Tutorial 8 – CSCA67

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0.0.1 2.1 For $n \geq 1, n^3 - n$ is divisible by 3

Base (n = 1): $1^3 - 1 = 0$, which is divisible by 3. Assume $k^3 - k = 3m$ for some integer m.

$$\begin{aligned}(k+1)^3 - (k+1) &= (k^3 = 3k^3 + 3k + 1) - (k+1) \\ &= (k^3 - k) + 3k^2 + 3k \\ &= 3m + 3(k^2 + k) \\ &= 3(m + k^2 + k)\end{aligned}$$

Which is divisible by 3, and therefore true by induction.