8.3.5 $\int \frac{dx}{x^2+10x+29} = \Big[x^2+10x+29$
$D = 10^2 - 4\cdot 29 = -16 \Rightarrow$

$\Rightarrow$ корней нет $\Rightarrow$ III тип простейшей дроби

$\frac{Ax+B}{x^2+px+q} = \frac{\frac{A}{2}(2x+p)+(B-\frac{Ap}{2})}{x^2+px+q}$; $A=0$ $B=1$ $p=10$ $q=29$

$= \Big[\int \frac{\frac{0}{2}(2x+10)+(1-\frac{0\cdot 10}{2})dx}{x^2+10x+29} \Big] = \Big[\int \frac{dx}{x^2+px+q},$

$y = x+\frac{p}{2}$; $y = x+\frac{10}{2} = x+5 \Rightarrow dy = dx$;

$x^2+px+q \to y^2+a^2$, $a = \sqrt{q-p^2/4}$

$A=0$, $B=1$

$p=10$, $q=29$; $y = x+\frac{p}{2} = x+\frac{10}{2} = x+5 \Rightarrow$

$\Rightarrow dy = dx$; $x^2+px+q = y^2+a^2 = y^2+$

$+(\sqrt{q-\frac{p^2}{4}})^2 = y^2+(\sqrt{29-\frac{10^2}{4}})^2 = y^2+2^2 = y^2+4\Big] =$

$= \int \frac{dy}{y^2+4} = \Big[\int \frac{dx}{x^2+a^2} = \underbrace{\frac{1}{a}}_{\frac{2}{\sqrt{4q-p^2}}} \cdot arctg \underbrace{\frac{x}{a}}_{\frac{2x+p}{\sqrt{4q-p^2}}} + C \Big] =$

$= \underbrace{\frac{1}{2}}_{} \cdot arctg \frac{y}{2} + C$

$\frac{2}{\sqrt{4\cdot 29-10^2}} = \frac{2}{\sqrt{16}} = \frac{1}{2}$

8.3.6 $\int \frac{(x+6)dx}{x^2-2x+17} = \Big[x^2-2x+17=0$
$D = (-2)^2 - 4\cdot 17 = 4-68 = -64$

$A=1$ $B=6$

$p=-2$ $q=17$; $x+6 = \frac{1}{2}(2x-2)+6+1 = \frac{1}{2}(2x-2)+7$

$$= \frac{1}{2}\int \frac{2x-2}{x^2-2x+17}\,dx + 7\int \frac{dx}{x^2-2x+17} = \Big[\ 1)\ t = x^2 + px + q$$

$t = x^2 - 2x + 17 \Rightarrow dt = (2x-2)\,dx$

2) $y = x + \frac{p}{2} \Rightarrow dy = dx$

$y = x + \frac{-2}{2} = x - 1 \Rightarrow dy = dx$; $x^2 + px + q = y^2 + a^2 =$

$= y^2 + \left(\sqrt{q - \frac{p^2}{4}}\right)^2$; $x^2 - 2x + 17 = y^2 + 16\ \Big] =$

$$= \frac{1}{2}\int \frac{dt}{t} + 7\int \frac{dy}{y^2+16} = \frac{1}{2}\ln|t| + 7 \cdot \frac{1}{4}\,\mathrm{arctg}\,\frac{y}{4}$$

$$= \frac{1}{2}\ln(x^2 - 2x + 17) + \frac{7}{4}\,\mathrm{arctg}\,\frac{x-1}{4} + C$$

8.3.7 $\displaystyle\int \frac{(4x-1)\,dx}{x^2+x+1} = \Big[\ x^2 + x + 1$ $\quad A = 4\ \ B = -1$
$D = 1 - 4 = -3 \quad p = 1\ \ q = 1$

$4x - 1 = \frac{4}{2}(2x+1) + (-1 - 2) = \frac{4}{2}(2x+1) - 3\ \Big] = \frac{4}{2}\ \cdot$

$$\cdot \int \frac{(2x+1)\,dx}{x^2 + x + 1} - 3\int \frac{dx}{x^2 + x + 1} = \Big[\ t = x^2 + x + 1 \Rightarrow$$

$dt = (2x+1)\,dx \quad y = x + \frac{1}{2} \Rightarrow x^2 + x + 1 = y^2 + \frac{3}{4}$

$$= 2\ln|t| - \frac{\sqrt{3}}{2}\,\mathrm{arctg}\,\frac{2y}{\sqrt{3}} = 2\ln(x^2 + x + 1) -$$

$$- 2\sqrt{3}\,\mathrm{arctg}\,\frac{2x+1}{\sqrt{3}} + C$$

8.3.9 $\displaystyle\int \frac{dx}{(x^2+1)^3} = \Big[\ x^2 + 1 = 0 \Rightarrow x^2 = -1 \Rightarrow$ нет корней
$\Rightarrow$ IV тип: $\displaystyle\int \frac{Ax+B}{(x^2+px+q)^n}\,dx$

$$\int \frac{dx}{(x^2+px+q)^n} = \Big[\ y = x + \frac{p}{2},\ a = \sqrt{q - \frac{p^2}{4}}\ \Big] \quad A = 0,\ B = 1$$

$x^2 + px + q = y^2 + a^2 \quad p = 0,\ q = 1$

$y = x + \frac{p}{2} = x + \frac{0}{2} = x,\ a = \sqrt{1 - \frac{0^2}{4}} = \sqrt{1} = 1$

$x^2 + 1 = y^2 + a^2 = y^2 + 1^2;\ dy = dx\ \Big] = \displaystyle\int \frac{dy}{(y^2+1^2)^3}$

$$= \left[\int \frac{dy}{(y^2+a^2)^n} = \frac{1}{2(n-1)\cdot a^2}\cdot\frac{y}{(y^2+a^2)^{n-1}} + \frac{1}{a^2}\cdot\frac{2n-3}{2n-2}\cdot\int\frac{dy}{(y^2+a^2)^{n-1}}\right] = \frac{1}{4}\cdot\frac{y}{(y^2+1^2)^2} + \frac{3}{4}\cdot\int\frac{dy}{(y^2+1^2)^2} =$$

$$= \frac{1}{4}\cdot\frac{y}{(y^2+1)^2} + \frac{3}{4}\cdot\left(\cancel{\int dy}\ \frac{1}{2}\cdot\frac{y}{y^2+1} + \frac{1}{2}\cdot\int\frac{dy}{y^2+1}\right) =$$

$$= \frac{y}{4(y^2+1)^2} + \frac{3y}{8(y^2+1)} + \frac{3}{8}\int\frac{dy}{y^2+1} = \frac{y}{4(y^2+1)^2} + \frac{3y}{8(y^2+1)} +$$

$$+ \frac{3}{8}\cdot\cancel{\frac{1}{2y}\ln\left|\frac{y+1}{y}\right|}\ \frac{1}{y}\,\mathrm{arctg}\,\frac{1}{y} + C = \frac{x}{4(x^2+1)^2} + \frac{3x}{8x^2+8} +$$

$$+ \frac{3}{8x}\,\mathrm{arctg}\,\frac{1}{x} + C$$