Часть 5 Гриценко Павел ИВТ 2 курс

11.4.42 $z = \operatorname{arctg}\frac{y}{x}$, $x = e^{2t}+1$, $y = e^{2t}-1$

$$\frac{\partial z}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2}\left(\frac{y}{x}\right)'_x = -\frac{yx^{-2}}{1+\frac{y^2}{x^2}}$$

$$\frac{\partial z}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{y}{x}\right)'_y = \frac{x^{-1}}{1+\frac{y^2}{x^2}}$$

$$\frac{dx}{dt} = 2e^{2t}\ ;\ \frac{dy}{dt} = 2e^{2t}$$

$$\frac{dz}{dt} = \left(-\frac{yx^{-2}}{1+\frac{y^2}{x^2}} + \frac{x^{-1}}{1+\frac{y^2}{x^2}}\right)\cdot 2e^{2t} = \frac{x^{-1}-yx^{-2}}{1+\frac{y^2}{x^2}}\cdot$$

$$\cdot 2e^{2t} = \frac{(e^{2t}+1)^{-1}-(e^{2t}-1)\cdot(e^{2t}+1)^{-2}}{1+\frac{(e^{2t}-1)^2}{(e^{2t}+1)^2}}\cdot 2e^{2t}$$

11.4.43 $z = x^4+y^4-4x^2y^2$, $x = e^{2t}$, $y = e^{2t}$

$$\frac{\partial z}{\partial x} = 4x^3-8xy^2\ ;\ \frac{\partial z}{\partial y} = 4y^3-8x^2y$$

$$\frac{dx}{dt} = 2e^{2t}\ ;\ \frac{dy}{dt} = 2e^{2t}$$

$$\frac{dz}{dt} = 2e^{2t}(4x^3-8xy^2+4y^3-8x^2y) =$$

$$= 2e^{2t}(4e^{6t}-8e^{6t}-8e^{6t}+4e^{6t}) = 2e^{2t}\cdot$$

$$(-8e^{6t}) = -16e^{8t}$$

11.4.44 $z = xy + \frac{x}{y}$, $x = \operatorname{tg} t$, $y = \ln t$

$$\frac{\partial z}{\partial x} = y + \frac{1}{y}\ ;\ \frac{\partial z}{\partial y} = x - \frac{x}{y^2}$$

$$\frac{dx}{dt} = \frac{1}{\cos^2 t} \; ; \; \frac{dy}{dt} = \frac{1}{t}$$

$$\frac{dz}{dt} = \left(y + \frac{1}{y}\right)\frac{1}{\cos^2 t} + \left(x - \frac{x}{y^2}\right)\frac{1}{t} =$$

$$= \left(\ln t + \frac{1}{\ln t}\right)\frac{1}{\cos^2 t} + \left(\operatorname{tg} t - \frac{\operatorname{tg} t}{\ln^2 t}\right)\frac{1}{t}$$

11.4.45 $z = \frac{x}{y^2}$, $x = \operatorname{arctg} 2t$, $y = \arcsin t$

$$\frac{\partial z}{\partial x} = \frac{1}{y^2} \; ; \; \frac{\partial z}{\partial y} = -\frac{2x}{y^3}$$

$$\frac{dx}{dt} = \frac{2}{1+4t^2} \; ; \; \frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}}$$

$$\frac{dz}{dt} = \frac{1}{y^2} \cdot \frac{2}{1+4t^2} + -\frac{2x}{y^3} \cdot \frac{1}{\sqrt{1-t^2}} =$$

$$= \frac{2}{\arcsin^2(t)(1+4t^2)} - \frac{2\operatorname{arctg} 2t}{\arcsin^3(t) \cdot \sqrt{1-t^2}}$$

11.4.46 $z = \frac{x}{\sqrt{x^2+y^2}}$, $x = 5^{t^2}$, $y = \arccos 2t$

$$\frac{\partial z}{\partial x} = \frac{\sqrt{x^2+y^2} - (\sqrt{x^2+y^2})'_x x}{x^2+y^2} =$$

$$= \frac{\sqrt{x^2+y^2} - \frac{x}{\sqrt{x^2+y^2}} \cdot x}{x^2+y^2}$$

$$\frac{\partial z}{\partial y} = \frac{\sqrt{x^2+y^2} - \frac{y}{\sqrt{x^2+y^2}} x}{x^2+y^2}$$

$$\frac{dx}{dt} = 5^{t^2} \cdot \ln 5 \cdot 2t$$

$$\frac{dy}{dt} = -\frac{2}{\sqrt{1-(2t)^2}}$$

$$\frac{dz}{dt} = \frac{\sqrt{x^2+y^2} - \frac{x^2}{\sqrt{x^2+y^2}}}{x^2+y^2} \times 5^{t^2} \cdot \ln 5 \cdot 2t \; \times -$$

$$-\frac{\sqrt{x^2+y^2}-\frac{yx}{\sqrt{x^2+y^2}}}{x^2+y^2}\cdot\frac{2}{\sqrt{1-4t^2}}=\frac{1}{5^{2t^2}+\arccos^2 2t}\cdot$$

$$\cdot\left(\left(\sqrt{5^{2t^2}+\arccos^2 2t}-\frac{x5^{2t^2}}{\sqrt{5^{2t^2}+\arccos^2 2t}}\right)\cdot 5^{t^2}\cdot\ln 5\cdot 2t-\right.$$

$$\left.-\left(\sqrt{5^{2t^2}+\arccos^2 2t}-\frac{5^{t^2}\cdot\arccos 2t}{\sqrt{5^{2t^2}+\arccos^2 2t}}\right)\cdot\frac{2}{\sqrt{1-4t^2}}\right)$$

11.4.47 $z=x\sin(x+y),\ x=\frac{1}{t^3},\ y=(t-1)^2$

$$\frac{\partial z}{\partial x}=\sin(x+y)+x(\sin(x+y))'_x=\sin(x+y)+x\cos(x+y)$$

$$\frac{\partial z}{\partial y}=x'_y\sin(x+y)+x\cos(x+y)=x\cos(x+y)$$

$$\frac{dx}{dt}=-\frac{3}{t^4};\quad \frac{dy}{dt}=2(t-1)\cdot(t-1)'_t=2(t-1)$$

$$\frac{dz}{dt}=(\sin(x+y)+x\cos(x+y))\left(-\frac{3}{t^4}\right)+x\cos(x+y)\cdot$$

$$\cdot 2(t-1)=\frac{\cos\left(\frac{1}{t^3}+(t-1)^2\right)}{t^3}\cdot 2(t-1)-\frac{3}{t^4}\cdot$$

$$\cdot\left(\sin\left(\frac{1}{t^3}+(t-1)^2\right)+\frac{\cos\left(\frac{1}{t^3}+(t-1)^2\right)}{t^3}\right)$$

11.4.48 $z=\frac{\cos x^2}{y},\ x=\ln(t+2),\ y=\operatorname{tg}t$

$$\frac{\partial z}{\partial x}=\frac{1}{y}(\cos x^2)'_x=-\frac{\sin x^2\cdot 2x}{y}$$

$$\frac{\partial z}{\partial y}=\left(\frac{1}{y}\right)'_y\cos x^2+(\cos x^2)'_y\frac{1}{y}=-\frac{\cos x^2}{y^2}$$

$$\frac{dx}{dt}=(\ln(t+2))'=\frac{1}{t+2}(t+2)'=\frac{1}{t+2}$$

$$\frac{dy}{dt}=(\operatorname{tg}t)'=\frac{1}{\cos^2 t}$$

$$\frac{dz}{dt} = -\frac{\sin x^2 \cdot 2x}{y(t+2)} - \frac{\cos x^2}{y^2\cos^2 t} = -\frac{\sin \ln^2(t+2)\cdot 2\ln(t+2)}{tg\, t\cdot(t+2)} -$$

$$-\frac{\cos \ln^2(t+2)}{tg^2 t\cdot\cos^2 t}$$

11.4.49 $z = tg\frac{x^2}{y}$, $x=\cos^2 t$, $y=\sin 2t$

$$\frac{\partial z}{\partial x} = \frac{1}{\cos^2\frac{x^2}{y}}\cdot\left(\frac{x^2}{y}\right)'_x = \frac{2xy^{-1}}{\cos^2\frac{x^2}{y}}$$

$$\frac{\partial z}{\partial y} = \frac{1}{\cos^2\frac{x^2}{y}}\cdot\left(-\frac{x^2}{y^2}\right) = -\frac{x^2y^{-2}}{\cos^2\frac{x^2}{y}}$$

$$\frac{dx}{dt} = -2\cos t\sin t\ ; \quad \frac{dy}{dt} = 2\cos 2t$$

$$\frac{dz}{dt} = \frac{-2xy^{-1}\cdot 2\cos t\sin t}{\cos^2\frac{x^2}{y}} - \frac{x^2y^{-2}\,2\cos 2t}{\cos^2\frac{x^2}{y}} =$$

$$= -\frac{4\cos^2 t\sin^{-1}2t\cdot\cos t\sin t}{\cos^2\frac{\cos^4 t}{\sin 2t}} - \frac{2\cos^4 t\sin^{-2}2t\cos 2t}{\cos^2\frac{\cos^4 t}{\sin 2t}}$$

11.4.62 $y^4 - 6x^2y^2 + arctg\,2x = 0$

$$F(x;y) = y^4 - 6x^2y^2 + arctg\,2x$$

$$y' = -\frac{(y^4-6x^2y^2+arctg\,2x)'_x}{(y^4-6x^2y^2+arctg\,2x)'_y} = -\frac{-12xy^2+\frac{2}{1+x^2}}{4y^3-12x^2y}$$

$$y = \pm\sqrt[4]{6x^2y^2(x)+arctg\,2x}$$

11.4.63 $e^{-x+y^3} - 2x - 18x^3 - 1 = 0$

$$F(x;y(x)) = e^{-x+y^3(x)} - 2x - 18x^3 - 1$$

$$\cancel{y=e^{-x+y^3(x)}} - 2x - 18x^3 - 1$$

11.4.75 $z = u^2 v^2$, $x = u + v$, $y = u - v$

$x = u + v$ $\qquad$ $y = u - v$

$u = x - v$ $\qquad$ $y = x - v - v$

$y = x - 2v$

$v = \frac{x-y}{2}$

$u = x - \frac{x-y}{2}$

$\frac{\partial z}{\partial u} = 2uv^2$; $\frac{\partial z}{\partial v} = 2vu^2$

$dz = 2uv^2 du + 2vu^2 dv = (2x-(x-y)) \cdot$

$\cdot \frac{(x-y)^2}{4} d\left(x - \frac{x-y}{2}\right) + (x-y)\left(x - \frac{x-y}{2}\right)^2 d\left(\frac{x-y}{2}\right) =$

$= (x-y)\left(\frac{(x+y)(x-y)}{2} d\left(x - \frac{x-y}{2}\right) + \left(x - \frac{x-y}{2}\right) \cdot\right.$

$\left. \cdot d\left(\frac{x-y}{2}\right)\right)$

11.4.76 $x = u\cos v$, $y = u \sin v$ $\quad z = u^2$

$\frac{\partial z}{\partial u} = 2u$ $\quad \frac{\partial z}{\partial v} = 0$

$dz = 2u\,du$

$u = \frac{x}{\cos v} = \frac{y}{\sin v}$

$x \sin v = y \cos v$

$\frac{y}{x} = \frac{\sin v}{\cos v} = \operatorname{tg} v$

$v = \operatorname{arctg} \frac{y}{x} \Rightarrow u = \frac{x}{\cos(\operatorname{arctg} \frac{y}{x})}$

$$dz = \frac{2x}{\cos \operatorname{arctg} \frac{y}{x}} \, d\left(\frac{x}{\cos \operatorname{arctg} \frac{y}{x}}\right)$$

11.4.77. $x = v\cos u - u\cos u + \sin u$

$y = v\sin u - u\sin u - \cos u$

$z = (u-v)^2$

$$\frac{\partial z}{\partial u} = ((u-v)^2)'_u = 2(u-v)(u-v)'_u = 2(u-v)$$

$$\frac{\partial z}{\partial v} = ((u-v)^2)'_v = 2(u-v)(u-v)'_v = -2(u-v)$$

$$dz = 2(u-v)du - 2(u-v)dv$$

$$x = \cos u(v-u) + \sin u$$

$$\sin u = x - \cos u(v-u)$$

$$\cos u = -y - \sin u(v-u)$$

$$v - u = \frac{x - \sin u}{\cos u} = \frac{y - \cos u}{\sin u}$$

$$\frac{x - \sin u}{\cos u} = \frac{x}{\cos u} - \operatorname{tg} u$$

$$\frac{y - \cos u}{\sin u} = \frac{y}{\sin u} - \operatorname{ctg} u$$

$$\frac{x}{\cos u} - \frac{y}{\sin u} = \operatorname{tg} x - \operatorname{ctg} u$$

$$x\sin u - y\cos u = \frac{(\operatorname{tg} u - \operatorname{ctg} u)}{1} \cdot \sin u \cos u$$

$$x\sin u - y\cos u = \sin^2 u - \cos^2 u$$

$$x = \sin u \ ; \ y = \cos u \ \Rightarrow$$

$$\Rightarrow y = vx - ux - y, \quad x = vy^2 - uy + x$$

$$2y = vx - ux$$

$$vy - uy = 0$$

$$\frac{2y}{x} = (v - u)$$

$$y = 0 \text{ или } v = u$$

$$u - v = -\frac{2y}{x}$$

$$u = \arcsin x = v$$

$$dz = 2 \cdot \left(-\frac{2y}{x}\right) \cdot d(\arcsin x) + 2 \cdot \frac{2y}{x} \cdot$$

$$d(\arcsin x) = \frac{4y}{x} d(\arcsin x) - \frac{4y}{x} \cdot$$

$$\cdot d(\arcsin x) = 0$$

$F(x; y(x)) = e^{-x} \cdot e^{y^3(x)} - 2x - 18x^3 - 1$

$$e^{y^3 x} = \frac{2x + 18x^3 + 1}{e^{-x}}$$

$$y = \sqrt[3]{\log_e \frac{2x + 18x^3 + 1}{e^{-x}}}$$

11.4.64 $tg(x^2 + y^4) - 3x^2 - 17 = 0$

$F(x; y) = tg(x^2 + y^4) - 3x^2 - 17$

$\cancel{F(x; y)} y' = \cancel{\frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial}} - \frac{F_x'(x; y)}{F_y'(x; y)} =$

$$= - \frac{\frac{2x}{\cos^2(x^2 + y^4)} - 6x}{\frac{4yx^3}{\cos^2(x^2 + y^4)}} +$$

$$y = -\int \frac{\frac{2x}{\cos^2(x^2 + y^4)} - 6x}{\frac{4x^3y^3}{\cos^2(x^2 + y^4)}} dx$$

11.4.65 $x^2 y^4 - 3y^3 - 6y^2 + 3y + x^2 = 0$

11.6.67 $z = u^2 \ln v$, $u = y/x$ $v = x^2+y^2$

$z = \left(\frac{y}{x}\right)^2 \ln(x^2+y^2)$

$\frac{\partial z}{\partial y} = \frac{2y}{x^2} \cdot \frac{2y}{x^2+y^2}$

$\frac{\partial z}{\partial x} = -2\frac{y^2}{x^3} \quad \frac{2x}{x^2+y^2}$

11.6.68 $z = f(u; v)$, $u = \frac{2y}{x+y}$, $v = x^2+y^2$

$\frac{\partial z}{\partial x} = f'_x\left(\frac{2y}{x+y}; x^2+y^2\right)$

$\frac{\partial z}{\partial y} = f'_y\left(\frac{2y}{x+y}; x^2+y^2\right)$

$dz = f'_x\left(\frac{2y}{x+y}; x^2+y^2\right)dx + f'_y\left(\frac{2y}{x+y}; x^2+y^2\right)dy$

11.6.69 $z = f(u; v)$ $u = \ln(x^2-y^2)$, $v = xy^2$

$\frac{\partial z}{\partial x} = f'_x(\ln(x^2-y^2); xy^2)$

$\frac{\partial z}{\partial y} = f'_y(\ln(x^2-y^2); \cancel{v} xy^2)$

11.6.70 $z = u^2 v - uv^2$, $u = x\sin y$, $v = y\cos x$

$\frac{\partial z}{\partial u} = 2vu - v^2$

$\frac{\partial z}{\partial v} = u^2 - 2uv$

$dz = (2vu - v^2)du + (u^2 - 2uv)dv =$

$= (2xy\sin y\cos x - y^2\cos^2 x)\,d(x\sin y) +$

$+ (x^2\sin^2 y - 2xy\sin y(\cos x)\, d(y\cos x)$

11.6.71 $z = f(u;v)$, $u = \cos(xy)$, $v = x^5 - 7y$

$\frac{\partial z}{\partial u} = f'_u(u;v)$; $\frac{\partial z}{\partial v} = f'_v(u;v)$

$dz = f'_{\cos xy}(\cos xy; x^5 - 7y)\, d(\cos xy) +$

$+ f'_{x^5-7y}(\cos(xy); x^5-7y)\, d(x^5-7y)$

11.6.72 $z = f(u;v)$; $u = \sin\frac{x}{y}$, $v = \sqrt{\frac{x}{y}}$

$\frac{\partial z}{\partial u} = f'_u(u;v)$; $\frac{\partial z}{\partial v} = f'_v(u;v)$

$dz = f'_u(u;v)\,du + f'_v(u;v)\,dv =$

$= f'_{\sin\frac{x}{y}}(\sin\frac{x}{y}; \sqrt{\frac{x}{y}})\, d(\sin\frac{x}{y}) + f'_{\sqrt{\frac{x}{y}}}(\sin\frac{x}{y}; \sqrt{\frac{x}{y}})\, d\sqrt{\frac{x}{y}}$

11.6.73 $x = \frac{u^2+v^2}{2}$, $y = \frac{u^2-v^2}{2}$, $z = uv$

$2x = u^2 + v^2$ $\qquad$ $2y = u^2 - v^2$

$2x - v^2 = u^2$ $\qquad$ $2y + v^2 = u^2$

$\pm\sqrt{2x - v^2} = u$ $\qquad$ $\pm\sqrt{2y + v^2} = u$

$2x - v^2 = 2y + v^2$

$2x - 2y = 2v^2$

$v = \pm\sqrt{x-y}$

$u = \pm\sqrt{2y + x - y} = \pm\sqrt{y+x}$

$\frac{\partial z}{\partial u} = v$

$\frac{\partial z}{\partial v} = u$

$dz = v\,du + u\,dv = \pm\sqrt{x-y}\,d(\pm\sqrt{x+y}) \pm$
$\pm\sqrt{x+y}\,d(\pm\sqrt{x-y})$

11.4.74 $x = \sqrt{a}(\sin u + \cos v)$, $y = \sqrt{a}(\cos u -$
$- \sin v)$, $z = 1 + \sin(u - v)$

$\frac{\partial z}{\partial u} = \cos u$; $\frac{\partial z}{\partial v} = \cos v$

$x = \sqrt{a}(\sin u + \cos v)$

$(\sin u + \cos v) = \frac{x}{\sqrt{a}}$

$\cos v = \frac{x}{\sqrt{a}} - \sin u \Rightarrow v = \arccos\left(\frac{x}{\sqrt{a}} - \sin u\right)$

$y = \sqrt{a}(\cos u - \sin v)$

$\cos u - \sin v = \frac{y}{\sqrt{a}}$

$\cos u = \frac{y}{\sqrt{a}} + \sin v \Rightarrow u = \arccos\left(\frac{y}{\sqrt{a}} + \sin v\right)$

$\cos u = \frac{y}{\sqrt{a}} + \sqrt{1 - \left(\frac{x}{\sqrt{a}} - \sin u\right)^2}$