3. RRC can be used to eliminate inter-symbol interference. It can be seen that when RRC is adopted, the high frequency component present in rectangular pulse conditions is filtered out and the transmission bandwidth is narrowed.



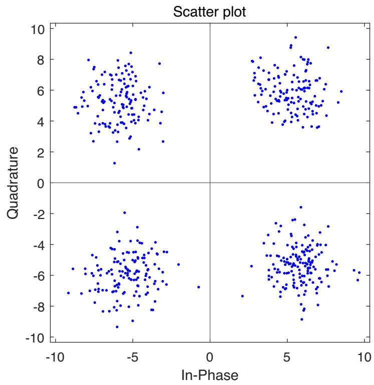

(a) $E_b/N_0 = 0dB$




(b) $E_b/N_0 = 3dB$

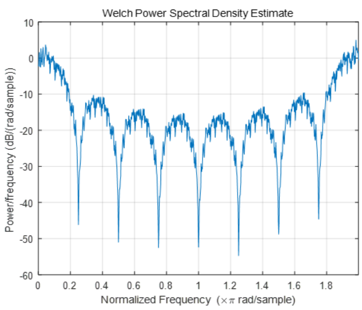



(c) $E_b/N_0 = 6dB$

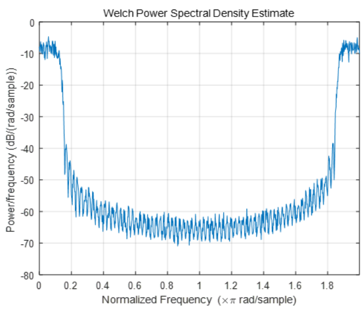



(d) $E_b/N_0 = 10dB$

Figure 2: Signal Constellation under Different $E_b/N_0$




(a) Rectangular Pulse Shape




(b) Root Raise Cosine Pulse Shape

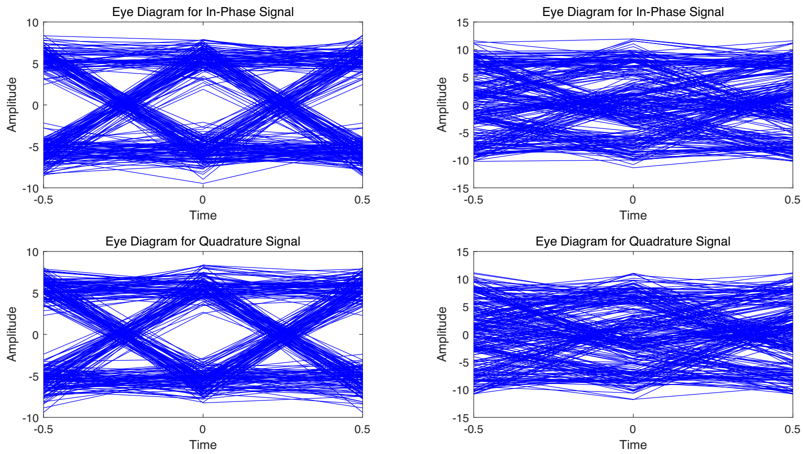
Figure 3: Power Density Spectrum with Different Pulse Shapes

## 4.4 Eye Diagram

Figure 4 shows the eye diagrams of AWGN channel and two-path ISI channel. It can be seen clearly that ISI caused by multipath distorts the eye diagram.

Figure 5, Figure 6, Figure 7 reflect how ISI affects BER performance, phase estimation and synchronization accuracy respectively. We can tell that, apart from the poor eye diagram result, ISI also leads to worse BER performance and decreased phase estimation and synchronization accuracy compared to the AWGN channel.

It's clear that BER of two-path ISI channel is much larger than that of AWGN channel, especially when SNR is high. Besides, even if phase estimation error occurs in AWGN channel, it is smaller than that in two-path ISI channel. Moreover, when we set the size of training sequence as 100, both of their synchronization errors are small. But when we decrease the size to 10, it's clear that accuracy in two-path ISI channel is worse. This can be explained in subsection 4.5.




(a) AWGN channel (b) Two-path ISI channel

Figure 4: Eye diagram of two channels

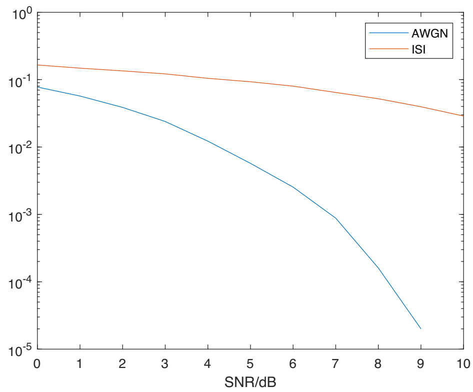



Figure 5: BER of two channels

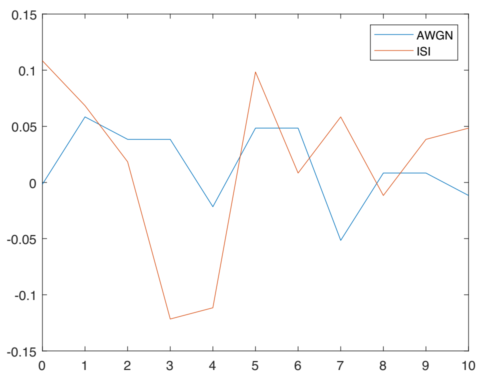
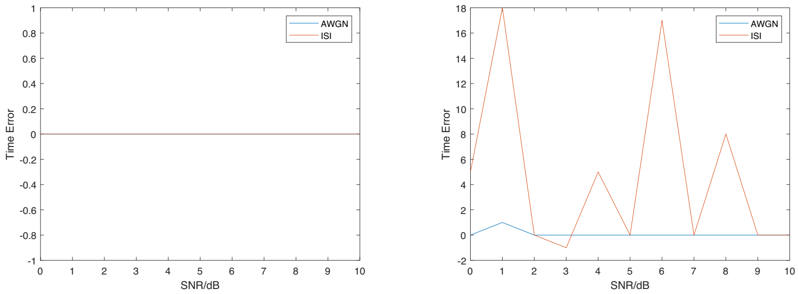



Figure 6: Phase error of two channels




(a) nr_training_bits = 100 (b) nr_training_bits = 10

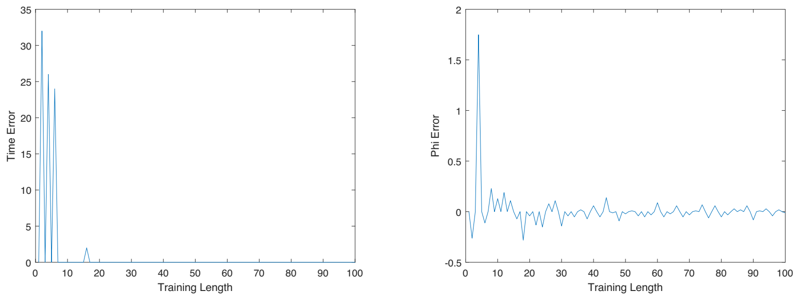
Figure 7: Synchronization error of two channels

### 4.5 Training Sequence Length

We fix the $E_b/N_0$ as 5dB and set the training sequence length from 2 to 100 to obtain the estimation error of each length. The result in Figure 8 shows how training sequence length influences phase estimation and synchronization performance.

For one thing, Figure 8(a) shows that the synchronization error is relatively great when the training length is less than 16 while it is almost 0 when the training length becomes larger. For another, Figure 4.5 shows that the phase estimation error is relatively significant when the training length is less than 18 while it is reduced when the training length becomes larger.

Then, when we set $E_b/N_0$ from 0 to 20dB, the result basically corresponds to what we found above. So, we can conclude that a training sequence length of 20 can achieve comparatively good phase estimation and synchronization, and the following sequence may be “useless”.




(a) Synchronization error (b) Phase error

Figure 8: Error under different training sequence length

## 5 Conclusions

In this project [1], we realized the simulation of a QPSK system. We first investigated the BER performance as a function of $E_b/N_0$ with perfect phase estimation and synchronization and found the simulations results totally agreed with the theoretical results. Then, we drew the signal constellation, power density spectrum and eye diagram under different circumstances. And we also quantitatively analyzed the impact of training sequence length and different types of channels on phase estimation and synchronization. In short, we enhanced our knowledge of communication theory for further research.

## Appendix

The missing functions in matlab are as follows:

```
function d = qpsk(b)
d = zeros(1, length(b)/2);
for n = 1:length(b)/2
odd = b(2*n);
even = b(2*n-1);
if (even == 0) && (odd == 0)
d(n) = exp(1i*pi/4);   % This denotes 45'
end
if (even == 1) && (odd == 0)
d(n) = exp(1i*3*pi/4); % This denotes 135'
end
if (even == 1) && (odd == 1)
d(n) = exp(1i*5*pi/4); % This denotes 225'
end
if (even == 0) && ( odd== 1)
d(n) = exp(1i*7*pi/4); % This denotes 315'
end
end
end

function t_samp = sync(mf, b_train, Q, t_start, t_end)
time=t_end-t_start;
r=zeros(1,time+1);
qpsk_train=qpsk(b_train);
len=length(qpsk_train);
for i=t_start:t_end
a=mf(i+((0:len-1).*Q));
r(i-t_start+1)=abs(a*qpsk_train');
end
[M,I]=max(r);
t_samp=I+t_start-1;
end

function bhat = detect(r)
N=length(r);
bhat=zeros(1,2*N);
for i=1:N
if(real(r(i))>=0)
bhat(2*i-1)=0;
end
if(real(r(i))<0)
bhat(2*i-1)=1;
end
if(imag(r(i))>=0)
bhat(2*i)=0;
end
if(imag(r(i))<0)
bhat(2*i)=1;
end
end
end
```

```
function phihat = phase_estimation(r, b_train)
phihat = 0;
qpsk_train=qpsk(b_train);
len=length(qpsk_train);
r_train=r(1:len);
e = norm(r_train-qpsk_train);
for phi=-pi:0.01:pi
r_phi=r_train*exp(-1j*phi);
delta=norm(r_phi-qpsk_train);
if delta<e
e=delta;
phihat=phi;
end
end
end
```

## References


[1] Upamanyu Madhow. *Fundamentals of Digital Communication.* Cambridge University Press, 2008.