Homework #1: Morlet Wavelets

Due February 9th , 2017

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The goal of this assignment is to have your environment set up with the libraries we will be using during the semester. We will have Feb 9th to be a class lab and you are expected to show your system and debug it further if needed.

MORLET WAVELETS (ONE PEAK)

$$\psi_{\sigma,\theta}(u) = \frac{C_1}{\sigma}\left(e^{i\,\frac{\pi}{2\,\sigma}(u\,.e_\theta)} - C_2\right)e^{-\frac{u^2}{2\sigma^2}} \quad ; \qquad u = (x,y), e_\theta = (\cos\theta\,, \sin\theta)$$

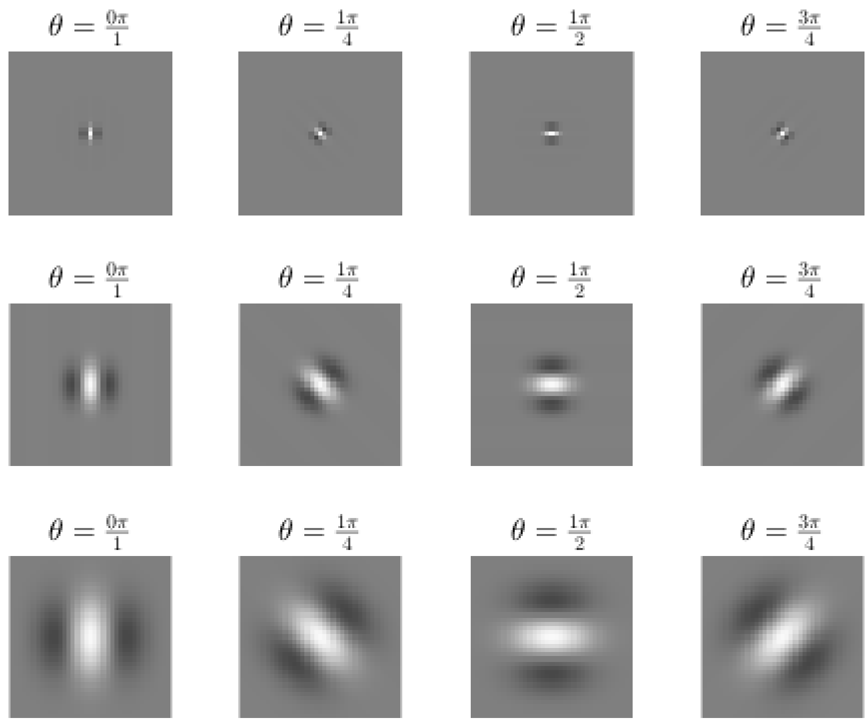
1. Let us first create the Morlet Wavelets, for different parameters,

   (a) $\sigma = 1$, (b) $\sigma = 3$, (c) $\sigma = 6$,

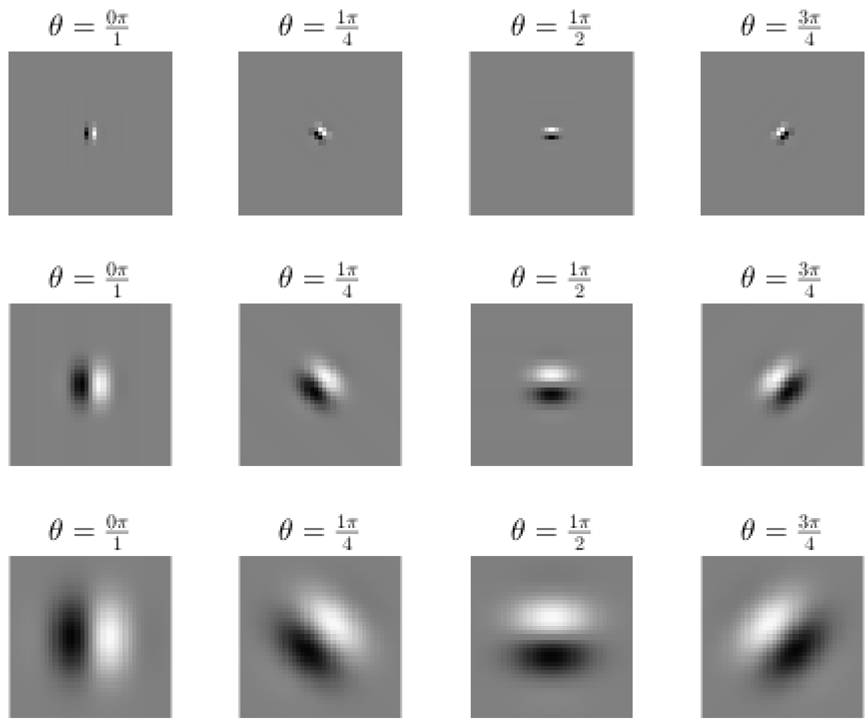
   (i) $\theta = 0$ (ii) $\theta = \frac{\pi}{4}$ (iii) $\theta = \frac{\pi}{2}$ (iv) $\theta = \frac{3\pi}{4}$

   These are 12 combinations of parameters. Let us refer to the parameters as $\lambda = (\sigma, \theta)$, i.e., there are $\lambda_1, \dots, \lambda_{12}$ different values. To represent each filter, plot these wavelets, the real part and the complex part using windows of size 37 x 37 pixels (since the largest scale is $\sigma = 6$, so the window is 3 x 6=18, i.e., from -18 to 18 → 37 pixels.)

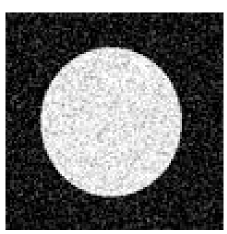
   We show them (see next page) for more angles



The real part (above) and the complex part (below) of Morlet 2D for σ=1,3,6 and θ values are shown on windows of size 37 pixels. The function values are shown as “0” → grey, negative numbers → dark, positive numbers →bright.



2. For this images below,



noisy circle

and for each λ_i, perform a convolution using a convolution function (either with the complex wavelet at once, or with the real part of the wavelet and separately with the complex part). Produce the results, i.e., show the “24 images” associated to $\boldsymbol{W}_{\lambda_i}\boldsymbol{I}(\boldsymbol{u})$**,** where

$$\boldsymbol{W}_{\lambda_i}\boldsymbol{I}(\boldsymbol{u}) = \psi_{\lambda_i} * I(u) = \sum_{x'}\sum_{y'} I(x-x', y-y')\ \psi_{\lambda_i}(x', y') \qquad i = 1, \dots, 24$$

12 are associated to the convolution with the real part and 12 with the convolution of the complex part or

$$\boldsymbol{W}_{\lambda_i}^{\boldsymbol{Real}}\boldsymbol{I}(\boldsymbol{u}) = \psi_{\lambda_i}^{Real} * \ I(u) = \sum_{x'}\sum_{y'} I(x-x', y-y')\,\psi_{\lambda_i}^{Real}(x', y') \qquad i = 1, \ldots, 24$$

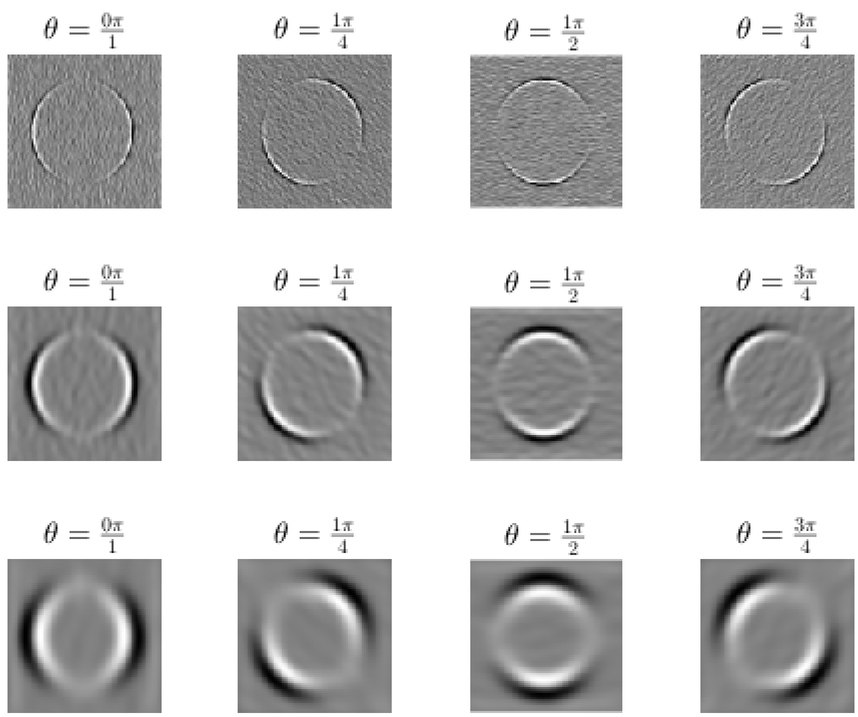
$$\boldsymbol{W}_{\lambda_i}^{\boldsymbol{Im}}\boldsymbol{I}(\boldsymbol{u}) = \psi_{\lambda_i}^{Im} * \ I(u) = \sum_{x'}\sum_{y'} I(x-x', y-y')\ \ \psi_{\lambda_i}^{Im}(x', y') \qquad i = 1, \ldots, 24$$

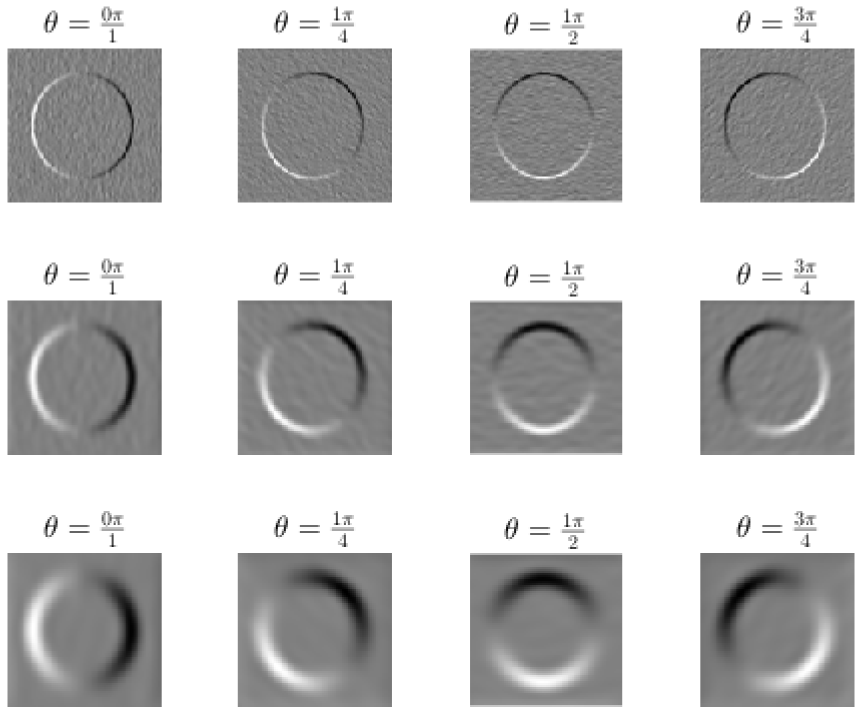
NOTE: Again, your wavelets are represented by a 37 x 37 pixels window which you use to perform the convolution with the image. You can use convolution libraries in Python, don't need to implement here.

Lastly, convolve the image with a Gaussian Blur

$$G_{\sigma=6} * I(u) = C\ e^{-\frac{u^2}{2\sigma^2}} * \ I(u) = \sum_{x'}\sum_{y'} I(x', y')C\ e^{-\frac{(x'-x)^2+(y'-y)^2}{2\sigma^2}}$$

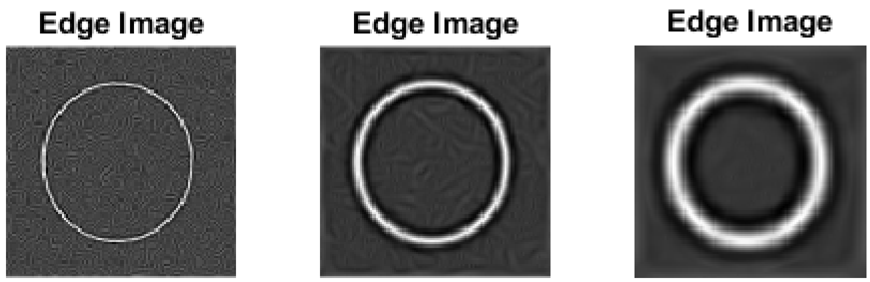
This Gaussian function also is described by a 37 x 37 pixels window So all together, we have 25 images.





3. Summary Images and Histograms

Construct two "new images" per scale, one from the "real" data and the other from the "imaginary" data as follows: The real image is constructed from each pixel u as $W_{max}^{Real} I(u) = max_{\theta_i} \left| W_{\theta_i}^{Real} I(u) \right|$ and the imaginary image is constructed as $W_{max}^{Im} I(u) = max_{\theta_i} \left| W_{\theta_i}^{Im} I(u) \right|$. Each of these new images values are real, non negative, but they are not integers. Show these images by converting them to integer values between 0 and 255. Plot the histogram of $W_{max}^{Im} I(u)$ and $W_{max}^{Real} I(u)$. The histogram of $W_{max}^{Im} I(u)$ allows us to choose a threshold to decide what is a large "intensity difference" for each image. The histogram of $W_{max}^{Real} I(u)$ allow us to understand how small are these values compared to $W_{max}^{Im} I(u)$.



4. Edge Detection

Edges are locations of high $\left|W_{\lambda_i}^{Im} I(u)\right|$ and small $\left|W_{\lambda_i}^{Real} I(u)\right|$. How to create edge detection from this observation?

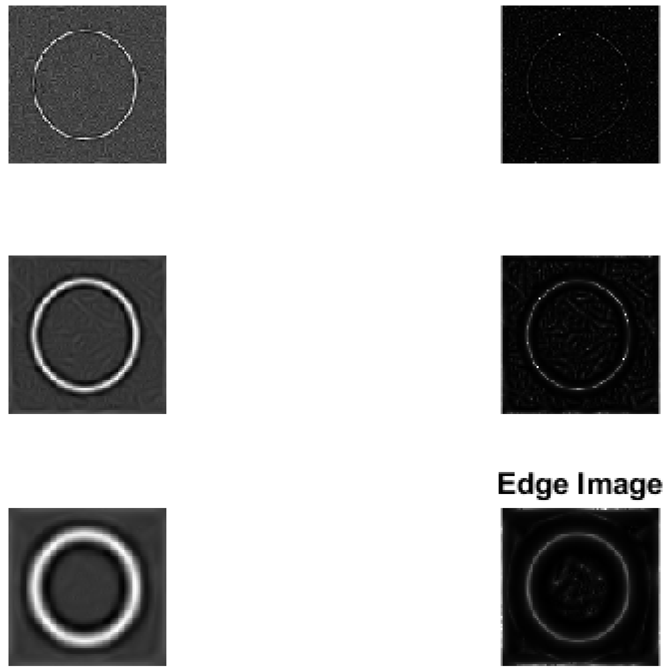
Two ideas:

$$ratio(\boldsymbol{u}) = \frac{W_{max}^{Real} I(u) + 0.001 * \varepsilon}{W_{max}^{Im} I(u) + \varepsilon} \qquad diff(\boldsymbol{u}) = W_{max}^{Im} I(u) - W_{max}^{Real} I(u)$$

where $C = max_u \, W_{max}^{Im} I(u)$ - maximum contrast in the image. So $diff(\boldsymbol{u}) \geq 0$. Then

$$edge(\boldsymbol{u}) = 1 - \frac{ratio(u)}{D} \qquad or \qquad edge(\boldsymbol{u}) = e^{-\alpha \, diff(u)}$$

where $D = max_u ratio(u)$. Note $0 \leq edge(\boldsymbol{u}) \leq 1$. Choose an image and apply these two detectors. Explain how the parameters $\varepsilon, \beta, \alpha$, being larger or smaller affect your final result.



DELIVERY: In class. Please create your own display. Make sure to put captions indicating the parameters associated to the images you display. Also, please, make sure to also show the filters. The TAs and myself will see your results in class.