Swarm consensus

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Abstract

The strength of gnomes lies in their coordinated action. Being small and subtle creatures themselves, the forest gnomes can form large swarms acting as one giant creature. This unusual defense strategy requires a lot of skill and training. Directing a swarm is not an easy task! Initially, gnomes used leader-based control algorithms, although those have been proven to be vulnerable to abuse and failure.

After thorough research and study, gnomes developed their own leaderless consensus algorithm based on very simple rules. It is based on gossip in a network of a known diameter d. One of the gnomes proposes a plan which then spreads gnome to gnome. If there is an agreement, gnomes act *all at once*. If there are conflicting plans (an extreme rarity), they try again. The resulting upper bound on the swarm's reaction time is its round-trip time $2dt$, where t is the command relay time. The original algorithm is non-Byzantine; all gnomes must be sane and sober.

While working on the algorithm, gnomes discovered *swarm time*, a sibling concept to L. Lamport's logical time. That led to a Byzantine-ready version of the algorithm.

Running a swarm requires perfect coordination and consensus. It has to adapt rapidly to the changing environment, as well as its own changing form and composition. As certain incidents have shown, the centralized mode of coordination is prone to abuse and failure. Namely, the leader may prioritize his own personal interest, while disappearance of the leader surely disorganizes the swarm. Gnomes had to invent a better way!

They started by thinking their assumptions and limitations through. First of all, gnomes can join a swarm at any time, then fall off and rejoin at any rate. Hence, any complex division of roles is impractical; especially, everything election-based. Also, a gnome's communication ability is limited and the processing ability is even more limited. Any sophisticated algorithms are definitely out of question. Most of the gnomes can only repeat and relay commands, while also doing their physical work.

Luckily, gnomes can relay messages very efficiently, in uniform time t. While they did not want a central leader anymore, they have pretty good ethics. So, they may rely on the existence of naturally emerging leaders who will propose necessary maneuvers at the right time. They rarely have many proposers at once, but there is always someone. Finally, a swarm requires that all gnomes act synchronously, without a time lag. That was the most tricky requirement.

Figure 1: An artist's impression of a gnome swarm. © Disney

Gnomes studied the literature on databases and distributed systems, as well as on swarm behavior of fishes, birds and humans. They studied 2 phase commit [1], 3 phase commit [2], Paxos [3] and Raft [4]. Sadly, all these algorithms focus on a consensus between a small number of known participants. Similarly, they imply the existence of a leader giving orders unilaterally. Even if the leader is elected, that poses a significant communication overhead and delay. Gnomes can tolerate a temporary confusion, but voting, counting and doing complex moves all at once is just too difficult. By the end of such an election the gnomes will be laying on the ground, in the most optimistic case! Whom that leader will command then?

Proof-of-work [5] (aka the Nakamoto consensus) was laughed off. Gnomes will not burn their limited energy simply to prove they are decent gnomes! Gnomes know each other, to start with, so the problem itself is non-existant, not to mention the price of the solution. Similarly, the Byzantine generals problem was found to be irrelevant at first. Gnomes know who their friends are. What gnomes actually needed was a non-Byzantine consensus algorithm for larger dynamic groups where participants may come and go. Also, their communication ability is limited, so a gnome can only talk to his swarm neighbors, not all the gnomes at once. Luckily, any corruption of a message is very easy to detect when everybody is relaying it and you can hear it from all sides.

The work led by late S. Q. Locke Sr resulted in a consensus algorithm based on buddycast message relay in an open network. Neither the full list of participants, nor their number is known at any time. The exact network topology is quite dynamic, so better to consider it an unknown as well. The only postulated requirement is that the diameter of the network is bounded and the upper bound d is known. In other words, any gnome can reach any other gnome through a neighbor chain of d steps or less. In a swarm, that mostly occurs naturally.

The consensus algorithm is simple: one gnome proposes an action; other gnomes relay it if they consider it reasonable. If all gnomes agree, they act all at once. The tricky part is to know whether all gnomes agree on an action. Preferably, quickly. Preferably, at once. The exact algorithm is as follows:

1. Gnome $p \in \mathbb{G}$ proposes an action to his swarm neighbors $n \in N(p)$, where $N(g)$ is the set of gnomes who can hear g, including himself. Note that $h \in N(g) \iff g \in N(h)$. We define a k-neighborhood of gnome g as a set of gnomes being at most k steps away: $N^{-1}(g) := \varnothing$, $N^0(g) := \{g\}$, and recursively for $k \geq 0$, $N^{k+1}(g) := \bigcup_{h \in N^k(g)} N(h)$. Note that $N^d(g) = \mathbb{G}$ where d is the upper bound for the diameter of the swarm and $\mathbb{G}$ is all the swarm's gnomes.

2. Every gnome g hearing an action proposed, relays it to his neighbors $N(g)$. Note that gnomes have no difficulties relaying and listening to all the neighbors at once, and for each gnome, it takes exactly the same time to make an announce. For that reason, we speak of *turns* of conversation.

3. Each gnome g tracks the spread of the proposal in the following way: once all his neighbors $n \in N(g)$ say their k-neighborhood (or bigger) is aware of the proposal, g realizes that his $(k+1)$-neighborhood is now aware and announces that on the next turn. Recursively, that announce may trigger *awareness neighborhood* increases for the neighbors.

4. Let us define a function α that tracks the radius of that I-know-that-they-know neighborhood. For gnomes unaware of the action, $\alpha_0(g) := -1$. For the proposer, we set $\alpha_0(p) := 0$. Then recursively, $\alpha_{t+1}(g) := 1 + \min\{\alpha_t(n) : n \in N(g)\}$ if $\exists n \in N(g), \alpha_t(n) \geq 0$. Otherwise, for the unaware gnomes, $\alpha_{t+1}(g)$ is still -1. The awareness neighborhood itself is thus $A_t(g) := N^{\alpha_t(g)}(g)$.

5. Once a gnome's d-neighborhood agrees on the proposal, he realizes that everyone now agrees. Then, he acts immediately! $\alpha_t(g) = d \Rightarrow A_t(g) = N^d(g) = \mathbb{G}$. That completes a consensus round; gnomes become ready for new proposals.

6. Once a gnome hears of two contradicting proposals at once, he gets confused and announces that $\alpha_t(g) = -\infty$.

7. Confusion spreads as gnomes relay it. That precludes the d-neighborhood consensus. The proposers have to think again and maybe retry proposing after a timeout, in the new round.

The key finding is that gnomes reach d-neighborhood consensus all at once, so they act all at once! That fact is completely independent of the number of the gnomes in the swarm or the way they are arranged. The only requirement is any two gnomes being connected by a neighbor-to-neighbor chain of length d or less. That is the Schmebulock's consensus theorem, so let us prove it for the humans. Unless noted otherwise, we assume no confusion happens.

Lemma 1. *The awareness neighborhood grows,* $\alpha_{t+1}(g) \geq \alpha_t(g)$.

Lemma 2. *Once* $\alpha_t(g) \geq d$, *g knows that all gnomes are aware of the proposal.*

Definition 3. The swarm's *bottom* is the gnomes who progressed the least in reaching the consensus: their awareness neighborhood radius is the smallest, $b_t := \min\{\alpha_t(g) : g \in \mathbb{G}\}$, $B_t := \{g \in \mathbb{G} : \alpha_t(g) = b_t\}$.

Suppose $r(p)$ is the maximum distance from the proposer p to any other gnome, $r(p) = \min\{k : N^k(p) = \mathbb{G}\}$. Then, p's most remote gnomes are $R_p = \mathbb{G} - N^{r(p)-1}(p)$. Note that at turn $r(p) - 1$, the remote gnomes are not aware of the proposal yet, $\forall g \in R_p, \alpha_{r(p)-1}(g) = -1$, while some their neighbors just became aware. Next turn is $r(p)$ when all remote gnomes become aware, while all their neighbors stay with $\alpha_{r(p)}(g) = 0$. The rest of the swarm will have $\alpha_{r(p)}(g) > 0$ already. Following this dynamics, we can see that

Lemma 4. *Initially,* $b_{r(p)-1} = -1$ *and* $B_{r(p)-1} = R_p$*. Later, for* $t \geq r(p) - 1$, $b_{t+1} = b_t + 1$ *and* $B_{t+1} = \bigcup_{i \in B_t} N(i)$*.*

In other words, $b_{r(p)+k} = k$ and all the bottom's neighbors also join the bottom. Naturally, in another d turns $b_{r(p)+d} = d$ and $B_{r(p)+d} = \mathbb{G}$, so all gnomes act on turn $r(p) + d$ the latest. Then, the question is: can any gnome act earlier than turn $r(p) + d$? None can, but let us suppose by contradiction that there is a gnome h such that $\alpha_{r(p)+d-j}(h) \geq d$ for $j \geq 1$. By applying the definition of α recursively we get:

Lemma 5. *If* $\alpha_k(h) \geq l$, $0 \leq i \leq l$ *and* $e \in N^i(h)$, *then* $\alpha_{k-i}(e) \geq l - i$.

Then, if substituting i, l for d, k for $r(p) + d - j$, we get that $\forall e \in N^d(h)$, $\alpha_{r(p)+d-j-d}(e) \geq d - d$, so $\alpha_{r(p)-j}(e) \geq 0$, where e might be in R_p as $N^i(h) = N^d(h) = \mathbb{G}$ which contradicts the fact that $\alpha_{r(p)-j}(e) \leq \alpha_{r(p)-1}(e) = -1$.

Theorem 6. *On turn* $2d$ *the latest, gnomes will become aware that they all agree, all at once.* $\exists t \leq 2d : \forall g \in \mathbb{G},\ \alpha_t(g) = d$ *and* $\forall t' < t : \alpha_{t'}(g) < d$. □

A gnome swarm rarely reaches a thousand members, but we made a computer simulation for a million-strong swarm and the algorithm works well in that case, see Fig. 2. For many common graph topologies, the diameter is logarithmic to the size. For the social graph of humanity, d is believed to be 6 ("six degrees of separation"). Hence, the swarm reaction time can be small for very large graphs. Also, the number of messages a node has to process is linear to the number of its edges. In other words, the algorithm is very scalable. Many consensus algorithms require every node to talk to or at least to be aware of every other node. That makes the number of messages quadratic to the graph size $O(N^2)$. In our case, a feasible upper bound is $O(Ne \log N)$ messages, where e is the number of edges per node.

Swarm time is a wonderful concept which is both similar to and the exact opposite of L. Lamport's *logical time* [6]. Remember that we defined $\alpha_t(g)$ as the radius of a gnome's I-know-that-they-know neighborhood. After $2d$ turns, the swarm synchronizes so $\forall g, h \in \mathbb{G} : \alpha_{2d+k}(g) = \alpha_{2d+k}(h)$. Still, nothing prevents the gnomes from counting indefinitely. Beyond d steps, the notion of *neighborhood* has no meaning: $N^{d+k}(g) = N^{d+l}(h) = \mathbb{G}$. Surprisingly, we can also see α_t as a time metric as it is monotonous for each gnome and roughly synchronous for the swarm. The Schmebulock's algorithms lets a swarm create a shared clock! In the simplest and most popular case, Lamport's logical time is defined as a *maximum* of incoming time values plus 1. The swarm time is a *minimum* of incoming time values plus 1. The mission of both metrics is exactly the same: establish a shared clock based solely on message passing

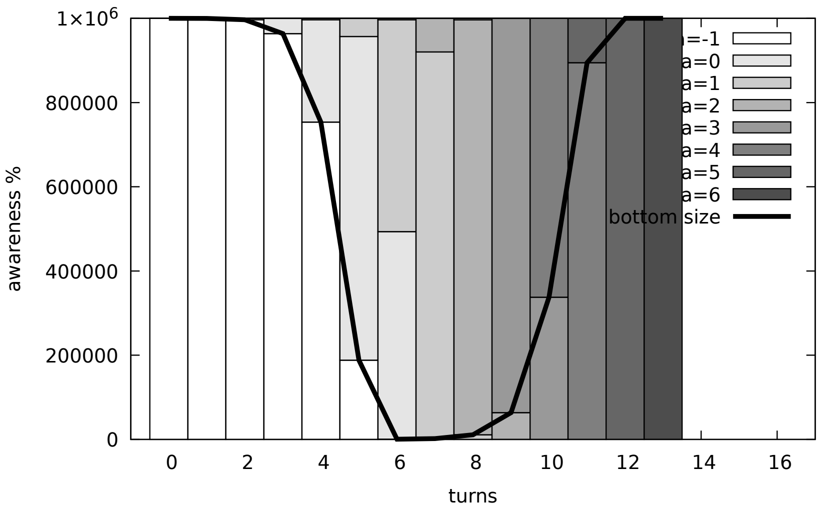


Figure 2: Simulation: one-million gnome swarm synchronizes in 14 turns. Bars show the percentage of gnomes progressed to a certain awareness neighborhood radius $\alpha_t(g)$. The line shows the number of bottom gnomes.

between distributed processes. The way of timekeeping is a cornerstone of any distributed architecture. The BitCoin paper [5] describes most of its machinery as a way to "implement a distributed timestamp server on a peer-to-peer basis". The Google Spanner paper refers to its satellite-based TrueTime system as "the key enabler" [7]. Authors believe that swarm time is the best fit for massively distributed self-synchronizing swarmed systems that we yet have to build.

Real-world considerations for the algorithm are manifold. Three main concerns are node churn, varying transmission times, faulty and Byzantine behavior. Indeed, gnomes may fall off the swarm, some gnomes might be slower than others, and finally, some may be unsober. To address that, we make three separate generalizations to the algorithm.

To account for node churn, we add two rules. First, a newly joining gnome must be ignored till the start of the next round. Second, the diameter upper bound d must hold. Re-assessing d in a moving swarm is difficult, so gnomes must keep the actual diameter under that pre-agreed bound at all times.

To account for the varying speed of message passing, we have to separate the physical time τ measured in seconds from the logical turns of the message exchange. As before, $\tau = 0$ is the moment of the proposer's initial announce. Assuming every gnome is able to convey his state change in $\tau_{\max}$ seconds or less, the physical swarm diameter is $D \leq d\tau_{\max}$. Then, $\alpha_\tau(g)$ is defined as 1 plus the minimum of $\alpha_{\tau'}(n)$ so far received from $n \in N(g)$. This change does not affect the dynamics of the algorithm much as *min* is not sensitive to the order of arguments. Hence, the relative speed of updates does not change much compared to the discrete case.

Finally, to address Byzantine faults we would need both swarm time and a multiphase consensus strategy. We define a gnome's *consensus phase* as his degree of knowledge about the swarm's knowledge. Namely, a gnome has phase 0 if he is aware of the proposal. A gnome has phase $k+1$, iff he is aware of the proposal and knows that all other gnomes reached phase k on that proposal. Note that a gnome reaching phase k also has all the earlier phases. Let us formulate the Schmebulock's theorem with all those generalizations in mind.

Definition 7. The swarm's *bottom time* is the minimum swarm time at a given moment τ of physical time, $\mu_\tau := \min\{\alpha_\tau(g) : g \in \mathbb{G}\}$.

Lemma 8. *Similarly to Lemma 4, if* $\mu_\tau \geq 0$*, then* $\mu_{\tau+\tau_{\max}} \geq \mu_\tau + 1$.

Corollary 9. $\mu_\tau \geq \lfloor \frac{\tau}{\tau_{\max}} \rfloor - d$.

Lemma 10. *For any two gnomes, the swarm time differs at most by* d.

Lemma 11. *At time* τ*,* $g, h \in \mathbb{G}$*, then* g *knows that* $\alpha_\tau(h) \geq \alpha_\tau(g) - d$.

Theorem 12. *At time* $\tau \geq 0$*, each gnome has consensus phase* $\lfloor \frac{\tau}{d\tau_{\max}} \rfloor - 1$.

Byzantine attacks can be a serious issue for a swarm once some gnomes abuse certain mushrooms and consequently become unsober and unreasonable. As a first mitigation, there is a list of simple sanity checks swarm neighbors use to expel violators:

1. a gnome can not backtrack by announcing a smaller neighborhood number than before, $\alpha_{t+1}(g) \geq \alpha_t(g)$,
2. a gnome can not stay at the same neighborhood number for more than 2 turns (unless the gnome is unaware of any new actions), $\alpha_{t+2}(g) > \alpha_t(g)$,
3. a neighbor can not announce a number greater than the number we announced to him, plus one $n \in N(g)$ then $\alpha_{t+1}(n) \leq \alpha_t(g) + 1$,
4. a gnome can not announce a different action unless the previous action was completed or the confusion timeout has passed,
5. a gnome can not keep progressing after being told that there is a conflicting proposal (must become confused).

But, it becomes much worse if the unsober gnome feels ironic and starts to sabotage the consensus in a smart way. Various scenarios of sabotage are shown on Fig. 3. There are three phases of consensus pictured. After phase 0, all gnomes know the proposal. After phase 1, all gnomes know that all other gnomes know. After phase 2, gnomes know that all know that all know. Note that any event ripples through the swarm in d turns.

The unsober gnome j (the *joker*) can inject a competing proposal on turn t_c during phase 0 to *confuse* the swarm and prevent p's proposal from achieving consensus. Alternatively, he can inject on turn t_f during phase 1 to *fool* some part of the gnomes into believing there are confused gnomes. It is forbidden to make proposals in phase 1, but the joker may say his *silly friend* is confused. That would look plausible as normal confusion spreads in phases 0 and 1. Finally, a joker may inject a competing proposal in phase 2 to *trick* some

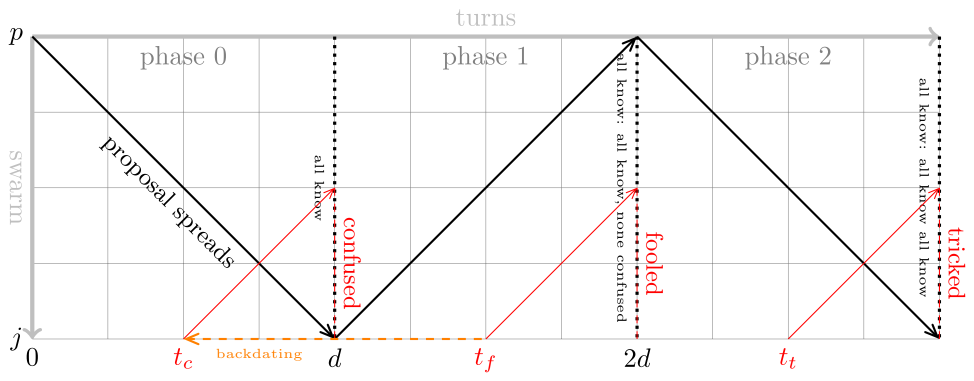


Figure 3: Byzantine sabotage: a competing proposal is injected under different phases of consensus. Propagation of the valid proposal is the solid black line, the malicious proposal is red. The drawing is effectively a Minkowski diagram.

gnomes into believing he is fooled. In each case, the swarm would divide into parts: agreeing and confused, acting and fooled, non-tricked and tricked.

One way to handle competing proposals in blockchain architectures is to use some lottery to limit the ability to propose. That might be a “happy hash” in proof-of-work architectures or a “happy second” in some proof-of-stakes. The idea of lottery is in serious conflict with the efficiency of a swarm. What if the happy ticket falls to the sleepy gnome, not to the wise one?

More sophisticated techniques depend on swarm time being tracked continuously across rounds. That is not too difficult. Just $2d$ turns after the start, gnomes synchronize and the swarm sounds like it is buzzing a rhythmic tune. That puts all events on a shared time/space grid, very much like Fig. 3 shows. Turn numbers can now be compared between different proposals.

Then, it is possible to apply a technique reminiscent of the Lamport’s *arbitrary total order* [6]. Except, this one is based on the swarm time, not logical time. Once proposals can be ranked by their creation time, any later injections can be ignored. Ties are resolved based on the proposer’s *rank*. A gnome’s rank is not a static value, but the details are irrelevant here. It is sufficient to say that gnomes can uniformly choose between two competing proposals. That leaves one opportunity though: the joker may backdate his bogus proposal. That can be done in the phase 0 only. If the proposal is backdated more than d turns, gnomes would realize: they should have heard of it before but they did not. Such a joker would be thrown out immediately. Similarly, a backdated proposal may never reach consensus: on its nominal turn $2d$, its actual turn would be less than that, so the awareness neighborhood size may be too small $\alpha_{2d}(g) < d$. The bogus proposal would not be acted on, but it can outcompete the valid proposal while propagating.

That leads us to the last anti-Byzantine technique gnomes call a “merry swarm”. They mostly do it for fun to see how long it can hold while more and more gnomes misbehave. In a merry swarm, gnomes let each proposal spread and act on the one that reaches $\alpha_t(g) = d$ sooner. If there is a tie, they resolve it in a uniform arbitrary way.

As a conclusion, we can only praise the ingenuity of the gnomes. Differently from the past consensus algorithms preoccupied with leader elections and majorities, the Schmebulock's algorithm can reach (non-Byzantine) consensus in an open network of arbitrary topology, where each gnome is only aware of his immediate neighbors and can only communicate with them. Neither the total number of gnomes nor the exact topology of the network are known to the participants. Nevertheless, gnomes are perfectly able to synchronize their behavior and act all at once!

References

[1] C. Mohan and B. Lindsay. 1985. Efficient commit protocols for the tree of processes model of distributed transactions. SIGOPS Oper. Syst. Rev. 19, 2 (April 1985), 40–52. DOI:https://doi.org/10.1145/850770.850772

[2] Skeen, Dale (February 1982). A Quorum-Based Commit Protocol (Technical report). Department of Computer Science, Cornell University. `https://ecommons.cornell.edu/handle/1813/6323`

[3] Leslie Lamport. 1998. The part-time parliament. ACM Trans. Comput. Syst. 16, 2 (May 1998), 133–169. DOI:https://doi.org/10.1145/279227.279229

[4] Ongaro, Diego; Ousterhout, John (2013). "In Search of an Understandable Consensus Algorithm" `https://raft.github.io/raft.pdf`

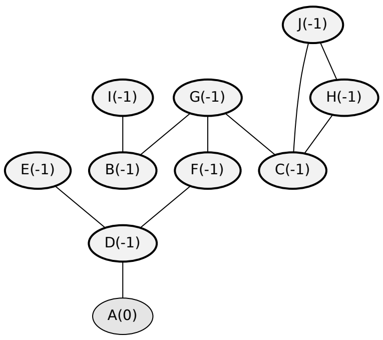
[5] Satoshi Nakamoto. "Bitcoin: A Peer-to-Peer Electronic Cash System" `https://bitcoin.org/bitcoin.pdf`

[6] Leslie Lamport. Time, clocks, and the ordering of events in a distributed system. In *Concurrency: the Works of Leslie Lamport*, pages 179–196. 2019.

[7] James C. Corbett, Jeffrey Dean, Michael Epstein et al. "Spanner: Google's globally distributed database", Proceedings of OSDI 2012

A An example

Here we show a consensus progression in a really small swarm of 10 gnomes of unusually large diameter 5.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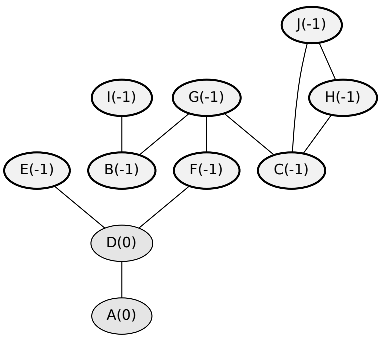
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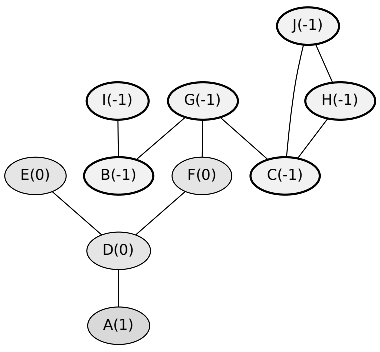
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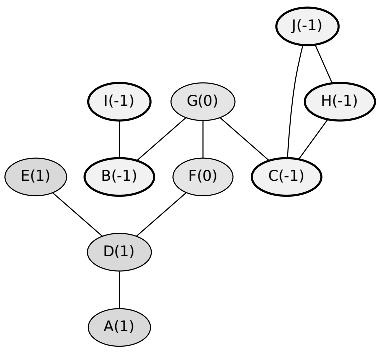
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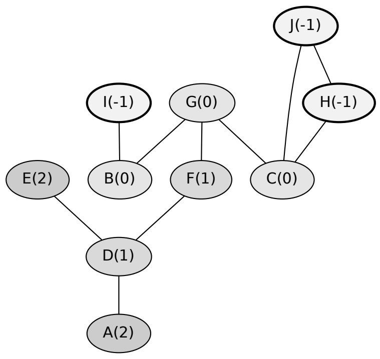
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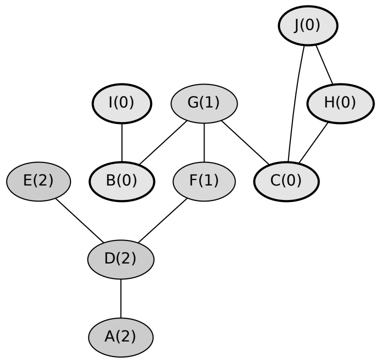
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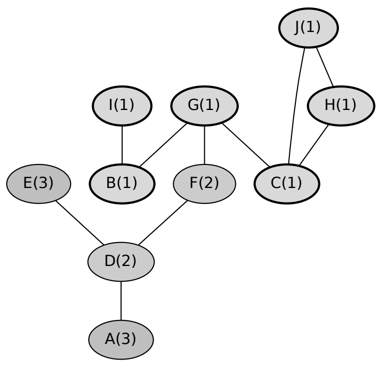
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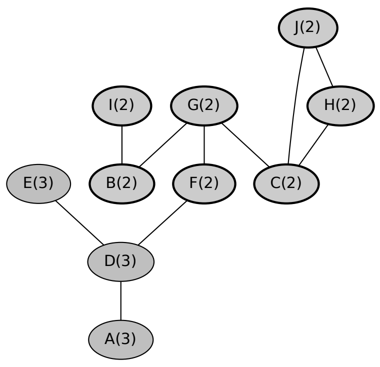
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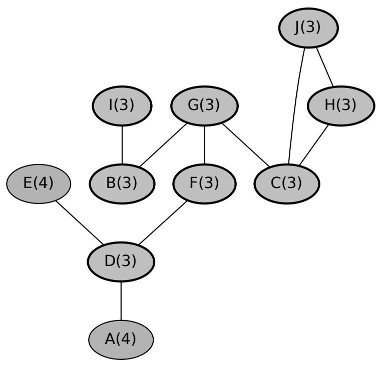
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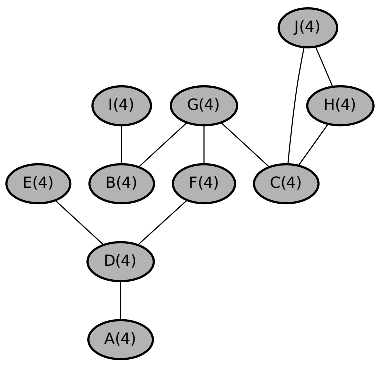
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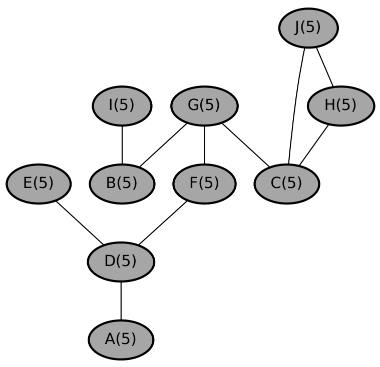
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