Why is the complexity of BA cubic with respect to the number of unknowns?

Lecture 10, Slide 29

Bundle Adjustment vs Pose-graph Optimization

- BA is **more precise** than pose-graph optimization because it adds additional constraints (*landmark constraints*)
- But **more costly**: $\boxed{O\left((qN + lm)^3\right)}$ with N being the number of points, m the number of cameras poses and q and l the number of parameters for points and camera poses. Workarounds:

Nonlinear Least Squares

$$P^i, C_1, \dots, C_n = argmin_{X^i, C_1, \dots, C_n,} \sum_{k=1}^{n} \sum_{i=1}^{N} \rho\left(p_k^i - \pi(P^i, K_k, C_k)\right)$$

$$\Big\downarrow \text{Reformulate}$$

$$E_{NLS} = \sum_i \| f(x_i; p) - x_i' \|^2$$

https://dellaert.github.io/19F-4476/Slides/S07-SFM-B.pdf, Slide 23

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$$\Big\downarrow \text{Linearize}$$

$$f(x; p + \Delta p) = f(x; p) + J(x; p)\Delta p$$

$$r = x' - f(x; p) = J(x; p)\Delta p$$

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Nonlinear Least Squares

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$$\Big\downarrow \text{ Reformulate}$$

$$E_{NLS} = \sum_i \|f(x_i; p) - x'_i\|^2$$

$$\Big\downarrow \text{ Linearize}$$

$$f(x; p + \Delta p) = f(x; p) + J(x; p)\Delta p$$

$$r = x' - f(x; p) = J(x; p)\Delta p$$

$$E_{NLS} = \sum_i \|f(x; p) + J(x; p)\Delta p - x'_i\|^2 = \sum_i \|J(x; p)\Delta p - r_i\|^2$$

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Nonlinear Least Squares

$$E_{NLS} = \sum_i \|f(x;p) + J(x;p)\Delta p - x_i'\|^2 = \sum_i \|J(x;p)\Delta p - r_i\|^2$$

$$\Big\downarrow \text{Differentiate and set to zero}$$

$$\left[\sum_i J^T(x_i;p)J(x_i;p)\right]\Delta p = \sum_i J^T(x_i;p)r_i$$

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Nonlinear Least Squares

$$E_{NLS} = \sum_i \| f(x;p) + J(x;p)\Delta p - x_i' \|^2 = \sum_i \| J(x;p)\Delta p - r_i \|^2$$

$$\Big\downarrow \text{ Differentiate and set to zero}$$

$$\left[\sum_i J^T(x_i;p) J(x_i;p) \right] \Delta p = \sum_i J^T(x_i;p) r_i$$

$$\Big\downarrow$$

$$A \Delta p = b$$

$$\Delta p* = \underbrace{A^{-1}}_{} b$$

Cholesky decomposition requires
O(N^3) for a NxN matrix

https://dellaert.github.io/19F-4476/Slides/S07-SFM-B.pdf, Slide 23