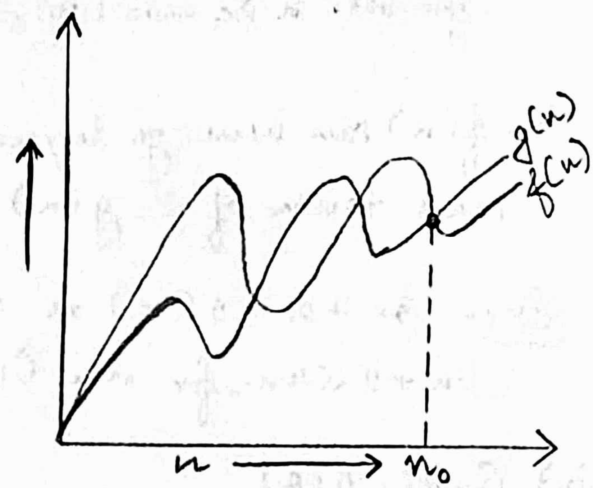
Q1. What do you understand by Asymptotic notation, define different asymptotic notation with example.

i) Big O(n)



$$f(n) \Rightarrow O(g(n))$$

if $f(n) \leq g(n) \times C \ \forall \ n \geq n_0$

for some constant, $C > 0$

g(n) is 'tight' upper bound of f(n)

eg:- $f(n) \Rightarrow n^2 + n$

$g(n) \Rightarrow n^3$

$n^2 + n \leq C * n^3$

$n^2 + n = O(n^3)$

ii) Big Omega (Ω)

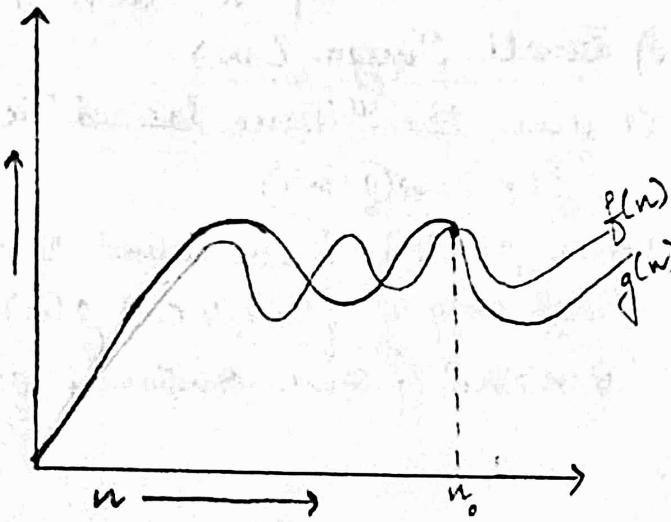
When $f(n) = \Omega(g(n))$

means g(n) is "tight" lowerbound of f(n) ie f(n) can go beyond g(n)

ie $f(x) = \Omega \, g(n)$

if and only if

$$f(n) \geq C.g(n)$$



$\forall \ n_2 > n_0$ and C = constant > 0

Ex: $f(n) \Rightarrow n^3 + 4n^2$

$g(n) \Rightarrow n^2$

ie $f(n) \geq C * g(n)$

$n^3 + 4n^2 = \Omega(n^2)$

iii) Big Theta (θ)

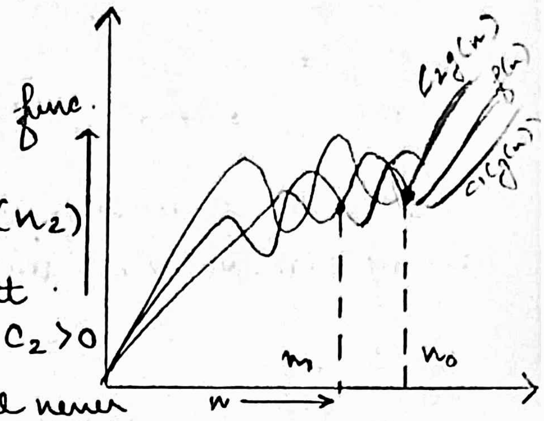
When $f(n) = \theta(g(n))$ gives the tight upperbound and lowerbound both.

ie $f(n) = \theta(g(n))$

if and only if

$$c_1 * g(n_1) \leq f(n) \leq c_2 * g(n_2)$$

for all $n \geq \max(n_1, n_2)$, some constant $c_1 > 0$ & $c_2 > 0$



ie. $f(n)$ can never go beyond $c_2\, g(n)$ and will never come down of $c_1\, g(n)$.

Ex:- $3n+2 = \theta(n)$ as $3n+2 \geq 3n$ & $3n+2 \leq 4n$ for n, $c_1 = 3$, $c_2 = 4$ & $n_0 = 2$

iv) Small o (o)

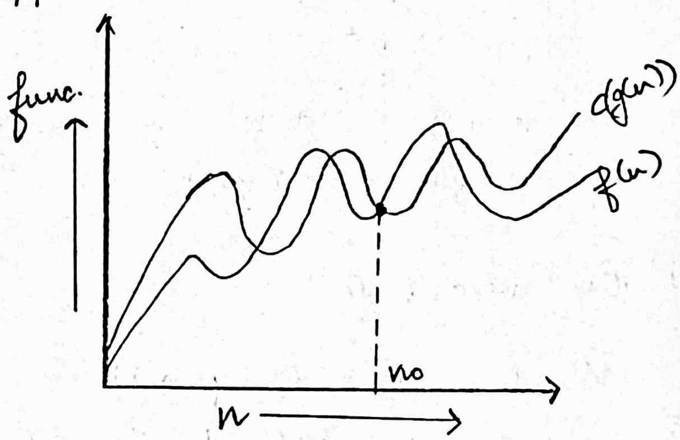
when $f(n) = o\, g(n)$ gives the upper bound

ie $f(n) = o\, g(n)$

if and only if

$$f(n) < c * g(n)$$

$\forall\ n > n_0$ & $n > 0$



Ex:- $f(n) = n^2$; $g(n) = n^3$

$$f(n) < c * g(n)$$

~~$\forall\ n > n_0$~~ $n^2 = o(n^3)$

v) Small Omega (ω)

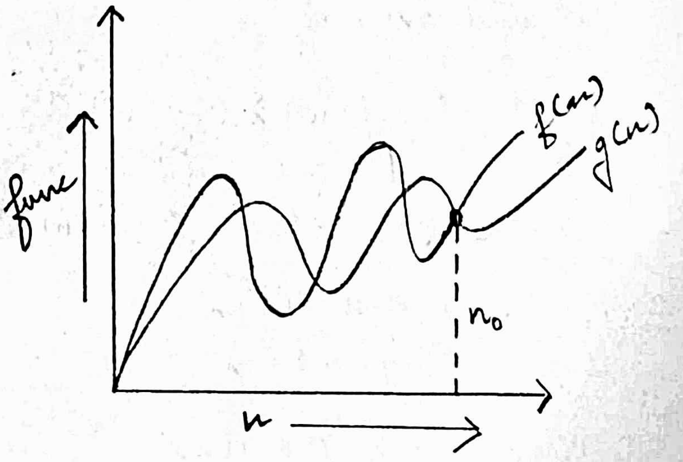
It gives the 'lower bound' ie

$$f(n) = \omega(g(n))$$

where $g(n)$ is lower bound of $f(n)$

if and only if $f(n) > c * g(n)$

$\forall\ n > n_0$ & some constant, $c > 0$



Q2. What should be time complexity of:

```
for (int i = 1 to n)
  {
     i = i * 2;   → O(1)
  }
```

↳ for $i \Rightarrow 1, 2, 4, 6, 8 \dots n$ times

ie series is a GP

So $a = 1$, $r = 2/1$

K^{th} value of GP:

$$t_k = a r^{k-1}$$

$$t_k = 1(2)^{k-1}$$

$$2n = 2^k$$

$$\log_2(2n) = k \log 2$$

$$\log_2 2 + \log_2 n = k$$

$$\log_2 n + 1 = k \quad \text{(Neglecting '1')}$$

So, Time Complexity $T(n) \Rightarrow \underline{O(\log_2 n)}$ → Ans.

Q3. $T(n) = \{ 3T(n-1)$ if $n>0$

otherwise 1

$\}$

↳ ie $T(n) \Rightarrow 3T(n-1)$ — (1)

$T(n) \Rightarrow 1$

put $n \Rightarrow n-1$ in (1)

$T(n-1) \Rightarrow 3T(n-2)$ — (2)

put (2) in (1)

$T(n) \Rightarrow 3 \times 3T(n-2)$

$T(n) \Rightarrow 9T(n-2)$ → (3)

put $n \Rightarrow n-2$ in (1)

$T(n-2) = 3T(n-3)$

put in (3).

$T(n) = 27T(n-3)$ → 4)

Generalising series,

$$T(k) = 3^k\, T(n-k) \quad \text{— (5)}$$

for k^{th} terms, let $n-k=1$ ‹ <u>Base case</u> ›

$$k = n-1$$

put in (5)

$$T(n) = 3^{n-1}\, T(1)$$

$$T(n) = 3^{n-1} \qquad \text{(}\underline{\text{neglecting } 3'}\text{)}$$

$$\underline{T(n) = O(3^n)}$$

Q4. $T(n) = \{\ 2T(n-1)-1$ if $n>0$,
otherwise 1
}

$$T(n) = 2T(n-1) - 1 \quad \rightarrow (1)$$

put $n = n-1$

$$T(n-1) = 2T(n-2) - 1 \quad \rightarrow (2)$$

put in (1)

$$T(n) = 2\times(2T(n-2)-1)-1$$

$$= 4T(n-2) - 2 - 1 \quad \text{— (3)}$$

put $n = n-2$ in (1)

$$T(n-2) = 2T(n-3) - 1$$

Put in (1)

$$T(n) = 8T(n-3) - 4 - 2 - 1 \quad \text{— (4)}$$

Generalising series

$$T(n) = 2^k\, T(n-k) - 2^{k-1} - 2^{k-2} \ldots 2^0$$

$\Rightarrow$ <u>k^{th} term</u> Let $n-k=1$
$k = n-1$

$$T(n) = 2^{n-1}\, T(1) - 2^k\left(\frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^k}\right)$$

$$= 2^{n-1} - 2^{n-1}\left(\frac{1}{2} + \frac{1}{2^2} + \cdots \frac{1}{2^{n-1}}\right)$$

ie Series in GP.

$$a = 1/2 \quad , \quad r = 1/2.$$

So,

$$T(n) = 2^{n-1}\left(1 - \left(\frac{1}{2}\left(\frac{1-(1/2)^{n-1}}{1-1/2}\right)\right)\right)$$

$$= 2^{n-1}\left(1 - 1 + (1/2)^{n-1}\right)$$

$$= \frac{2^{n-1}}{2^{n-1}}$$

$$T(n) = O(1) \text{ Ans}$$

Q5. What should be time complexity of

```
int i=1, s=1;
while (s<=n)
{
    i++;
    s = s+i;
    printf ("#");
}
```

$\rightarrow$ $i = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad \ldots$

$s = 1 + 3 + 6 + 10 + 15 + \ldots$

Sum of $s = 1 + 3 + 6 + 10 + \ldots + n \rightarrow 1)$

Also $s = 1 + 3 + 6 + 10 + \ldots T_{n-1} + T_n \rightarrow 2)$

$0 = 1 + 2 + 3 + 4 + \ldots n - T_n$

$T_k = 1 + 2 + 3 + 4 + \ldots + k$

$T_k = \frac{1}{2}k(k+1)$

for k iterations

$1 + 2 + 3 + \ldots k <= n$

$\frac{k(k+1)}{2} <= n$

$\frac{k^2 + k}{2} <= n$

$O(k^2) <= n$

$k = O(\sqrt{n})$

$\boxed{T(n) = O(\sqrt{n})}$ Ans.

Q6. Time Complexity of

```
void f(int n)
{
  int i, count=0;
  for(i=1; i*i<=n; ++i)
}
```

↳ As $i^2 = n$

$i = \sqrt{n}$

$i = 1, 2, 3, 4, \ldots \sqrt{n}$

$\sum_{i=1} 1+2+3+4+\ldots+\sqrt{n}$

$T(n) = \frac{\sqrt{n} * (\sqrt{n}+1)}{2}$

$T(n) = \frac{n * \sqrt{n}}{2}$

$\underline{T(n) = O(n)}$ → Ans.

Q7. Time Complexity of

```
void f(int n)
{
  int i, j, k, count=0;
  for(int i = n/2 ; i<=n ; ++i)
   for (j=1; j<=n; j=j*2)
   for( k=1; k<=n; k=k*2)
       count++;
}
```

↳ Since, for $k=k^2$

$k = 1, 2, 4, 8, \ldots n$

∵ Series is in GP

So, $a=1$, $r=2$

$$\frac{a(r^n-1)}{r-1}$$

$$= \frac{1(2^k-1)}{1}$$

$n = 2^k - 1$

$n+1 = 2^k$

$\log_2(n) = k$

i	j	k
1	$\log(n)$	$\log(n) * \log(n)$
2	$\log(n)$	$\log(n) * \log\{n\}$
$\vdots$	$\vdots$	$\vdots$
n	$\log(n)$	$\log\{n\} * \log(n)$

$T.C \Rightarrow O(n * \log n * \log n)$

$\Rightarrow O\underline{(n \log^2(n))} \rightarrow$ Ans

Q8. Time Complexity of

```
void function (int n)
 {
    if (n==1) return;
    for(i=1 to n) {
    for(j=1 to n) {
       printf("*");
     }
 }
   function(n-3);
}
```

↳ for(i = 1 to n)

we get $j = n$ times every Turn

$\therefore i * j = n^2$

k^{th}, Now,

$T(n) = n^2 + T(n-3);$

$T(n-3) = (n^2 3)^2 + T(n-6);$

$T(n-6) = (n^3 6)^2 + T(n-9);$

and $T(1) = 1;$

Now, substitute each value in $T(n)$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$

Let

$\cancel{k}^{n} - 3k = 1$

$k = (n-1)/3$ total terms $= k+1$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$

$T(n) \simeq k n^2$

$T(n) \simeq (k-1)/3 * n^2$

So, $\underline{T(n) = O(n^3)}$ → Ans

Q9. Time Complexity of :-

```
void function (int n)
{
    for (int i = 1 to n) {
        for (int j = 1; j <= n; j = j + i) {
            printf ("*");
        }
    }
}
```

↳ for $i = 1$ $\quad j = 1 + 2 + \dots (n >, j + i)$

$i = 2$ $\quad j = 1 + 3 + 5 \dots (n >, j + i)$

$i = 3$ $\quad j = 1 + 4 + 7 \dots (n >, j + i)$

n^{th} term of AP is

$T(n) = a + d \times m$

$T(m) = 1 + d \times m$

$(n-1)/d = m$

for $i = 1$ $\quad (n-1)/1$ times

$i = 2$ $\quad (n-1)/2$ times

$\vdots$

$i = n-1$

we get,

$T(n) = i_1 j_1 + i_2 j_2 + \dots i_{n-1} j_{n-1}$

$= \frac{(n-1)}{2} + \frac{(n-2)}{2} + \frac{(n-3)}{3} + \dots 1$

$= n + n/2 + n/3 + \dots n/n-1 - n \times 1$

$= n [1 + 1/2 + 1/3 + \dots 1/n-1] - n * 1$

$= n \times \log n - n + 1$

Since $\int 1/x = \log x$

$\underline{T(n) = O(n \log n)}$ → Ans.

Q10 (9)

For the Function n^{k} & c^{n}, what is the asymptotic Relationship b/w these functions?

Assume that $k >= 1$ & $c > 1$ are constants. Find out the value of c & n_0 of which relationship holds.

↳ As given n^k and c^n

Relationship b/w n^k & c^n is

$$n^k = O(c^n)$$

$$n^k \leqslant a(c^n)$$

$$\forall\ n \geq n_0 \text{ \& constant, } a > 0$$

for $n_0 = 1$; $c = 2$

$\Rightarrow$ $1^k < a^2$

$\Rightarrow$ $\underline{n_0 = 1 \ \& \ c = 2}$ → Ans