Force Control of an RPR manipulator
EL 522 Final Project

Griswald Brooks

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Abstract

While positioning of manipulator end effectors is an important task, often applying a particular force to the environment is needed. This paper describes a basic hybrid force control technique and presents simulation results.

Contents

Chapter 1

Introduction

1.1 Concept and Motivation

Many manipulator applications require specific amounts of force to be applied to the environment. Tasks include machining, object manipulation, and part assembly. Without force control, the robot would apply an arbitrary amount of force as a function of its joint error and interaction with the environment. This would lead to unpredictable results and possible manipulator instability, e.g. causing the end effector to bounce or damage the object or environment with which it's interacting.

One method of force control is a feed forward method called impedance control. In this method the environment is modeled as a mass-spring or a mass-spring-dampened system and the required displacement needed for a desired force is calculated (1.1). The manipulator is then commanded to its intended tracking plus this displacement. This can be though of as the manipulator tracking an inner virtual wall.

$$\delta x = \frac{F_d}{k} \tag{1.1}$$

Where F_d is the desired force, k is the modeled wall spring constant,

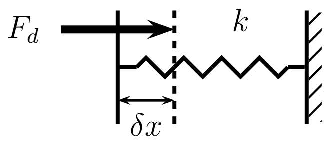

Figure 1.1: Mass-spring wall model with virtual wall.

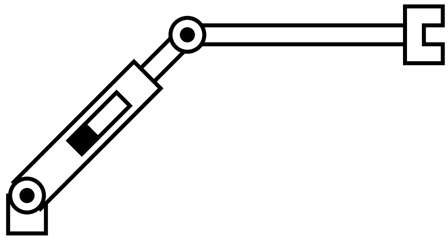

Figure 1.2: Schematic diagram of RPR manipulator

and δx is the wall displacement necessary to produce F_d. One drawback of this method is that it assumes perfect knowledge of the environment. If the wall stiffness varies the applied force will vary. To supplement this, a force/torque sensor can be added to the end effector which provides feedback to the controller allowing the applied force to be modulated. This method is known as Hybrid Force Control and is the technique used in this report.

1.2 Manipulator Model

The manipulator considered in this project is the RPR manipulator. It has a revolute joint at its base and wrist which are coplanar, and a prismatic joint in between, Figure 1.2.

1.2.1 Kinematics

The kinematic model of the manipulator is given by the transformation matrices (1.2), resulting in an end effector transformation of (1.3). The DH parameters for the manipulator are given in Table 1.1.

$$A_1 = \begin{bmatrix} cos\theta_1 & 0 & sin\theta_1 & 0 \\ sin\theta_1 & 0 & -cos\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.2a}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.2b}$$

$$A_3 = \begin{bmatrix} cos\theta_3 & -sin\theta_3 & 0 & a_3 cos\theta_3 \\ sin\theta_3 & cos\theta_3 & 0 & a_3 sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.2c}$$

Joint	α_i	a_i	d_i	θ_i
1	0	$90°$	0	θ_1
2	0	$-90°$	d_2	0
3	a_3	0	0	θ_3

Table 1.1: DH parameters for RPR manipulator.

$$A_1A_2A_3 = \begin{bmatrix} c_{13} & -s_{13} & 0 & a_3c_{13} + d_2s_1 \\ s_{13} & c_{13} & 0 & a_3s_{13} - d_2c_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.3}$$

Using the standard input tranformation matrix seen in (1.4),the inverse kinematics of the manipulator are given by (1.5). These were used to calculate the desired joint positions, given an end effector reference trajectory.

$$H_{in} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.4}$$

$$\theta_1 + \theta_3 = atan2(r_{21}, r_{11}) \tag{1.5a}$$
$$\theta_1 = atan2(d_x - a_3c_{13}, a_3s_{13} - d_y) \tag{1.5b}$$
$$d_2 = \sqrt{(d_x - a_3c_{13})^2 + (d_y - a_3s_{13})^2} \tag{1.5c}$$
$$\theta_3 = atan2(r_{21}, r_{11}) - \theta_1 \tag{1.5d}$$

The desired joint velocities $\dot{q}$ were given using the inverse Jacobian, (1.6). This involved first calculating the Jacobian, (1.7), and numerically calculating the Moore-Penrose pseudoinverse.

$$\dot{q} = J^{\#}v \tag{1.6}$$

Where v is a 6×1 column vector containing the three end effector translational velocities and three angular velocities.

$$J = \begin{bmatrix} -a_3 s_{13} + d_2 c_1 & s_1 & -a_3 s_{13} \\ a_3 c_{13} + d_2 s_1 & -c_1 & a_3 c_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \tag{1.7}$$

Following this, the desired joint accelerations $\ddot{q}$ were given by (1.9) and calculated using the aforementioned inverse Jacobian and the element-wise time derivative of the Jacobian (1.8).

$$\dot{J} = \begin{bmatrix} -a_3 c_{13}(\dot{\theta}_1 + \dot{\theta}_3) + \dot{d}_2 c_1 - d_2 s_1 \dot{\theta}_1 & c_1 \dot{\theta}_1 & -a_3 c_{13}(\dot{\theta}_1 + \dot{\theta}_3) \\ -a_3 s_{13}(\dot{\theta}_1 + \dot{\theta}_3) + \dot{d}_2 s_1 + d_2 c_1 \dot{\theta}_1 & s_1 \dot{\theta}_1 & -a_3 s_{13}(\dot{\theta}_1 + \dot{\theta}_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1.8}$$

$$\ddot{q} = J^{\#}(\dot{v} - \dot{J}\dot{q}) \tag{1.9}$$

1.2.2 Dynamics

Modeling the manipulator as a collection of point masses located at the revolute joints and end effector, the dynamics of the manipulator were given by (1.10).

$$D(q)\ddot{q} + C(q, \dot{q}) + G(q) = U + B(\dot{q}) + J(q)^T F \tag{1.10}$$

Where $D(q)$ is the inertia matrix (1.11), $C(q, \dot{q})$ is the centripetal vector (1.12), $G(q)$ are all the gravity terms (1.13), U is the control input, $B(\dot{q})$ are the dampening terms due to viscous joint friction (1.14), and F is the generalized force vector of forces/torques applied to the end effector (1.15).

$$D(q) = \begin{bmatrix} (m_2 + m_3)d_2^2 + m_3 a_3(1 - 2d_2 s_3) & -m_3 a_3 c_3 & m_3 a_3(1 - d_2 s_3) \\ -m_3 a_3 c_3 & m_2 + m_3 & -m_3 a_3 c_3 \\ m_3 a_3(1 - d_2 s_3) & -m_3 a_3 c_3 & m_3 a_3 \end{bmatrix} \tag{1.11}$$

$$C(q,\dot{q}) = \begin{bmatrix} 2d_2\dot{d}_2\dot{\theta}_1(m_2+m_3) + m_3a_3(\dot{d}_2s_3 - (\dot{d}_2s_3 + d_2c_3\dot{\theta}_3)(2\dot{\theta}_1+\dot{\theta}_3)) \\ m_3a_3s_3(\dot{\theta}_1+\dot{\theta}_3)^2 - (m_2+m_3)d_2\dot{\theta}_1^2 \\ m_3a_3(d_2\dot{\theta}_1\dot{\theta}_3c_3 - 2s_3\dot{d}_2\dot{\theta}_1) \end{bmatrix} \tag{1.12}$$

$$G(q) = g\begin{bmatrix} (m_2+m_3)d_2s_1 + m_3a_3c_{13} \\ -(m_2+m_3)c_1 \\ m_3a_3c_{13} \end{bmatrix} \tag{1.13}$$

Where m_2 and m_3 are the masses at the elbow (joint 3) and the end effector respectively and g is the acceleration due to gravity in $\frac{m}{s^2}$.

$$B(\dot{q}) = \begin{bmatrix} -b_1\dot{\theta}_1 \\ -b_2\dot{d}_2 \\ -b_3\dot{\theta}_3 \end{bmatrix} \tag{1.14}$$

For this simulation, the viscous dampening coefficients were $b_1 = b_2 = b_3 = 0.1\frac{Ns}{m}$.

$$F = \begin{bmatrix} f_x, f_y, f_z, \tau_x, \tau_y, \tau_z \end{bmatrix}^T \tag{1.15}$$

Where f_n represents a force in the n direction and τ_n represents a torque along the n axis. The details about the elements used for this simulation are discussed in Section 1.3.

1.3 Environmental Model

As mentioned in Section 1.1, hybrid force control is being simulated and as such, the environment with which the manipulator is going to interact, i.e. a wall, is modeled as a mass-spring system. In addition to a reactive force in the direction normal to the wall, a term for viscous friction c is added for when the manipulator is 'dragging' against the wall. This results in a force vector applied to the end effector when contacting the wall of (1.16).

$$F_w = \begin{bmatrix} k\delta x, -cv_y, 0, 0, 0, 0 \end{bmatrix}^T \tag{1.16}$$

Where δx in this case is always taken to be negative when the end effector is 'indenting' the wall. According to specification, k can take on any value from $6000\frac{N}{m}$ to $60000\frac{N}{m}$, so in simulation the wall stiffness k is taken as a random number within this range.

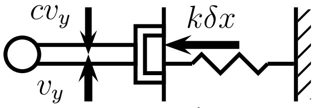

Figure 1.3: Wall modeled as a mass-spring system with viscous friction.

1.4 Feedback Linearization

1.5 Hybrid Force Control

Chapter 2

Results

2.1 Position Error

2.2 Force Error

Chapter 3

Conclusion