Часть 4 Грищенко Павел ИВТ 2 курс

11.3.29 $z = (5x^2y - y^3 + 7)^3$

$z'_x = 3(5x^2y - y^3 + 7)^2 \cdot (5x^2y - y^3 + 7)'_x =$

$= 3(5x^2y - y^3 + 7)^2 \cdot (10xy = 30xy(5x^2y -$

$y^3 + 7)^2$

$z'_y = 3(5x^2y - y^3 + 7)^2 \cdot (5x^2y - y^3 + 7)'_y =$

$= 3(5x^2y - y^3 + 7)^2 \cdot (5x^2 - 3y^2)$

$z' = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = 30xy(5x^2y - y^3 + 7)^2 dx +$

$+ 3(5x^2y - y^3 + 7)^2 \cdot (5x^2 - 3y^2) dy = 3(5x^2y - y^3 +$

$+ 7)^2 (10xy\,dx + (5x^2 - 3y^2) dy)$

11.3.30 $v = \operatorname{arctg} \frac{u}{t}$

$v'_u = \frac{\frac{1}{t}}{1 + \left(\frac{u}{t}\right)^2}$

$v'_t = \frac{1}{1 + \left(\frac{u}{t}\right)^2} \cdot \left(\frac{u}{t}\right)'_t = -\frac{\frac{u}{t^2}}{1 + \left(\frac{u}{t}\right)^2}$

$v' = \frac{1/t\,du}{1 + \left(\frac{u}{t}\right)^2} + \frac{u/t^2\,dt}{1 + \left(\frac{u}{t}\right)^2} = \frac{\frac{1}{t} du + \frac{u}{t^2} dt}{1 + \left(\frac{u}{t}\right)^2}$

11.3.31 $z = x\sqrt{y} + \frac{y}{\sqrt[3]{x}}$

$z'_x = \sqrt{y} - \frac{\frac{1}{3} \cdot y}{\sqrt[3]{x^4}}$; $z'_y = \frac{x}{2\sqrt{y}} + \frac{1}{\sqrt[3]{x}}$

$z' = \left(\sqrt{y} - \frac{y}{3\sqrt[3]{x^4}}\right) dx + \left(\frac{x}{2\sqrt{y}} + \frac{1}{\sqrt[3]{x}}\right) dy$

11.3.32 $z = \ln \operatorname{tg} \frac{x}{y}$

$z'_x = \left(\ln \operatorname{tg} \frac{x}{y}\right)'_x = \frac{1}{\operatorname{tg} \frac{x}{y}} \cdot \left(\operatorname{tg} \frac{x}{y}\right)'_x = \frac{1}{\operatorname{tg} \frac{x}{y}} \cdot$

$\cdot \frac{1}{\cos^2 \frac{x}{y}} \cdot \left(\frac{x}{y}\right)'_x = \frac{1}{\operatorname{tg} \frac{x}{y}} \cdot \frac{1}{\cos^2 \frac{x}{y}} \cdot \frac{1}{y} = \frac{1}{y \sin \frac{x}{y} \cos \frac{x}{y}}$

$z'_y = \left(\ln \operatorname{tg} \frac{x}{y}\right)'_y = \frac{1}{\operatorname{tg} \frac{x}{y}} \cdot \left(\operatorname{tg} \frac{x}{y}\right)_y = \frac{1}{\operatorname{tg} \frac{x}{y}} \cdot \frac{1}{\cos^2 \frac{x}{y}} \cdot$

$\cdot \left(\frac{x}{y}\right)'_y = \frac{\cos \frac{x}{y}}{\sin \frac{x}{y}} \cdot \frac{1}{\cos^2 \frac{x}{y}} \cdot \left(-\frac{x}{y^2}\right) = -\frac{x}{y^2 \sin \frac{x}{y} \cos \frac{x}{y}}$

$z' = \frac{dx}{y \sin \frac{x}{y} \cos \frac{x}{y}} - \frac{x\,dy}{y^2 \sin \frac{x}{y} \cos \frac{x}{y}} = \frac{1}{y \sin \frac{x}{y} \cos \frac{x}{y}} \cdot$

$\cdot \left(dx - \frac{x\,dy}{y}\right)$

11.3.33 $z = \sqrt{u + \sqrt{u^2 + v^2}}$

$z'_u = \left(\left(u + (u^2 + v^2)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)'_u = \frac{1}{2\sqrt{u + \sqrt{u^2+v^2}}} \cdot$

$(u + \sqrt{u^2+v^2})'_u = \frac{1}{2\sqrt{u+\sqrt{u^2+v^2}}} \cdot \left(1 + \frac{1}{2\sqrt{u^2+v^2}} \cdot\right.$

$\left.\cdot (u^2+v^2)'_u\right) = \frac{1}{2\sqrt{u+\sqrt{u^2+v^2}}} \cdot \left(1 + \frac{u}{\sqrt{u^2+v^2}}\right) =$

$= \frac{1}{2\sqrt{u + \sqrt{u^2+v^2}}} + \frac{u}{2\sqrt{u+\sqrt{u^2+v^2}}\sqrt{u^2+v^2}}$

$z'_v = \left(\sqrt{u + \sqrt{u^2+v^2}}\right)'_v = \frac{1}{2\sqrt{u+\sqrt{u^2+v^2}}} \cdot$

$\cdot (u + \sqrt{u^2+v^2})'_v = \frac{1}{2\sqrt{u+\sqrt{u^2+v^2}}} \cdot \frac{1}{2\sqrt{u^2+v^2}} \cdot$

$\cdot (u^2+v^2)'_v = \frac{1}{2\sqrt{u+\sqrt{u^2+v^2}}} \cdot \frac{2v}{2\sqrt{u^2+v^2}} =$

$= \frac{v}{2\sqrt{u+\sqrt{u^2+v^2}} \cdot \sqrt{u^2+v^2}}$

$z' = \left(\frac{1}{2\sqrt{u+\sqrt{u^2+v^2}}} + \frac{u}{2\sqrt{u+\sqrt{u^2+v^2}}\sqrt{u^2+v^2}}\right)du$

$+ \frac{v\,dv}{2\sqrt{u+\sqrt{u^2+v^2}} \cdot \sqrt{u^2+v^2}}$

11.3.34 $z = \ln \frac{\sqrt{x^2+y^2} - x}{\sqrt{x^2+y^2} + x}$

$z'_x = \frac{1}{\frac{\sqrt{x^2+y^2} - x}{\sqrt{x^2+y^2} + x}} \cdot \left(\frac{\sqrt{x^2+y^2} - x}{\sqrt{x^2+y^2} + x}\right)'_x =$

$\frac{\sqrt{x^2+y^2} + x}{\sqrt{x^2+y^2} - x} \cdot \left(\frac{(\sqrt{x^2+y^2} - x)'_x(\sqrt{x^2+y^2} + x)}{(\sqrt{x^2+y^2} + x)^2} - \right.$

$$-\frac{(\sqrt{x^2+y^2}+x)'_x(\sqrt{x^2+y^2}-x)}{(\sqrt{x^2+y^2}+x)^2}\Bigg) = \frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x}\cdot$$

$$\cdot\frac{\left(\frac{2x}{2\sqrt{x^2+y^2}}-1\right)(\sqrt{x^2+y^2}+x)-\left(\frac{x}{\sqrt{x^2+y^2}}+1\right)(\sqrt{x^2+y^2}-x)}{(\sqrt{x^2+y^2}+x)^2}=$$

$$=\frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x}\cdot\left(\frac{x-\sqrt{x^2+y^2}+\frac{x^2}{\sqrt{x^2+y^2}}-x-\sqrt{x^2+y^2}+\frac{x^2}{\sqrt{x^2+y^2}}}{(\sqrt{x^2+y^2}+x)^2}\right)$$

$$=\frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x}\cdot\left(\frac{\frac{2x^2}{\sqrt{x^2+y^2}}-2\sqrt{x^2+y^2}}{(\sqrt{x^2+y^2}+x)^2}\right)$$

$$z'_y=\frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x}\cdot\frac{\frac{y}{\sqrt{x^2+y^2}}(\sqrt{x^2+y^2}+x)-\frac{y}{\sqrt{x^2+y^2}}(\sqrt{x^2+y^2}-x)}{(\sqrt{x^2+y^2}+x)^2}=$$

$$=\frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x}\cdot\frac{\frac{y}{\sqrt{x^2+y^2}}(\sqrt{x^2+y^2}+x-\sqrt{x^2+y^2}+x)}{(\sqrt{x^2+y^2}+x)^2}=$$

$$=\frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x}\cdot\frac{\frac{2xy}{\sqrt{x^2+y^2}}}{(\sqrt{x^2+y^2}+x)^2}$$

$$z'=\frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x}\left(\frac{\frac{2x^2}{\sqrt{x^2+y^2}}-2\sqrt{x^2+y^2}}{(\sqrt{x^2+y^2}+x)^2}dx+\right.$$

$$\left.+\frac{2xy\,dy}{(\sqrt{x^2+y^2}+x)^2\sqrt{x^2+y^2}}\right)$$

11.3.35 $z=\arccos\frac{\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}}$

$$z'_x=\frac{1}{\sqrt{1-\left(\frac{\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}}\right)^2}}\cdot\left(\frac{\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}}\right)'_x=\frac{1}{\sqrt{1-\frac{x^2-y^2}{x^2+y^2}}}\cdot$$

$$\cdot\frac{(\sqrt{x^2-y^2})'_x(\sqrt{x^2+y^2})-(\sqrt{x^2+y^2})'_x(\sqrt{x^2-y^2})}{x^2+y^2}=\frac{1}{\sqrt{1-\frac{x^2-y^2}{x^2+y^2}}}\cdot$$

$$\cdot\frac{\frac{x\sqrt{x^2+y^2}}{\sqrt{x^2-y^2}}-\frac{x\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}}}{x^2+y^2}$$

$$z'_y=\frac{1}{\sqrt{1-\frac{x^2-y^2}{x^2+y^2}}}\cdot\frac{\frac{-y\sqrt{x^2+y^2}}{\sqrt{x^2-y^2}}-\frac{y\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}}}{x^2+y^2}$$

$$z' = \frac{1}{\sqrt{1-\frac{x^2-y^2}{x^2+y^2}}} \cdot \left(\frac{\frac{x\sqrt{x^2+y^2}}{\sqrt{x^2-y^2}} - \frac{x\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}}}{x^2+y^2} dx + \right.$$

$$\left. + \frac{\frac{-y\sqrt{x^2+y^2}}{\sqrt{x^2-y^2}} - \frac{y\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}}}{x^2+y^2} dy \right)$$

11.3.36 $z = \sin\frac{x}{y} \cdot \cos\frac{y}{x}$

$$z'_x = \left(\sin\frac{x}{y}\right)'_x \cos\frac{y}{x} + \left(\cos\frac{y}{x}\right)'_x \sin\frac{x}{y} = \cos\frac{x}{y}\cdot\left(\frac{x}{y}\right)'_x \cdot$$

$$\cdot \cos\frac{y}{x} - \sin\frac{y}{x}\left(\frac{y}{x}\right)'_x \sin\frac{x}{y} = \frac{1}{y}\cos\frac{x}{y}\cos\frac{y}{x} +$$

$$+ \frac{y}{x^2}\sin\frac{y}{x}\sin\frac{x}{y};$$

$$z'_y = \cos\frac{x}{y}\cdot\left(\frac{x}{y}\right)'_y \cos\frac{y}{x} - \frac{1}{x}\sin\frac{y}{x}\sin\frac{x}{y} = -\frac{x}{y^2}\cdot$$

$$\cdot\cos\frac{x}{y}\cos\frac{y}{x} - \frac{1}{x}\sin\frac{y}{x}\sin\frac{x}{y}$$

$$z' = \frac{1}{y}\cos\frac{x}{y}\cos\frac{y}{x}dx + \frac{y}{x^2}\sin\frac{y}{x}\sin\frac{x}{y}dx - \frac{1}{x}\cdot$$

$$\cdot\sin\frac{y}{x}\sin\frac{x}{y}dy - \frac{x}{y^2}\cdot\cos\frac{x}{y}\cos\frac{y}{x}dy$$

11.3.37 $z = (x^2+y^2)\frac{1-\sqrt{x^2+y^2}}{1+\sqrt{x^2+y^2}}$

$$z'_x = 2x\frac{1-\sqrt{x^2+y^2}}{1+\sqrt{x^2+y^2}} + \left(\frac{1-\sqrt{x^2+y^2}}{1+\sqrt{x^2+y^2}}\right)'_x (x^2+y^2) =$$

$$= \frac{2x(1-\sqrt{x^2+y^2})}{1+\sqrt{x^2+y^2}} + \frac{(1-\sqrt{x^2+y^2})'_x(1+\sqrt{x^2+y^2}) - (1+\sqrt{x^2+y^2})'_x(1-\sqrt{x^2+y^2})}{(1+\sqrt{x^2+y^2})^2}$$

$$\cdot(x^2+y^2) = \frac{2x(1-\sqrt{x^2+y^2})}{1+\sqrt{x^2+y^2}} + (x^2+y^2)\cdot$$

$$\cdot\frac{\frac{2x}{2\sqrt{x^2+y^2}}(1+\sqrt{x^2+y^2}) - \frac{x}{\sqrt{x^2+y^2}}(1-\sqrt{x^2+y^2})}{(1+\sqrt{x^2+y^2})^2} =$$

$$= \frac{2x(1-\sqrt{x^2+y^2})}{1+\sqrt{x^2+y^2}} + (x^2+y^2)\cdot\frac{\frac{x}{\sqrt{x^2+y^2}}(1+\sqrt{x^2+y^2}-1+\sqrt{x^2+y^2})}{(1+\sqrt{x^2+y^2})^2}$$

$$z'_y = \frac{2y(1-\sqrt{x^2+y^2})}{1+\sqrt{x^2+y^2}} + \left(\frac{1-\sqrt{x^2+y^2}}{1+\sqrt{x^2+y^2}}\right)'_y (x^2+y^2) =$$

$$= \frac{2y(1-\sqrt{x^2+y^2})}{1+\sqrt{x^2+y^2}} + \frac{\frac{y}{\sqrt{x^2+y^2}}(2\sqrt{x^2+y^2})}{(1+\sqrt{x^2+y^2})^2} \cdot (x^2+y^2)$$

$$z' = \frac{1-\sqrt{x^2+y^2}}{1+\sqrt{x^2+y^2}}(2x\,dx + 2y\,dy) + \frac{\frac{1}{\sqrt{x^2+y^2}}(2\sqrt{x^2+y^2})}{1+\sqrt{x^2+y^2}} \cdot$$

$$\cdot (x^2+y^2)(x\,dx + y\,dy)$$

11.3.38 $u = x^3 + yz^2 + 3yx - x + z$

$u'_x = 3x + 3y - 1$

$u'_y = z^2 + 3x$

$u'_z = 2yz + 1$

$u' = (3x+3y-1)dx + (z^2+3x)dy + (2yz+1)dz$

11.3.39 $u = x^{\frac{y}{z}}$

$u'_x = \frac{y}{z} \cdot x^{\frac{y}{z}-1}$

$u'_y = (x^{\frac{1}{z}} \cdot x^y)'_y = \cancel{y x^{\frac{1}{z}} \cdot x^{y-1}}\ x^{\frac{1}{z}} \cdot x^y \cdot$

$\cdot \ln x$

$u'_z = (x^{\frac{1}{z}} \cdot x^y)'_z = x^y \cdot x^{\frac{1}{z}} \cdot \ln x \cdot (-\frac{1}{z^2})$

$u' = \left(\frac{y}{z} \cdot x^{\frac{y}{z}-1}\right)dx + x^{\frac{1}{z}} \cdot x^y \cdot \ln x\left(dy\,\cdot -\right.$

$\left.\cdot - \frac{dz}{z^2}\right)$

11.3.40 $u = x^{y^z}$

$u'_x = y^z x^{y^z - 1}$

$u'_y = (x^{y^z})'_y = x^{y^z} \cdot \ln x \cdot (y^z)'_y = x^{y^z} \cdot \ln x \cdot$
$\cdot z y^{z-1}$

$u'_z = (x^{y^z})'_z = x^{y^z} \cdot \ln x \cdot (y^z)'_z = x^{y^z} \cdot \ln x \cdot$
$\cdot y^z \cdot \ln y$

$u' = y^z x^{y^z - 1} dx + x^{y^z} \cdot \ln x \cdot (z y^{z-1} dy +$
$y^z \cdot \ln y \, dz)$

11.3.41 $u'_x + u'_y + u'_z$ - ? при $x = y = z = 1$, если
$u = \ln(1 + x + y^2 + z^3)$.

$u'_x = (\ln(1 + x + y^2 + z^3))'_x = \frac{1}{1 + x + y^2 + z^3} \cdot$
$\cdot (1 + x + y^2 + z^3)'_x = \frac{1}{1 + x + y^2 + z^3}$

$u'_y = \frac{(1 + x + y^2 + z^3)'_y}{1 + x + y^2 + z^3} = \frac{2y}{1 + x + y^2 + z^3}$

$u'_z = \frac{(1 + x + y^2 + z^3)'_z}{1 + x + y^2 + z^3} = \frac{3z^2}{1 + x + y^2 + z^3}$

$u'_x + u'_y + u'_z = \frac{1}{1 + x + y^2 + z^3} + \frac{2y}{1 + x + y^2 + z^3} +$
$+ \frac{3z^2}{1 + x + y^2 + z^3} = \frac{1}{4} + \frac{2}{4} + \frac{3}{4} = \frac{6}{4} = 1{,}5$

11.3.42 $\frac{z'_x + z'_y}{z'_x z'_y}$; $x = 1$, $y = 2$, $z = x^3 y - x y^3$

$z'_x = 3 y x^2 - y^3 = 3 \cdot 2 \cdot 1^2 - 2^3 = 6 - 8 = -2$

$z'_y = x^3 - 3 x y^2 = 1^3 - 3 \cdot 1 \cdot 2^2 = 1 - 12 = -11$

$\frac{z'_x + z'_y}{z'_x z'_y} = \frac{-2 - 11}{-2 \cdot (-11)} = \frac{-13}{22}$

11.3.43 $\frac{\partial u}{\partial z}$; $x=0,\ y=0,\ z=\frac{\pi}{4}$, если

$$u=\sqrt{\sin^2 x+\sin^2 y+\sin^2 z}$$

$$u'_z=\frac{1}{2\sqrt{\sin^2 x+\sin^2 y+\sin^2 z}}\cdot(\sin^2 x+\sin^2 y+\sin^2 z)'_z=$$

$$=\frac{2\sin z\cdot\cos z}{2\sqrt{\sin^2 x+\sin^2 y+\sin^2 z}}$$

$$\frac{\sin z\cdot\cos z}{\sqrt{\sin^2 x+\sin^2 y+\sin^2 z}}=\frac{\sin\frac{\pi}{4}\cdot\cos\frac{\pi}{4}}{\sqrt{\sin^2\frac{\pi}{4}}}=$$

$$=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$$

11.3.44 $z=x+y-\sqrt{x^2+y^2}$; $x=3,\ y=4,\ \Delta x=0{,}1,$

$\Delta y=0{,}2.$

$$z'_x=1\cancel{+y}-(\sqrt{x^2+y^2})'_x=1\cancel{+y}-\frac{x}{\sqrt{x^2+y^2}}$$

$$z'_y=1-\frac{y}{\sqrt{x^2+y^2}}$$

$$z'=\left(1-\frac{x}{\sqrt{x^2+y^2}}\right)dx+\left(1-\frac{y}{\sqrt{x^2+y^2}}\right)dy\approx$$

$$\approx\left(1-\frac{x}{\sqrt{x^2+y^2}}\right)\Delta x+\left(1-\frac{y}{\sqrt{x^2+y^2}}\right)\Delta y=\left(1-\frac{3}{5}\right)\cdot$$

$$\cdot 0{,}1+\left(1-\frac{4}{5}\right)\cdot 0{,}2=0{,}04+0{,}04=0{,}08$$

11.3.45 $z=e^{xy}$; $x=1,\ y=1,\ \Delta x=0{,}15,$

$\Delta y=0{,}1.$

$$z'_x=ye^{xy}$$

$$z'_y=xe^{xy}$$

$$z'=ye^{xy}dx+xe^{xy}dy\approx ye^{xy}\Delta x+$$

$xe^{xy}\Delta y = e\cdot 0{,}15 + e\cdot 0{,}1 = 0{,}25e = \frac{e}{4}$

11.3.46 $z = \frac{x+3y}{y-3x}$; ~~x=1, y~~ от $x_1=2$ до $x_2=2{,}5$; от $y_1=4$ до $y_2=3{,}5$

$$z'_x = \frac{(x+3y)'_x(y-3x) - (y-3x)'_x(x+3y)}{(y-3x)^2} =$$

$$= \frac{(y-3x)+3(x+3y)}{(y-3x)^2} = \frac{10y}{(y-3x)^2}$$

$$z'_y = \frac{(x+3y)'_y(y-3x) - (y-3x)'_y(x+3y)}{(y-3x)^2} =$$

$$= \frac{3(y-3x)-(x+3y)}{(y-3x)^2} = \frac{-10x}{(y-3x)^2}$$

$$z' = \frac{10y}{(y-3x)^2}dx - \frac{10x}{(y-3x)^2}dy \approx \frac{10y}{(y-3x)^2}\Delta x -$$

$$- \frac{10x}{(y-3x)^2}\Delta y = \frac{10\cdot 4}{(4-6)^2}\cdot 0{,}5 - \frac{10\cdot 2}{(4-6)^2}\cdot(-0{,}5) =$$

$$= 10\cdot 0{,}5 + 5\cdot 0{,}5 = 7{,}5$$

11.3.47 $\sqrt{1{,}02^3 + 1{,}97^3}$

$x = 1,\ \Delta x = 0{,}02$

$y = 2,\ \Delta y = -0{,}03$

$$f'_x = \sqrt{x^3+y^3} = \frac{3x^2}{2\sqrt{x^3+y^3}} = \frac{3}{2\sqrt{9}} = \frac{1}{2}$$

$$f'_y = \sqrt{x^3+y^3} = \frac{3y^2}{2\sqrt{x^3+y^3}} = \frac{3\cdot 4}{2\cdot 3} = 2$$

$$\sqrt{1{,}02^3+1{,}97^3} \approx 3 + 0{,}01 + -0{,}06 \approx 2{,}95$$

11.3.48 $\sin 29^\circ \sin 46^\circ$

$x = 30 \quad \Delta x = -1$

$y = 45 \quad \Delta y = 1$

$f'_x = (\sin x \sin y)'_x = \sin y \cos x = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{4}$

$f'_y = (\sin x \sin y)'_y = \sin x \cos y = \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{4}$

$f(30°; 45° = \sin 30° \sin 45° = \frac{\sqrt{2}}{4}$

$\sin 29° \sin 46° \approx \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} \cdot \frac{\pi}{180} - \frac{\sqrt{2}}{4} \cdot \frac{\pi}{180} \approx$

$\approx \frac{\sqrt{2}}{4} + \frac{\pi}{180}\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) \approx 0{,}354 + 0{,}017 \cdot$

$\cdot \left(\frac{2{,}45 - 1{,}414}{4}\right) \approx 0{,}354 + 0{,}017 \cdot 0{,}26 \approx$

$0{,}354 + 0{,}0044 \approx 0{,}3584$

11.3.49 $\operatorname{arctg}\left(\frac{1{,}97}{102} - 1\right)$

$x = 2, \Delta x = -0{,}03$

$y = 1, \Delta y = 0{,}02$

$f_x' = \left(\operatorname{arctg}\left(\frac{x}{y} - 1\right)\right)'_x = \frac{1}{1+\left(\frac{x}{y}-1\right)^2} \left(\frac{x}{y}\right)'_x =$

$= \frac{1/y}{1+\left(\frac{x}{y}-1\right)^2} = \frac{\cancel{0,5}\ 1}{2}$

$f_y' = \left(\operatorname{arctg}\left(\frac{x}{y} - 1\right)\right)'_y = \frac{1}{1+\left(\frac{x}{y}-1\right)^2} \left(\frac{x}{y}\right)'_y =$

$= \frac{-x/y^2}{1+\left(\frac{x}{y}-1\right)^2} = \frac{-2}{2} = -1$

$\operatorname{arctg}\left(\frac{1{,}97}{1{,}02} - 1\right) \approx 0 + \frac{1}{2} \cdot (-0{,}03) +$

$+ (-1) \cdot 0{,}02 = -0{,}015 - 0{,}02 = -0{,}035$

11.3.50 $2{,}003^2 \cdot 3{,}998^3 \cdot 1{,}002^2$

$x = 2, \Delta x = 0{,}003$

$y = 4, \Delta y = -0{,}002$

$z = 1, \Delta z = 0{,}002$

$$f'_x = (x^2y^3z^2)'_x = 2y^3z^2x = 2\cdot4^3\cdot1\cdot2 = \cancel{4}\,256$$

$$f'_y = (x^2y^3z^2)'_y = 3x^2z^2y^2 = 2^2\cdot3\cdot4^2 = 192$$

$$f'_z = (x^2y^3z^2)'_z = 2x^2y^3z = \cancel{8}\,2\cdot4\cdot4^3 = 512$$

$$f(2;4;1) = \cancel{8}4\cdot64 = 256$$

$$2{,}003^2\cdot3{,}998^2\cdot1{,}002^2 \approx 256 + 256\cdot0{,}003 -$$

$$- 192\cdot0{,}002 + 512\cdot0{,}002 = 256 + 0{,}768 -$$

$$- 0{,}384 + 1{,}024 = 257{,}408$$