MATH 449 HW 1

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Note: All fractional values are accurate, but will be rounded to the hundreth's place in order to make monetary sense. Unrounded values will be used for calculations and put into lattice.

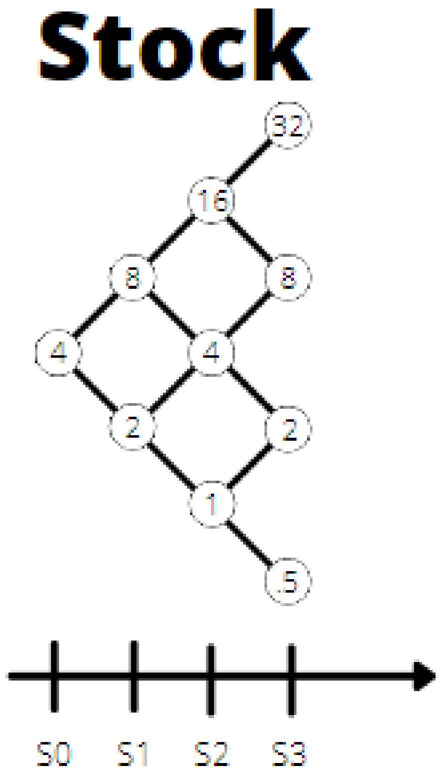
1 European Option

1.1 Dynamic Hedging Strategy for European Option

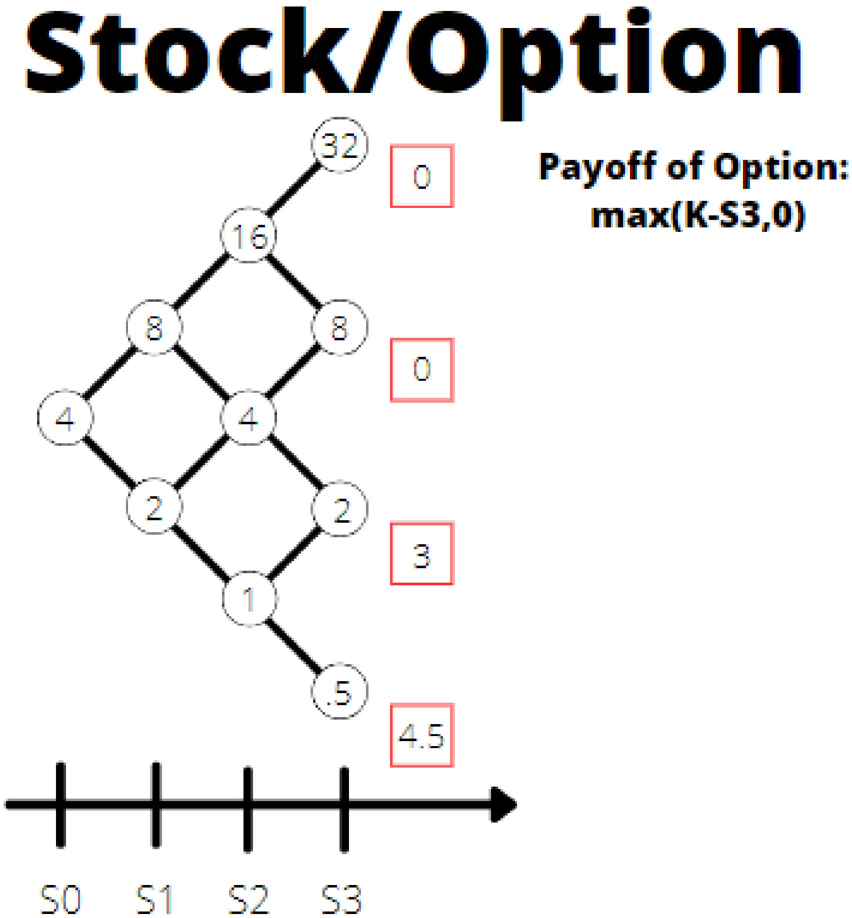
The risk-neutral probability is known to be:

$$\tilde{p} = \frac{R-d}{u-d} = \frac{1.25-.5}{2-.5} = \frac{1}{2}$$

We represent its inverse as $\hat{p} = 1 - \tilde{p} = 1/2$ We can demonstrate the lattice of the stock as follows:



Using the payoff of option calculation for puts ($\max(K - S_3, 0)$), we can begin to fill in our stock/option graph.



For reference, the equations were as follows, with N1 representing the topmost node of S3, and N4 the bottom node.
For N1: max(4-32,0) = 0
For N2: max(4-8, 0) = 0
For N3: max(4-2, 0) = 0
For N4: max(4-.5,0) = 0

We now will do the European pricing calculations to complete the graph.

$$P_2(uu) = \frac{1}{R}[\tilde{p} * P_3(uuu) + \hat{p} * P_3(uud)] = \frac{4}{5}[\frac{1}{2}(0) + \frac{1}{2}(0)] = 0.00$$

$$P_2(ud) = \frac{1}{R}[\tilde{p}*P_3(udu)+\hat{p}*P_3(udd)] = \frac{4}{5}[\frac{1}{2}(0)+\frac{1}{2}(3)] = \frac{3}{2}*\frac{4}{5} = \frac{6}{5} = 1.20$$

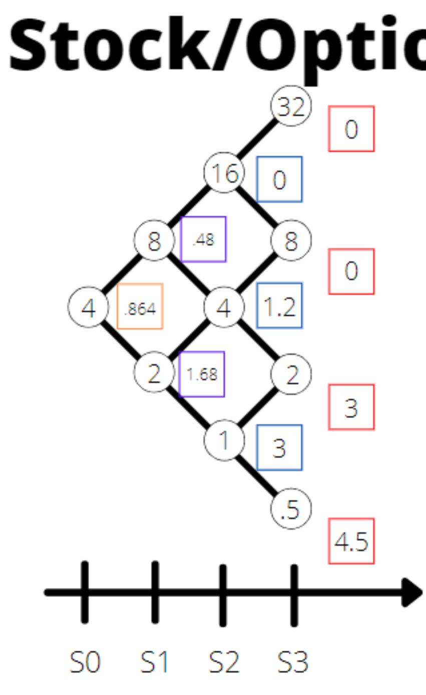
$$P_2(dd) = \frac{1}{R}[\tilde{p}*P_3(ddu)+\hat{p}*P_3(ddd)] = \frac{4}{5}[\frac{1}{2}(3)+\frac{1}{2}(4.5)] = \frac{4}{5}*(\frac{3}{2}+\frac{9}{4}) = \frac{4}{5}*(\frac{15}{4}) = 3.00$$

$$P_1(u) = \frac{1}{R}[\tilde{p}*P_2(uu)+\hat{p}*P_3(ud)] = \frac{4}{5}[\frac{1}{2}(0)+\frac{1}{2}(\frac{6}{5})] = \frac{4}{5}*\frac{3}{5} = \frac{12}{25} = 0.48$$

$$P_1(d) = \frac{1}{R}[\tilde{p}*P_2(du))+\hat{p}*P_2(dd))] = \frac{4}{5}[\frac{1}{2}(\frac{6}{5})+\frac{1}{2}(3)] = \frac{4}{5}*(\frac{3}{5})+\frac{3}{2}) = \frac{4}{5}*\frac{21}{10} = \frac{42}{25} = 1.68$$

$$P_0 = \frac{1}{R}[\tilde{p}*P_1(u))+\hat{p}*P_1(d)] = \frac{4}{5}[\frac{1}{2}(\frac{12}{25})+\frac{1}{2}(\frac{42}{25})] = \frac{4}{5}*(\frac{6}{25}+\frac{21}{25}) = \frac{4}{5}*\frac{27}{25} = \frac{108}{125} = .86$$

Now that we have the pricing calculations, we can fill in the rest of the graph.



We now know the no-arbitrage price is $\frac{108}{125} = \$0.86$.

1.2 Portfolio Values for European Option

In order to find the stock and bond prices of these, we must do the following calculations. The variable x represents the stock, and the variable y represents the bond. We also know that $u - d = 2 - \frac{1}{2} = \frac{3}{2}$ and $R(u-d) = \frac{5}{4} * \frac{3}{2} = \frac{15}{8} = 1.875$

For P_0:

$$x = \frac{P_1(u) - P_1(d)}{u-d} = \frac{\frac{12}{25} - \frac{42}{25}}{\frac{3}{2}} = \frac{-30}{25} * \frac{2}{3} = -\frac{20}{25} = -0.80$$

$$y = \frac{-d * P_1(u) + u * P_1(d)}{R(u-d)} = \frac{-\frac{1}{2} * \frac{12}{25} + 2 * \frac{42}{25}}{\frac{15}{8}} = \frac{-\frac{6}{25} + \frac{84}{25}}{\frac{15}{8}} = \frac{78}{25} * \frac{8}{15} = \frac{624}{375} = 1.66$$

For $P_1(u)$:

$$x = \frac{P_2(uu) - P_2(ud)}{u-d} = \frac{0 - \frac{6}{5}}{\frac{3}{2}} = -\frac{6}{5} * \frac{2}{3} = -\frac{4}{5} = -0.80$$

$$y = \frac{-d * P_2(uu) + u * P_2(ud)}{R(u-d)} = \frac{-\frac{1}{2} * 0 + 2 * \frac{6}{5}}{\frac{15}{8}} = \frac{\frac{12}{5}}{\frac{15}{8}} = \frac{12}{5} * \frac{8}{15} = \frac{32}{25} = 1.28$$

For $P_1(d)$:

$$x = \frac{P_2(du) - P_2(dd)}{u-d} = \frac{\frac{6}{5} - 3}{\frac{3}{2}} = -\frac{9}{5} * \frac{2}{3} = -\frac{6}{5} = -1.20$$

$$y = \frac{-d * P_2(du) + u * P_2(dd)}{R(u-d)} = \frac{-\frac{1}{2} * \frac{6}{5} + 2 * 3}{\frac{15}{8}} = \frac{-\frac{3}{5} + 6}{\frac{15}{8}} = \frac{27}{5} * \frac{8}{15} = \frac{216}{75} = 2.88$$

For $P_2(uu)$:

$$x = \frac{P_3(uuu) - P_3(uud)}{u-d} = \frac{0}{\frac{3}{2}} = 0$$

$$y = \frac{-d * P_3(uuu) + u * P_3(uud)}{R(u-d)} = \frac{0}{\frac{15}{8}} = 0$$

For $P_2(ud)$:

$$x = \frac{P_3(udu) - P_2(udd)}{u - d} = -3 * \frac{2}{3} = -2.00$$

$$y = \frac{-d * P_3(udu) + u * P_3(udd)}{R(u-d)} = \frac{-\frac{1}{2} * 0 + 2 * 3}{\frac{15}{8}} = 6 * \frac{8}{15} = \frac{48}{15} = 3.20$$

For $P_2(dd)$:

$$x = \frac{P_3(ddu) - P_2(ddd)}{u - d} = (3 - \frac{9}{2}) * \frac{2}{3} = -1.00$$

$$y = \frac{-d * P_3(ddu) + u * P_3(ddd)}{R(u-d)} = \frac{-\frac{1}{2} * 3 + 2 * \frac{9}{2}}{\frac{15}{8}} = \frac{15}{2} * \frac{8}{15} = \frac{8}{2} = 4.00$$

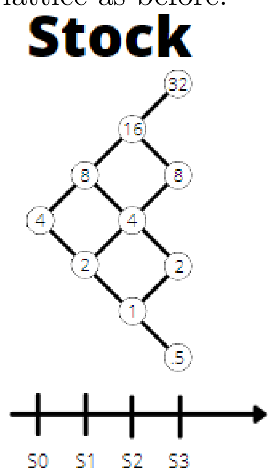
1.3 Riskless Profit in European Option

For a put on a unit of 100 shares of stock, there are two ways to make riskless profit. If the put is priced above the no-arbitrage price ($0.864), then we need to short the put and collect the premium. Now we'll use the premium to long the dynamic hedging portfolio, and thus we wouldn't gain or lose anything in the positions no matter where the market goes. So our riskless profit would be our option premium (which is greater than $86.40) minus $86.40. Similarly, if the put is priced below the no-arbitrage price, we long the put and short the dynamic hedging portfolio, collecting the premium to finance the long put position. We wouldn't gain or lose anything no matter what happens to the market here either. So our riskless profit would be $86.40 minus our option premium (which is less than $86.40).

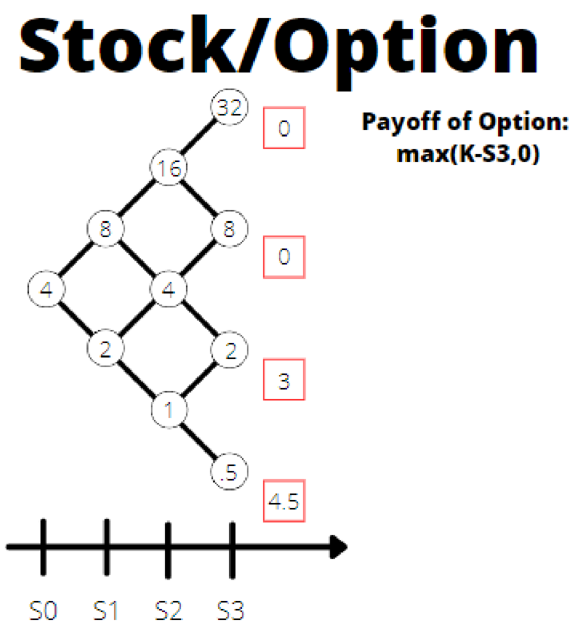
2 American Options

2.1 Finding the Dynamic Hedging Strategy

Under the same settings as problem 1, we will have the same stock lattice as before.



This will also initially result in the same stock/option lattice as earlier.



Now, however, we have a new calculation to determine the option payoff. We now must calculate two things at every node: A. The value of the put using the discounted risk-neutral formula and B. The value that would be obtained by immediate exercise of the put. Then we select the maximum of the two options.
The risk-neutral probability is known to be:

$$\tilde{p} = \frac{R-d}{u-d} = \frac{1.25-.5}{2-.5} = \frac{1}{2}$$

We represent its inverse as $\hat{p} = 1 - \tilde{p} = 1/2$
For $V_2(uu)$ our choice is 0:

(a)
$$\frac{1}{R}[\tilde{p} * P_3(uuu) + \hat{p} * P_3(uud)] = \frac{4}{5}[\frac{1}{2}(0) + \frac{1}{2}(0)] = 0.00$$

(b)
$$5 - 16 = -11 = 0.00$$

No payoff

For $V_2(ud)$ our choice is 1.20:

(a)
$$\frac{1}{R}[\tilde{p} * P_3(udu) + \hat{p} * P_3(udd)] = \frac{4}{5}[\frac{1}{2}(0) + \frac{1}{2}(3)] = \frac{3}{2} * \frac{4}{5} = \frac{6}{5} = 1.20$$

(b)
$$5 - 4 = 1.00$$

For $V_2(dd)$ our choice is 4.00:

(a)
$$\frac{1}{R}[\tilde{p} * P_3(ddu) + \hat{p} * P_3(ddd)] = \frac{4}{5}[\frac{1}{2}(3) + \frac{1}{2}(4.5)] = \frac{4}{5} * (\frac{3}{2} + \frac{9}{4}) = \frac{4}{5} * (\frac{15}{4}) = 3.00$$

(b)
$$5 - 1 = 4.00$$

For $V_1(u)$ our choice is 0.48:

(a)

$$\frac{1}{R}[\tilde{p} * P_2(uu) + \hat{p} * P_3(ud)] = \frac{4}{5}[\frac{1}{2}(0) + \frac{1}{2}(\frac{6}{5})] = \frac{4}{5} * \frac{3}{5} = \frac{12}{25} = 0.48$$

(b)

$$5 - 8 = -3 = 0.00$$

No payoff

For $V_1(d)$ our choice is 3.00:

(a)

$$\frac{1}{R}[\tilde{p}*P_2(du))+\hat{p}*P_2(dd))] = \frac{4}{5}[\frac{1}{2}*\frac{6}{5}+\frac{1}{2}*4] = \frac{4}{5}*[\frac{3}{5}+2] = \frac{4}{5}*\frac{13}{5} = \frac{52}{25} = 2.08$$

(b)

$$5 - 2 = 3.00$$

For V_0 our choice is 1.39:

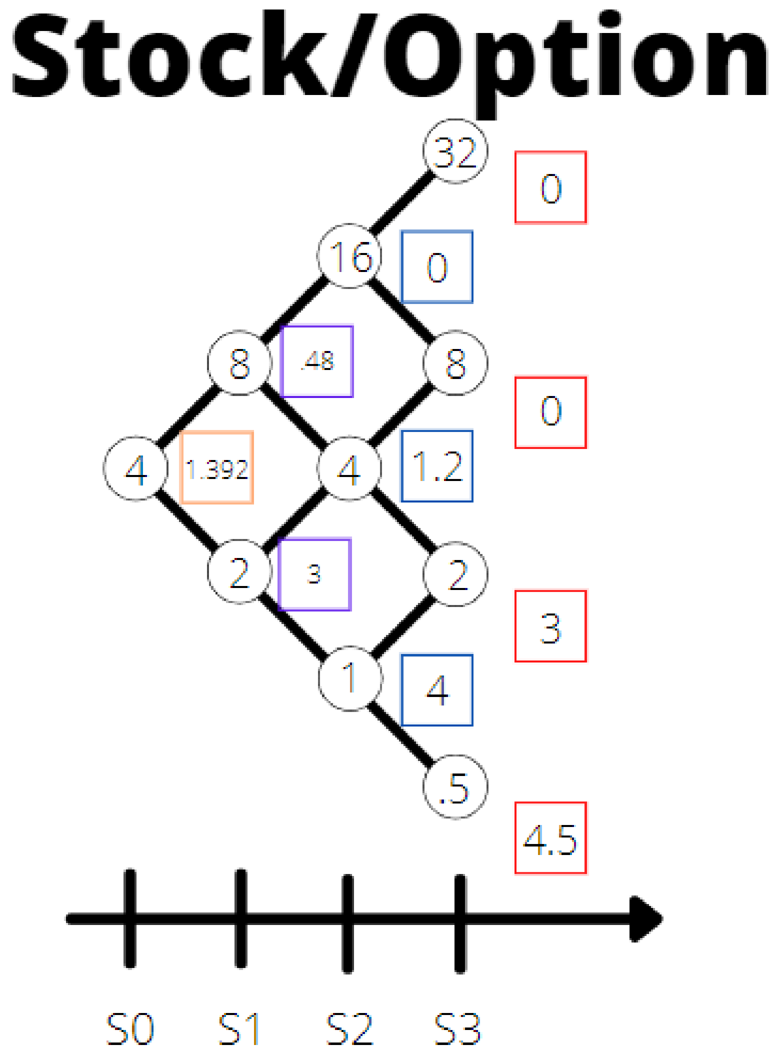
(a)

$$\frac{1}{R}[\tilde{p}*P_1(u))+\hat{p}*P_1(d)] = \frac{4}{5}[\frac{1}{2}*\frac{12}{25}+\frac{1}{2}*3] = \frac{4}{5}*[\frac{6}{25}+\frac{3}{2}] = \frac{4}{5}*\frac{87}{50} = \frac{348}{250} = 1.39$$

(b)

$$5 - 4 = 1.00$$

We now know that our no-arbitrage price for the option at time 0 is \$1.39. Our new stock/option graph now looks like this:



2.2 Portfolio Values for American Option

Now we calculate the portfolio values, where again, variable x represents the stock, the variable y represents the bond, $u - d = 2 - \frac{1}{2} = \frac{3}{2}$, and $R(u-d) = \frac{5}{4} * \frac{3}{2} = \frac{15}{8} = 1.875$
For V_0:

$$x = \frac{V_1(u) - V_1(d)}{u-d} = \frac{\frac{12}{25} - 3}{\frac{3}{2}} = -\frac{63}{25} * \frac{2}{3} = -\frac{42}{25} = -1.68$$

$$y = \frac{-d * V_1(u) + u * V_1(d)}{R(u-d)} = \frac{-\frac{1}{2} * \frac{12}{25} + 2 * 3}{\frac{15}{8}} = \frac{-\frac{6}{25} + 6}{\frac{15}{8}} = \frac{144}{25} * \frac{8}{15} = \frac{1152}{375} = 3.07$$

For $V_1(u)$:

$$x = \frac{V_2(uu) - V_2(ud)}{u-d} = \frac{0 - \frac{6}{5}}{\frac{3}{2}} = -\frac{6}{5} * \frac{2}{3} = -\frac{4}{5} = -0.80$$

$$y = \frac{-d * V_2(uu) + u * V_2(ud)}{R(u-d)} = \frac{-\frac{1}{2} * 0 + 2 * \frac{6}{5}}{\frac{15}{8}} = \frac{\frac{12}{5}}{\frac{15}{8}} = \frac{12}{5} * \frac{8}{15} = \frac{32}{25} = 1.28$$

For $V_1(d)$:

$$x = \frac{V_2(du) - V_2(dd)}{u-d} = \frac{\frac{6}{5} - 4}{\frac{3}{2}} = -\frac{14}{5} * \frac{2}{3} = -\frac{28}{15} = -1.87$$

$$y = \frac{-d * V_2(du) + u * V_2(dd)}{R(u-d)} = \frac{-\frac{1}{2} * \frac{6}{5} + 2 * 4}{\frac{15}{8}} = \frac{-\frac{3}{5} + 8}{\frac{15}{8}} = \frac{37}{5} * \frac{8}{15} = \frac{296}{75} = 3.95$$

For $V_2(uu)$:

$$x = \frac{V_3(uuu) - V_3(uud)}{u-d} = \frac{0}{\frac{3}{2}} = 0.00$$

$$y = \frac{-d * V_3(uuu) + u * V_3(uud)}{R(u-d)} = \frac{0}{\frac{15}{8}} = 0.00$$

For $V_2(ud)$:

$$x = \frac{V_3(udu) - V_2(udd)}{u-d} = -3 * \frac{2}{3} = -2.00$$

$$y = \frac{-d * V_3(udu) + u * V_3(udd)}{R(u-d)} = \frac{-\frac{1}{2} * 0 + 2 * 3}{\frac{15}{8}} = 6 * \frac{8}{15} = \frac{48}{15} = 3.20$$

For $V_2(dd)$:

$$x = \frac{V_3(ddu) - V_2(ddd)}{u-d} = (3 - \frac{9}{2}) * \frac{2}{3} = -1.00$$

$$y = \frac{-d * V_3(ddu) + u * V_3(ddd)}{R(u-d)} = \frac{-\frac{1}{2} * 3 + 2 * \frac{9}{2}}{\frac{15}{8}} = \frac{15}{2} * \frac{8}{15} = \frac{8}{2} = 4.00$$

2.3 Optimal Exercise for American Option

Whenever the stock goes down in a period, it's optimal to exercise, since it can not possibly be worse than holding to maturity.

2.4 Riskless Profit for American Option

Our riskless profit would be our option premium (which is greater than \$139.20) minus \$139.20. Similarly, if the put is priced below the no-arbitrage price, we long the put and short the dynamic hedging portfolio, collecting the premium to finance the long put position. We wouldn't gain or lose anything no matter what happens to the market here either. So our riskless profit would be \$139.20 minus our option premium (which is less than \$139.20).

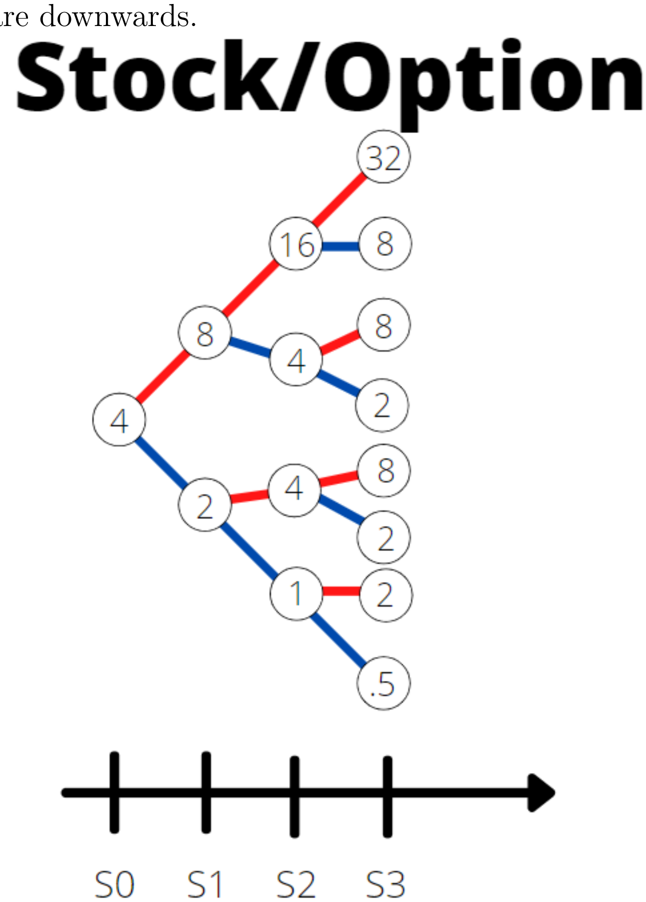
We can benefit from a dumb buyer who doesn't exercise at the optimal time by doing the following: first, we write them an American put and also long the same option. When the dumb buyer fails to exercise at the optimal time, we make our move by exercising our put and use part of it to create a replicating portfolio. We'll make a no-risk profit of \$1 every time.

3

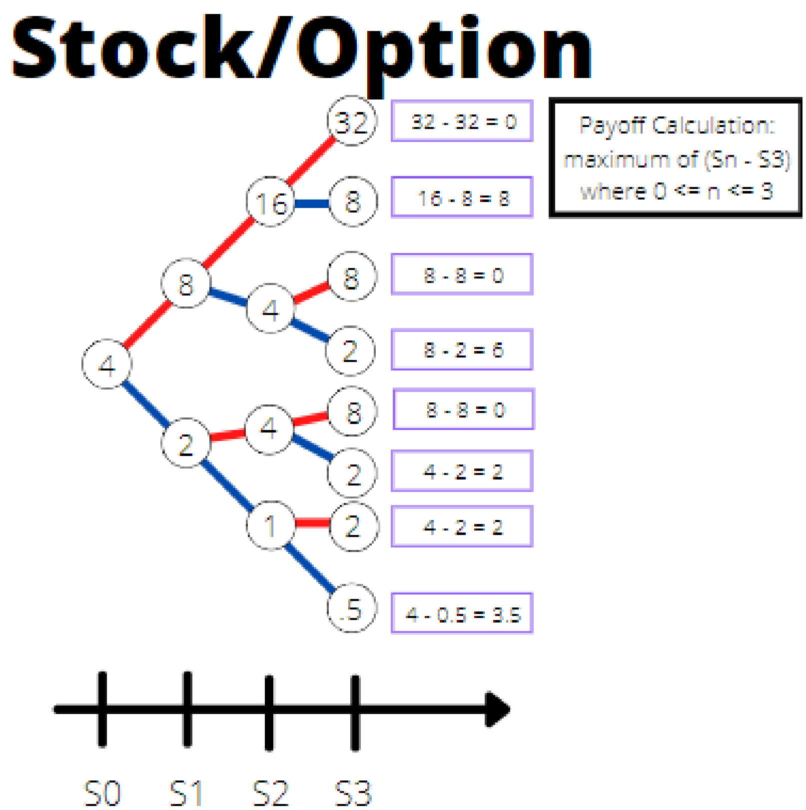
(a) Because these exotic derivatives have so many unique underlying conditions and such complex pricing, there may be arbitrage opportunity for the more experience investors to take advantage of. Higher risk, higher reward.

(b)
3.1 Dynamic Hedging for Lookback Option

Let us first observe the Lookback Option. Because the payoff is based upon the path taken, the stock lattice will be slightly altered from the one seen for American and European options. For clarity, red lines are upwards stock movement, and blue lines are downwards.



Now we can calculate the payoff in S3 using the function given in the problem. We must find the maximum value among the solutions to the equation $S_n - S_3$. The solution to these maximums are calculated in the purple boxes, but all calculations for each node (labeled N1 through N8, where N1 is at price 32 and N8 is price .5) can be found below the diagram.



For N1: max(4-32, 8-32, 16-32, 32-32) = 0.00
For N2: max(4-8, 8-8, 16-8, 8-8) = 8.00
For N3: max(4-8, 8-8, 4-8, 8-8) = 0.00
For N4: max(4-2, 8-2, 4-2, 2-2) = 6.00
For N5: max(4-8, 2-8, 4-8, 8-8) = 0.00
For N6: max(4-2, 2-2, 4-2, 2-2) = 2.00
For N7: max(4-2, 2-2, 1-2, 2-2) = 2.00
For N8: max(4-.5, 2-.5, 1-.5, .5-.5) = 3.50

Now we perform the pricing calculations ($Price = \frac{1}{R}[\tilde{p} * P_{up} + \hat{p} * P_{down}]$) to gather the data for S0, S1, and S2. As we're operating in the same three-period stock model as Q1 and Q2, we know that $R = \frac{5}{4}$, $\frac{1}{R} = \frac{4}{5}$, $\tilde{p} = \frac{R-d}{u-d} = \frac{1.25-.5}{2-.5} = \frac{1}{2}$, and $\hat{p} = 1 - \tilde{p} = \frac{1}{2}$.

$$P_2(uu) = \frac{1}{R}[\tilde{p}*P_3(uuu)+\hat{p}*P_3(uud)] = \frac{4}{5}[\frac{1}{2}(0)+\frac{1}{2}(8)] = \frac{4}{5}*4 = \frac{16}{5} = 3.20$$

$$P_2(ud) = \frac{1}{R}[\tilde{p}*P_3(udu)+\hat{p}*P_3(udd)] = \frac{4}{5}[\frac{1}{2}(0)+\frac{1}{2}(6)] = \frac{4}{5}*3 = \frac{12}{5} = 2.40$$

$$P_2(du) = \frac{1}{R}[\tilde{p}*P_3(duu)+\hat{p}*P_3(dud)] = \frac{4}{5}[\frac{1}{2}(0)+\frac{1}{2}(2)] = \frac{4}{5}*1 = \frac{4}{5} = 0.80$$

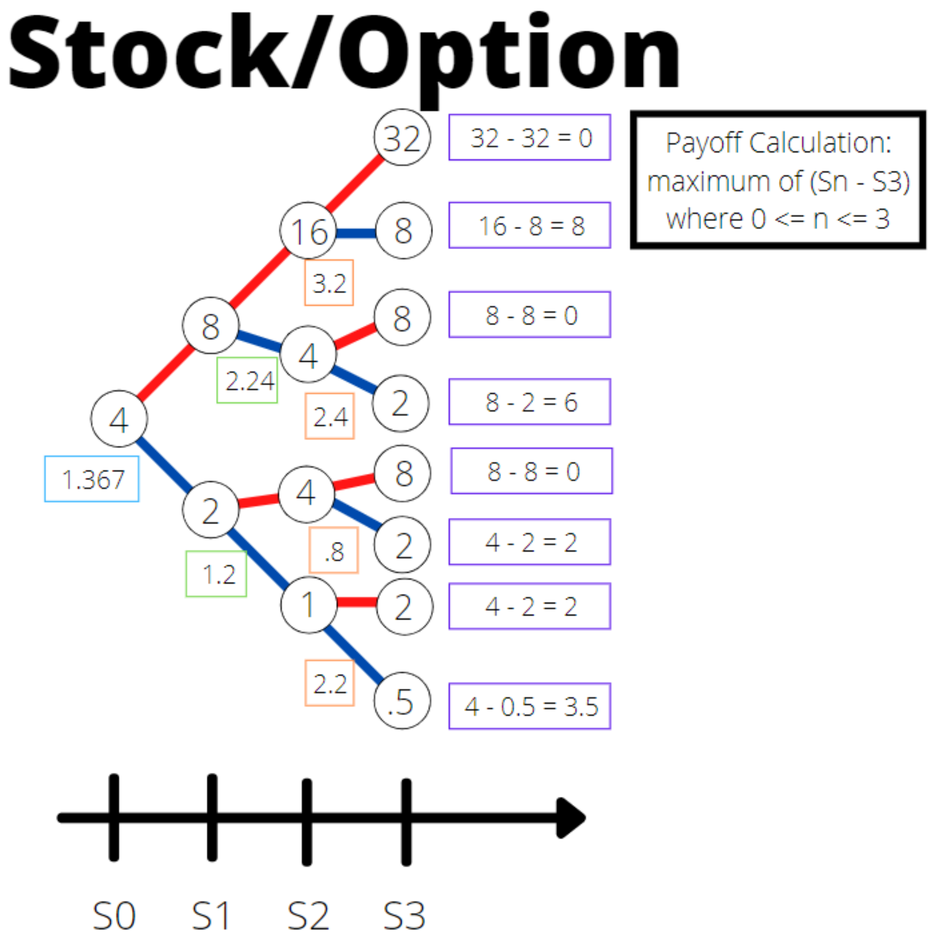
$$P_2(dd) = \frac{1}{R}[\tilde{p}*P_3(ddu)+\hat{p}*P_3(ddd)] = \frac{4}{5}[\frac{1}{2}(2)+\frac{1}{2}(3.5)] = \frac{4}{5}*(1+\frac{7}{4}) = \frac{11}{5} = 2.20$$

$$P_1(u) = \frac{1}{R}[\tilde{p}*P_2(uu)+\hat{p}*P_3(ud)] = \frac{4}{5}[\frac{1}{2}(\frac{16}{5})+\frac{1}{2}(\frac{12}{5})] = \frac{4}{5}*(\frac{8}{5}+\frac{6}{5}) = \frac{56}{25} = 2.24$$

$$P_1(d) = \frac{1}{R}[\tilde{p}*P_2(du))+\hat{p}*P_2(dd))] = \frac{4}{5}[\frac{1}{2}(\frac{4}{5})+\frac{1}{2}(\frac{11}{5})] = \frac{4}{5}*(\frac{2}{5}+\frac{11}{10}) = \frac{6}{5} = 1.20$$

$$P_0 = \frac{1}{R}[\tilde{p}*P_1(u))+\hat{p}*P_1(d)] = \frac{4}{5}[\frac{1}{2}(\frac{56}{25})+\frac{1}{2}(\frac{6}{5})] = \frac{4}{5}*(\frac{28}{25}+\frac{3}{5}) = \frac{4}{5}*\frac{43}{25} = \frac{172}{125} = 1.38$$

For clarity, the resulting lattice becomes as follows:



3.2 Portfolio Values for Lookback Option

Now that we've completed the pricing calculations, we can calculate the portfolio values. To reiterate from earlier, the variable x represents the stock, and the variable y represents the bond. We also know that $u - d = 2 - \frac{1}{2} = \frac{3}{2}$ and $R(u-d) = \frac{5}{4} * \frac{3}{2} = \frac{15}{8} = 1.875$. Stock price can be found by $x = \frac{P_{up} - P_{down}}{u-d}$, and bond by $y = \frac{-d*P_{up} + u*P_{down}}{R(u-d)}$.

For P_0:

$$x = \frac{P_1(u) - P_1(d)}{u-d} = \frac{\frac{56}{25} - \frac{6}{5}}{\frac{3}{2}} = \frac{26}{25} * \frac{2}{3} = \frac{52}{75} = 0.69$$

$$y = \frac{-d * P_1(u) + u * P_1(d)}{R(u-d)} = \frac{-\frac{1}{2} * \frac{56}{25} + 2 * \frac{6}{5}}{\frac{15}{8}} = \frac{-\frac{28}{25} + \frac{12}{5}}{\frac{15}{8}} = \frac{32}{25} * \frac{8}{15} = \frac{256}{375} = 0.68$$

For $P_1(u)$:

$$x = \frac{P_2(uu) - P_2(ud)}{u-d} = \frac{\frac{16}{5} - \frac{12}{5}}{\frac{3}{2}} = \frac{4}{5} * \frac{2}{3} = \frac{8}{15} = 0.53$$

$$y = \frac{-d * P_2(uu) + u * P_2(ud)}{R(u-d)} = \frac{-\frac{1}{2} * \frac{16}{5} + 2 * \frac{12}{5}}{\frac{15}{8}} = \frac{-\frac{8}{5} + \frac{24}{5}}{\frac{15}{8}} = \frac{16}{5} * \frac{8}{15} = \frac{128}{75} = 1.71$$

For $P_1(d)$:

$$x = \frac{P_2(du) - P_2(dd)}{u-d} = \frac{\frac{4}{5} - \frac{11}{5}}{\frac{3}{2}} = -\frac{7}{5} * \frac{2}{3} = -\frac{14}{15} = -0.93$$

$$y = \frac{-d * P_2(du) + u * P_2(dd)}{R(u-d)} = \frac{-\frac{1}{2} * \frac{4}{5} + 2 * \frac{11}{5}}{\frac{15}{8}} = \frac{-\frac{2}{5} + \frac{22}{5}}{\frac{15}{8}} = \frac{20}{5} * \frac{8}{15} = \frac{32}{15} = 2.13$$

For $P_2(uu)$:

$$x = \frac{P_3(uuu) - P_3(uud)}{u-d} = \frac{-8}{\frac{3}{2}} = \frac{-16}{3} = -5.33$$

$$y = \frac{-d * P_3(uuu) + u * P_3(uud)}{R(u-d)} = \frac{16}{\frac{15}{8}} = \frac{128}{15} = 8.53$$

For $P_2(ud)$:

$$x = \frac{P_3(udu) - P_2(udd)}{u - d} = -6 * \frac{2}{3} = -4.00$$

$$y = \frac{-d * P_3(udu) + u * P_3(udd)}{R(u - d)} = \frac{-\frac{1}{2} * 0 + 2 * 6}{\frac{15}{8}} = 12 * \frac{8}{15} = \frac{32}{5} = 6.40$$

For $P_2(du)$:

$$x = \frac{P_3(duu) - P_2(dud)}{u - d} = -2 * \frac{2}{3} = -\frac{4}{3} = -1.33$$

$$y = \frac{-d * P_3(duu) + u * P_3(dud)}{R(u - d)} = \frac{-\frac{1}{2} * 0 + 2 * 2}{\frac{15}{8}} = 4 * \frac{8}{15} = \frac{32}{15} = 2.13$$

For $P_2(dd)$:

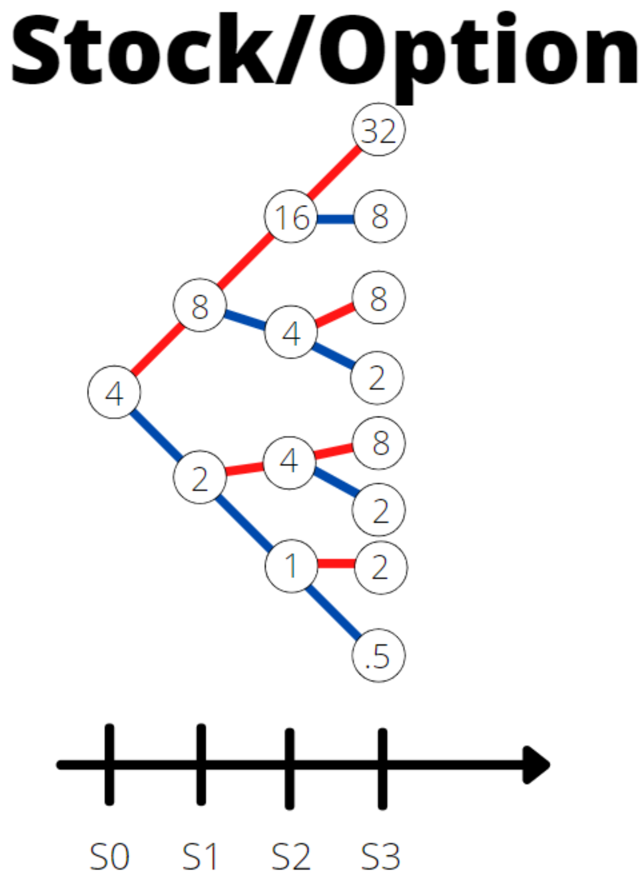
$$x = \frac{P_3(ddu) - P_2(ddd)}{u - d} = (2 - \frac{7}{2}) * \frac{2}{3} = -1.00$$

$$y = \frac{-d * P_3(ddu) + u * P_3(ddd)}{R(u - d)} = \frac{-\frac{1}{2} * 2 + 2 * \frac{7}{2}}{\frac{15}{8}} = 6 * \frac{8}{15} = \frac{28}{2} = 3.20$$

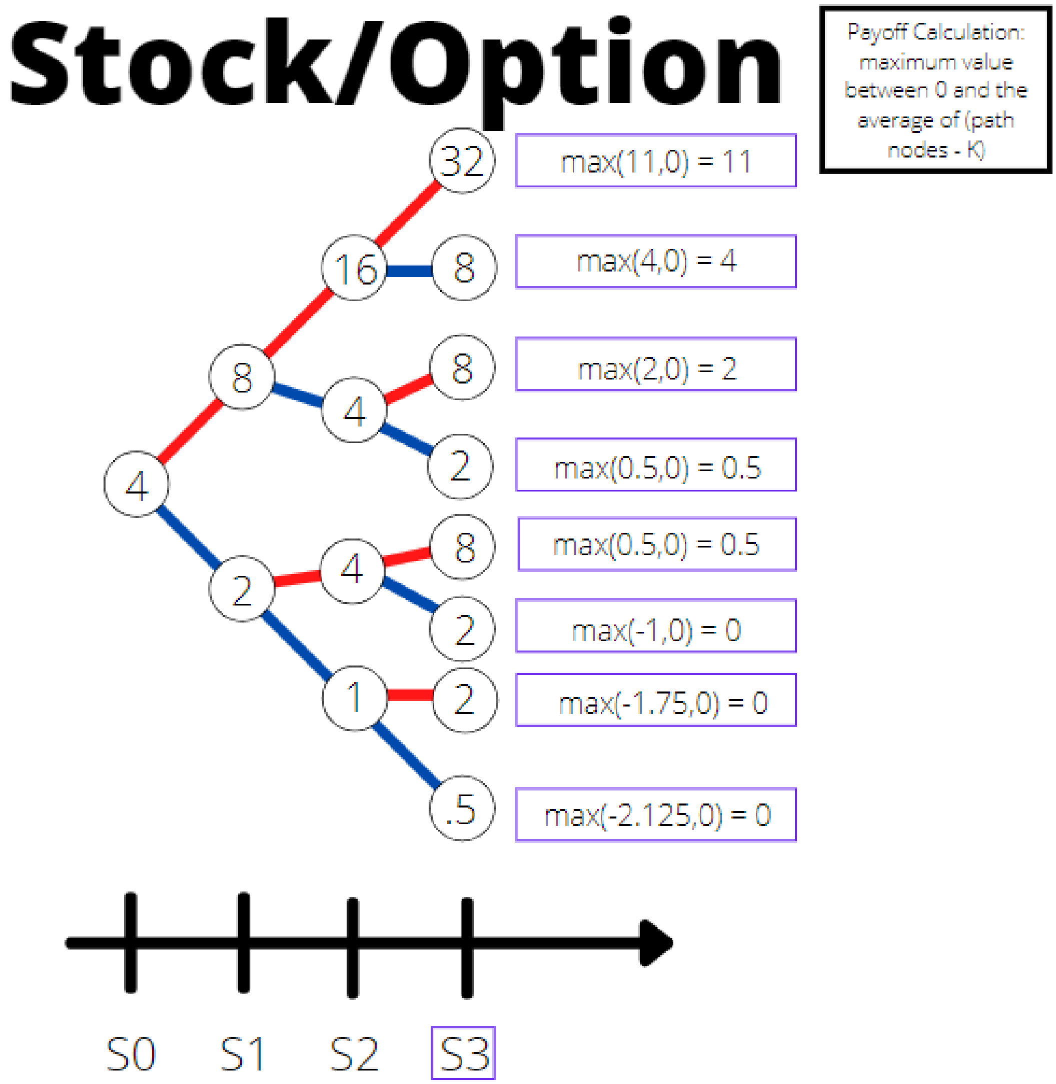
We've determined that the no-arbitrage price is \$1.38.

3.3 Dynamic Hedging Strategy for Asian Option

Now, looking at Asian options, the initial stock lattice is the same as the Lookback option lattice.



However, we now have a new payoff calculation; we must take the maximum value when comparing 0 to the average of the difference between each path node and the strike price (ie. max(0, avg. $S_n - K$). The solutions are included in the below graphic, with the actual equations posted below that. As before, N1 is the topmost node in S3, and N8 is the lowest in S3



N1: $\max(.25((32-4)+(16-4)+(8-4)+(4-4))=11,\ 0)$= 11
N2: $\max(.25((8-4)+(16-4)+(8-4)+(4-4))=5,\ 0)\ =\ 5$
N3: $\max(.25((8-4)+(4-4)+(8-4)+(4-4))=2,\ 0)$ =2
N4: $\max(.25((2-4)+(4-4)+(8-4)+(4-4))=.5,\ 0)\ =\ .5$
N5: $\max(.25((8-4)+(4-4)+(2-4)+(4-4))=.5,\ 0)\ =\ .5$
N6: $\max(.25((2-4)+(4-4)+(2)+(4-4))=0,\ 0)\ =\ 0$
N7: $\max(.25((2-4)+(1-4)+(2)+(4-4))=0,\ 0)\ =\ 0$
N8: $\max(.25((.5-4)+(1-4)+(2-4)+(4-4))=-2.125,\ 0)$=0

Again, we must complete the pricing calculations ($Price = \frac{1}{R}[\tilde{p} * P_{up} + \hat{p} * P_{down}]$), and we still know that $R = \frac{5}{4}$, $\frac{1}{R} = \frac{4}{5}$, $\tilde{p} = \frac{R-d}{u-d} = \frac{1.25-.5}{2-.5} = \frac{1}{2}$, and $\hat{p} = 1 - \tilde{p} = \frac{1}{2}$.

$$P_2(uu) = \frac{1}{R}[\tilde{p}*P_3(uuu)+\hat{p}*P_3(uud)] = \frac{4}{5}[\frac{1}{2}(11)+\frac{1}{2}(5)] = \frac{4}{5}*8 = \frac{32}{5} = 6.40$$

$$P_2(ud) = \frac{1}{R}[\tilde{p}*P_3(udu)+\hat{p}*P_3(udd)] = \frac{4}{5}[\frac{1}{2}(2)+\frac{1}{2}(\frac{1}{2})] = \frac{4}{5}*\frac{5}{4} = 1.00$$

$$P_2(du) = \frac{1}{R}[\tilde{p}*P_3(duu)+\hat{p}*P_3(dud)] = \frac{4}{5}[\frac{1}{2}(\frac{1}{2})] = \frac{4}{5}*\frac{1}{4} = \frac{1}{5} = 0.20$$

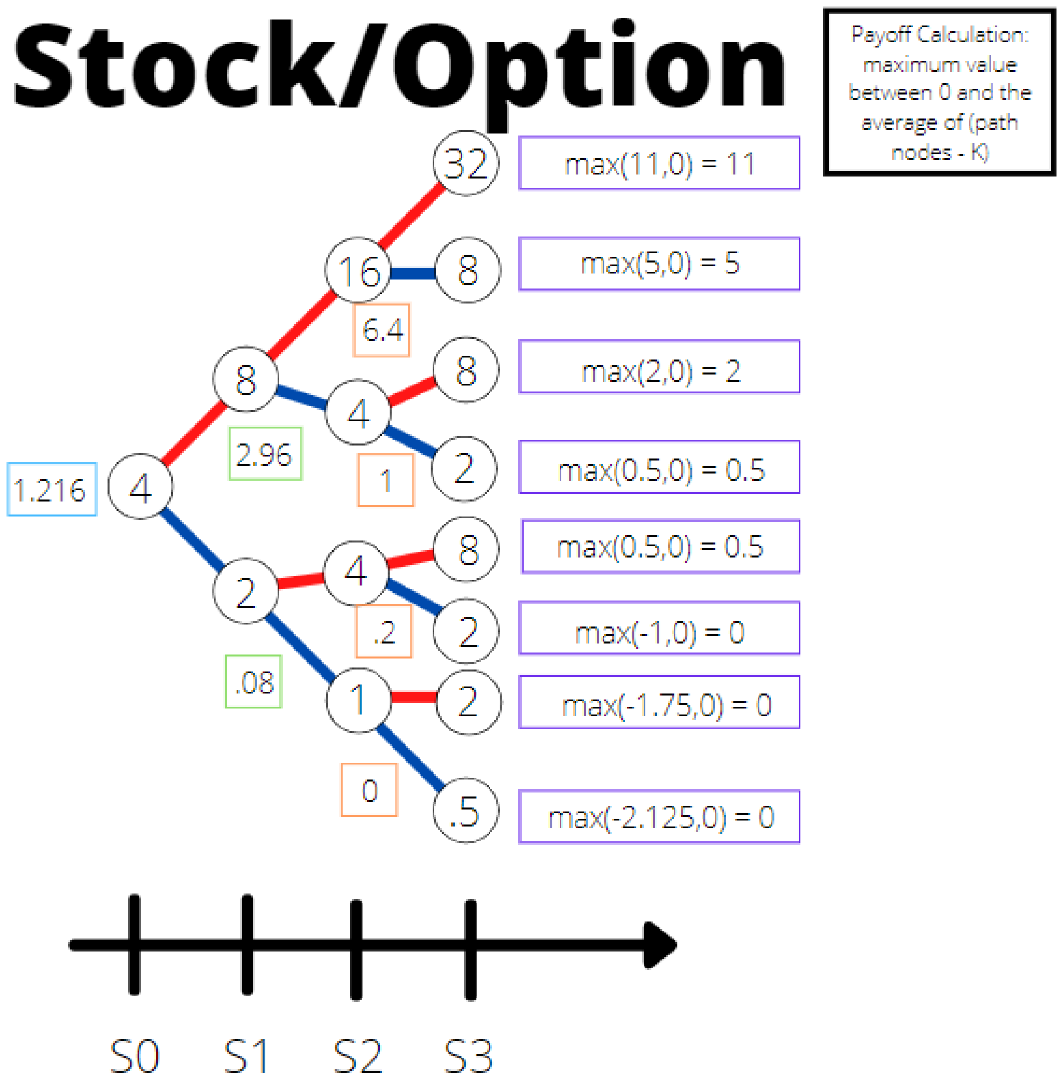
$$P_2(dd) = \frac{1}{R}[\tilde{p} * P_3(ddu) + \hat{p} * P_3(ddd)] = \frac{4}{5}[0] = 0.00$$

$$P_1(u) = \frac{1}{R}[\tilde{p}*P_2(uu)+\hat{p}*P_3(ud)] = \frac{4}{5}[\frac{1}{2}(\frac{32}{5})+\frac{1}{2}(1)] = \frac{4}{5}*(\frac{16}{5}+\frac{1}{2}) = \frac{4}{5}*\frac{37}{10} = \frac{74}{25} = 2.96$$

$$P_1(d) = \frac{1}{R}[\tilde{p}*P_2(du))+\hat{p}*P_2(dd))] = \frac{4}{5}[\frac{1}{2}(\frac{1}{5})] = \frac{4}{5}*(\frac{1}{10}) = \frac{2}{25} = 0.08$$

$$P_0 = \frac{1}{R}[\tilde{p}*P_1(u))+\hat{p}*P_1(d)] = \frac{4}{5}[\frac{1}{2}(\frac{74}{25})+\frac{1}{2}(\frac{2}{25})] = \frac{4}{5}*(\frac{37}{25}+\frac{1}{25}) = \frac{4}{5}*\frac{38}{25} = \frac{152}{125} = 1.22$$

The resulting lattice looks like this:



We've calculated the no-arbitrage price to be $1.22.

3.4 Portfolio Values for Asian Options

Now that we've completed the pricing calculations, we can again calculate the portfolio values. Like before, the variable x represents the stock, and the variable y represents the bond. We again know that $u - d = 2 - \frac{1}{2} = \frac{3}{2}$ and $R(u-d) = \frac{5}{4} * \frac{3}{2} = \frac{15}{8} = 1.875$. Stock price can be found by $x = \frac{P_{up} - P_{down}}{u-d}$, and bond by $y = \frac{-d*P_{up}+u*P_{down}}{R(u-d)}$.

For P_0:

$$x = \frac{P_1(u) - P_1(d)}{u-d} = \frac{\frac{74}{25} - \frac{2}{25}}{\frac{3}{2}} = \frac{72}{25} * \frac{2}{3} = \frac{48}{25} = 1.92$$

$$y = \frac{-d * P_1(u) + u * P_1(d)}{R(u-d)} = \frac{-\frac{1}{2} * \frac{74}{25} + 2 * \frac{2}{25}}{\frac{15}{8}} = \frac{-\frac{37}{25} + \frac{4}{25}}{\frac{15}{8}} = -\frac{264}{375} = -.70$$

For $P_1(u)$:

$$x = \frac{P_2(uu) - P_2(ud)}{u-d} = \frac{\frac{32}{5} - 1}{\frac{3}{2}} = \frac{27}{5} * \frac{2}{3} = \frac{18}{5} = 3.60$$

$$y = \frac{-d * P_2(uu) + u * P_2(ud)}{R(u-d)} = \frac{-\frac{1}{2} * \frac{32}{5} + 2 * 1}{\frac{15}{8}} = \frac{-\frac{16}{5} + 2}{\frac{15}{8}} = -\frac{6}{5} * \frac{8}{15} = -\frac{16}{25} = -0.64$$

For $P_1(d)$:

$$x = \frac{P_2(du) - P_2(dd)}{u-d} = \frac{\frac{1}{5}}{\frac{3}{2}} = \frac{1}{5} * \frac{2}{3} = \frac{2}{15} = 0.13$$

$$y = \frac{-d * P_2(du) + u * P_2(dd)}{R(u-d)} = \frac{-\frac{1}{2} * \frac{1}{5} + 2 * 0}{\frac{15}{8}} = \frac{-\frac{1}{10}}{\frac{15}{8}} = -\frac{1}{10} * \frac{8}{15} = -\frac{4}{75} = -0.05$$

For $P_2(uu)$:

$$x = \frac{P_3(uuu) - P_3(uud)}{u-d} = \frac{11 - 5}{\frac{3}{2}} = \frac{12}{3} = 4.00$$

$$y = \frac{-d * P_3(uuu) + u * P_3(uud)}{R(u-d)} = \frac{-\frac{1}{2}(11) + 2(5)}{\frac{15}{8}} = \frac{9}{2} * \frac{8}{15} = \frac{12}{5} = 2.40$$

For $P_2(ud)$:

$$x = \frac{P_3(udu) - P_2(udd)}{u-d} = (2 - \frac{1}{2}) * \frac{2}{3} = 1$$

$$y = \frac{-d * P_3(udu) + u * P_3(udd)}{R(u-d)} = \frac{-\frac{1}{2} * 2 + 2 * \frac{1}{2}}{\frac{15}{8}} = 0 * \frac{8}{15} = \frac{32}{5} = 0$$

For $P_2(du)$:

$$x = \frac{P_3(duu) - P_2(dud)}{u-d} = \frac{1}{2} * \frac{2}{3} = \frac{1}{3} = 0.33$$

$$y = \frac{-d * P_3(duu) + u * P_3(dud)}{R(u-d)} = \frac{-\frac{1}{2} * \frac{1}{2}}{\frac{15}{8}} = -\frac{1}{4} * \frac{8}{15} = -\frac{2}{15} = -0.13$$

For $P_2(dd)$:

$$x = \frac{P_3(ddu) - P_2(ddd)}{u-d} = (0) * \frac{2}{3} = 0.00$$

$$y = \frac{-d * P_3(ddu) + u * P_3(ddd)}{R(u-d)} = \frac{0}{\frac{15}{8}} = 6 * \frac{8}{15} = 0.00$$

(c) As stated earlier, the no-arbitrage price for the Lookback option is \$1.38 and for the Asian option it is \$1.22.

(d) Our riskless profit would be our option premium (which is greater than \$138.00) minus \$138.00. Similarly, if the put is priced below the no-arbitrage price, we long the put and short the dynamic hedging portfolio, collecting the premium to finance the long put position. We wouldn't gain or lose anything no matter what happens to the market here either. So our riskless profit would be \$138.00 minus our option premium (which is less than \$138.00).