

$$8.2.11 \int \frac{4x+3}{\sqrt{x^2-5}} dx = \int \frac{4x dx}{\sqrt{x^2-5}} + \int \frac{3 dx}{\sqrt{x^2-5}} \rightarrow$$

$$dt = d(x^2-5) = (x^2-5)' dx = 2x dx$$

$$2) \text{ by substitution } \int \frac{dx}{\sqrt{x^2+a}} = \ln|x+\sqrt{x^2+a}| + C, a=5$$

$$= \int \frac{2 dt}{\sqrt{t}} + 3 \int \frac{dx}{\sqrt{x^2+(-5)}} = 2 \cdot 2\sqrt{t} + 3 \ln|x+\sqrt{x^2-5}| + C$$

8.2.12

$$\int e^{\sin^2 x} \sin 2x dx = \left[\begin{aligned} \sin 2x &= 2 \sin x \cos x \\ t &= \sin^2 x \Rightarrow dt = d(\sin^2 x) \\ &= (\sin^2 x)' dx = 2 \sin x \cdot \cos x dx \end{aligned} \right] =$$

$$= \int e^t dt = e^t + C = e^{\sin^2 x} + C$$

$$8.2.13 \int \frac{1-2 \sin x}{\cos^2 x} dx = \int \frac{dx}{\cos^2 x} - \int \frac{2 \sin x dx}{\cos^2 x}$$

$$= \left[\begin{aligned} 1) \int \frac{dx}{\cos^2 x} &= \tan x + C \\ 2) t &= \cos x \Rightarrow dt = \end{aligned} \right]$$

$$d(\cos x) = (\cos x)' dt = -\sin x dx \Rightarrow \sin x dx = -dt$$

$$= \int \frac{dx}{\cos^2 x} - \int \frac{2 \cdot (-dt)}{t^2} = \int \frac{dx}{\cos^2 x} + 2 \int t^{-2} dt = \tan x + 2 \cdot \frac{t^{-2+1}}{-2+1} + C = \tan x - \frac{2}{t} + C = \tan x - \frac{2}{\cos x} + C$$

$$8.2.14 \int \frac{3x-4}{x^2-4} dx = \int \frac{3x dx}{x^2-4} - \int \frac{4 dx}{x^2-4} =$$

$$= 3 \int \frac{x dx}{x^2-4} - 4 \int \frac{dx}{x^2-4} = \left[\begin{aligned} 1) t &= x^2-4 \Rightarrow dt = 2x dx \\ 2) \int \frac{dx}{x^2-4} &= \frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + C \end{aligned} \right]$$

$$x dx = \frac{1}{2} dt \Rightarrow \int \frac{x dx}{x^2-4} = \frac{1}{2} \int \frac{dt}{t-4} = \frac{1}{2} \ln|t-4| = \frac{1}{2} \ln|x^2-4|$$

$$\ln \left| \frac{x-2}{x+2} \right| + C = \frac{3}{2} \ln |x^2-4| - \ln \left| \frac{x-2}{x+2} \right| + C$$

$$\begin{aligned} 8.2.16. \int \sqrt{9-x^2} dx &= \left[\begin{aligned} x &= 3 \sin t \Rightarrow x^2 = 9 \sin^2 t \\ dx &= d(3 \sin t) = \\ &= (3 \sin t)' dt = 3 \cos t dt = \int \sqrt{9(1-\sin^2 t)} \cdot 3 \cos t dt = \\ &= \int \sqrt{9 \cdot \cos^2 t} \cdot 3 \cos t dt = \int 3 \cos t \cdot 3 \cdot \cos t dt = \\ &= 9 \int \cos t \cdot \cos t dt = 9 \int \cos^2 t dt = \int \cos^2 x dx = \\ &= \frac{x}{2} + \frac{1}{4} \sin 2x + C \end{aligned} \right] = \left[\cos^2 x = \frac{1 + \cos 2x}{2} \right] = \\ &= 9 \int \frac{1 + \cos 2t}{2} dt = 9 \left(\int \frac{1}{2} dt + \int \frac{1}{2} \cdot \cos 2t dt \right) = \\ &= \frac{9}{2} \left(\int dt + \int \cos 2t dt \right) = \frac{9}{2} \left(\int dt + \frac{1}{2} \cdot 2 \cdot \right. \\ &\quad \cdot \int \cos 2t dt \Big) = \frac{9}{2} \left(\int dt + \frac{1}{2} \int \cos 2t d(2t) \right) = \\ &= \frac{9}{2} \left(t + \frac{1}{2} \sin 2t \right) + C = \left[\begin{aligned} x &= 3 \sin t \Rightarrow \\ \sin t &= \frac{x}{3} \Rightarrow t = \arcsin \left(\frac{x}{3} \right) \end{aligned} \right] = \frac{9}{2} \left(\arcsin \frac{x}{3} + \right. \\ &\quad \left. \frac{1}{2} \cdot \sin \left(2 \cdot \arcsin \frac{x}{3} \right) \right) + C = \left[\sin \left(2 \cdot \arcsin \frac{x}{3} \right) = \right. \\ &\quad \left. 2 \cdot \sin \left(\arcsin \frac{x}{3} \right) \cdot \cos \left(\arcsin \frac{x}{3} \right) = 2 \cdot \frac{x}{3} \cdot \right. \\ &\quad \left. \cdot \sqrt{1 - \frac{x^2}{9}} = \frac{2x}{3} \cdot \sqrt{\frac{9-x^2}{9}} = \frac{2x}{9} \sqrt{9-x^2} \right] = \frac{9}{2} \cdot \\ &\quad \cdot \left(\arcsin \frac{x}{3} + \frac{1}{2} \cdot \frac{2x \sqrt{9-x^2}}{9} \right) + C = \frac{9}{2} \left(\arcsin \frac{x}{3} + \right. \\ &\quad \left. + \frac{x \sqrt{9-x^2}}{9} \right) + C \end{aligned}$$

$$2.17 \int \frac{dx}{x(\sqrt{x+1})} = \int \frac{dx}{x}$$