The Foundation of Bridges

The Mathematical Basis of Bridge Design and Construction

Overview


History


All Bridges


Material Properties


Bridge Designs


Failure of Bridges

History

- Romans
- Middle Ages
- Renaissance
- Industrial Revolution (1740-1840)

All Bridges

- Settlement of Supports
- Loads
- Factor of Safety

- Dead Load

$$F_{L,D} = \int w(x)dx$$

$$L_D = \sum_{i=1}^{n} A_i l_i(\mu_i)$$

- Live Load

$$F_l = (F_y)\left(\frac{dV}{dt}\right)$$

Partial Safety Factors

$$P_f = \int \cdots \int f_x(x_1, x_2, \ldots, x_n) dx_1, dx_2, \ldots, dx_n$$

$$P_f = \int_{-\infty}^{\infty} F(B) \cdot f(M) dM$$

$$P_f = \Phi\left[\frac{-m_Z}{\sigma_Z}\right] = \Phi\left[-\frac{m_B - m_M}{\sqrt{\sigma_B^2 + \sigma_M^2}}\right] = \Phi[-\beta]$$

Material Properties

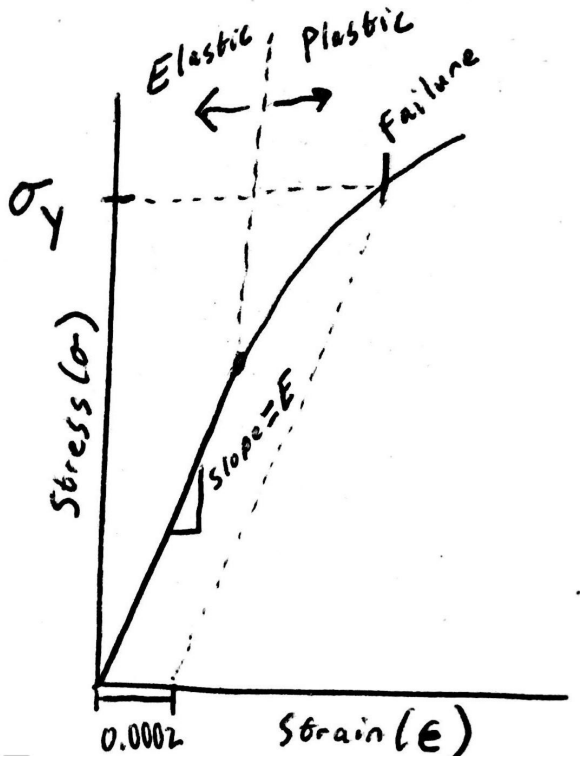
Stress-Strain

Properties of Specific Materials

Stress-Strain

$$E = \frac{\sigma_E}{\epsilon}$$

$$\sigma_E = \frac{F_I}{A_0} \qquad \epsilon = \frac{\Delta l}{l_0}$$



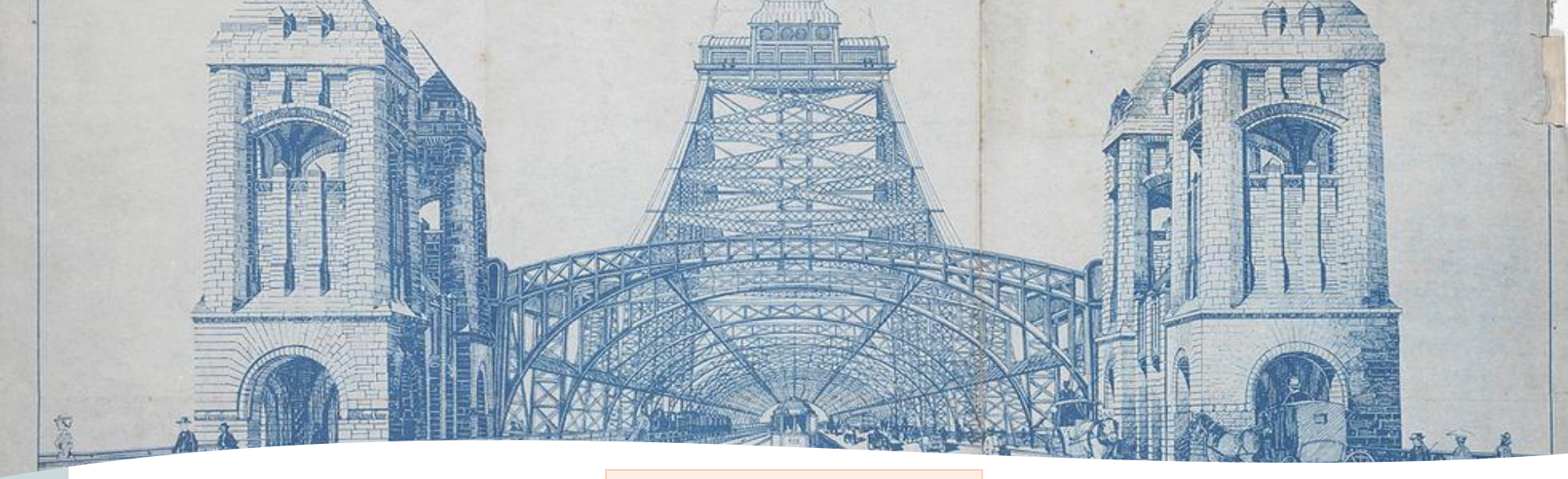
Steel

Iron

Concrete

Timber

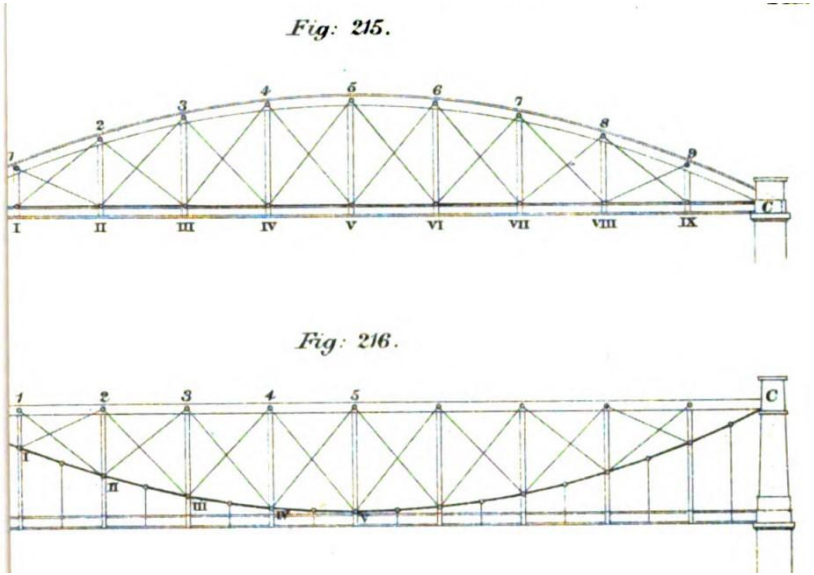
Stone



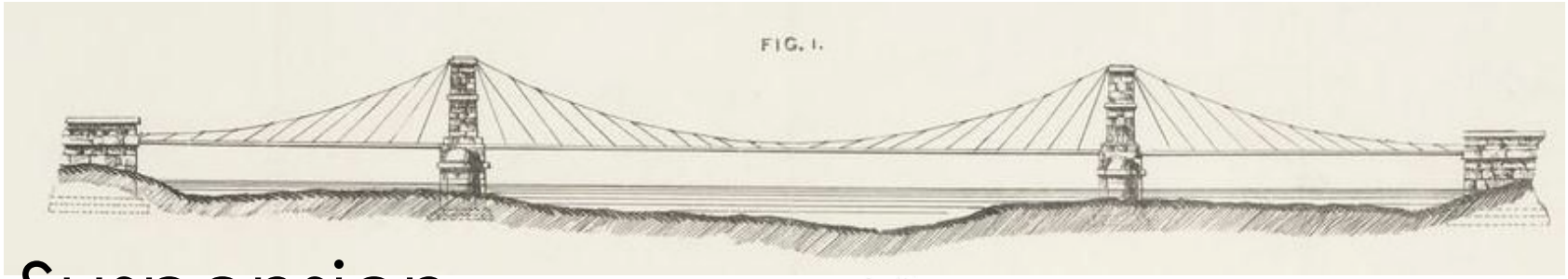
Bridge Designs

- Arch
- Suspension
- Cable-Stayed
- Truss

Arch Bridge

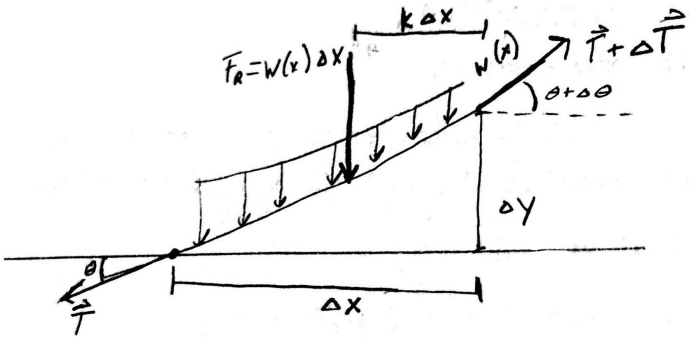


$$\frac{5}{24}\frac{L^2}{dE}$$



Suspension

Curve of Draping Cable



$$\sum M = 0 = w(x)(\Delta x)k(\Delta x) - Tcos\theta\Delta y + Tsin\theta\Delta x$$

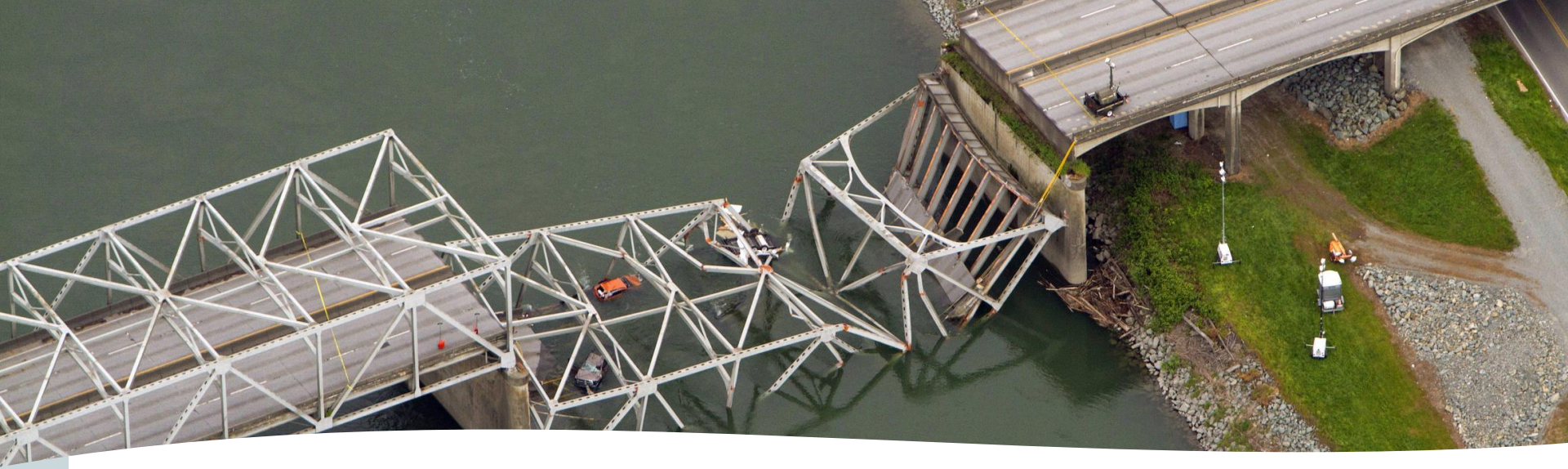
$$\sum F_x = 0 = -Tcos\theta + (T + \Delta T)cos(\theta + \Delta\theta)$$

$$\sum F_y = 0 = -Tsin\theta - w(x)(\Delta x) + (T + \Delta T)sin(\theta + \Delta\theta)$$

$$y = \frac{1}{F_H} \int \left(\int w(x) dx \right) dx$$

Cable-Stayed

Truss



Failure of Bridges

Lateral Buckling of Beams

Elastic Buckling of Plates

Lateral Buckling of Beams

$$M_{cr} = \frac{\pi^2 E}{l_e^2}\sqrt{\frac{I_y I_w}{\alpha}}\beta$$

$$M_{cr} = P_E\left(\frac{h\gamma}{2\sqrt{\alpha}}\right)$$

$$\gamma = \sqrt{1 + \frac{l_e^2 GJ}{\pi^2 E I_w}}$$

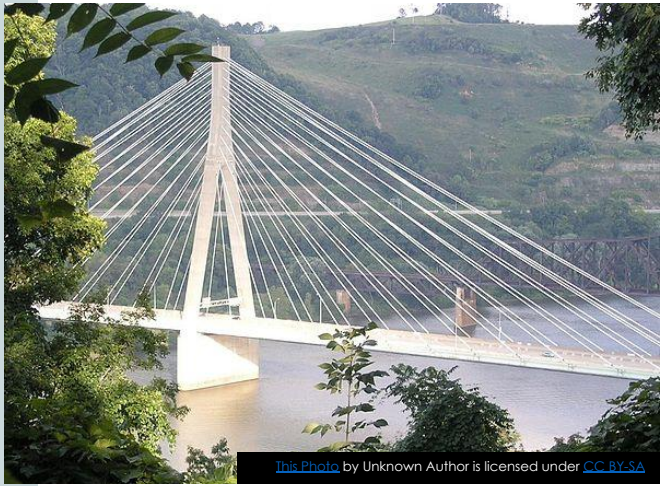
Elastic Buckling of Plates

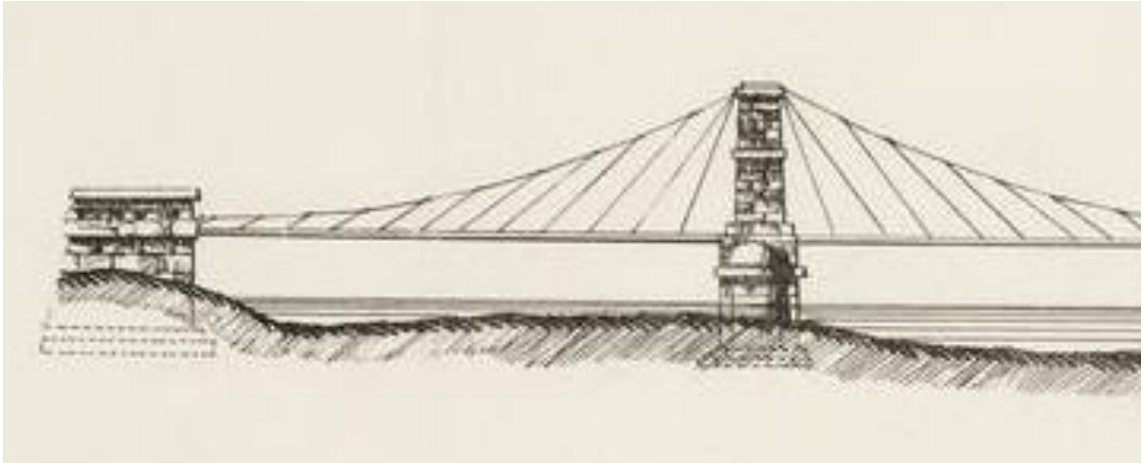
$$\frac{\partial^4 \omega}{\partial x^4} + \frac{2\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} = -\frac{F_c}{D}\frac{\partial \omega}{\partial x^2}$$

$$\frac{\partial^4}{\partial x^4}\left(\delta sin\frac{m\pi x}{a} sin\frac{m\pi y}{b}\right) + \frac{2\partial^4}{\partial x^2 \partial y^2}\left(\delta sin\frac{m\pi x}{a} sin\frac{m\pi y}{b}\right) + \dots$$

$$\dots + \frac{\partial^4}{\partial y^4}\left(\delta sin\frac{m\pi x}{a} sin\frac{m\pi y}{b}\right) = -\frac{F_c}{\frac{Et^3}{12(1-\mu^2)}}\frac{\partial}{\partial x^2}\left(\delta sin\frac{m\pi x}{a} sin\frac{m\pi y}{b}\right)$$

$$F_c = \frac{\pi^2 D}{b^2}\left(\frac{ma}{b} + \frac{b}{ma}\right)^2$$





Questions?

References

[1] William Callister and David Rethwisch. Material Science and Engineering. New Jersey, USA: John Wiley & Sons, Inc, 2014.

[2] Sukhen Chatterjee. The Design of Modern Steel Bridges. Oxford, UK: Blackwell Science Ltd, 2003.

[3] "Engraving. (Suspension Bridge Diagrams)". In: (1917). url: https://jstor.org/stable/community.26396505.

[4] T. Claxton Fidler. A Practical Treatise on Bridge-Construction. London, England: Charles Griffin and Company, 1887.

[5] John F. Fleming. "Nonlinear static analysis of cable-stayed bridge structures". In: Computers & Structures 10.4 (1979), pp. 621–635. doi: https://doi.org/10.1016/0045-7949(79)90006-3.

[6] Alain Goriely, Rebecca Vandiver, and Michel Destrade. "Nonlinear Euler Buckling". In: Proceedings: Mathematical, Physical and Engineering Sciences 464.2099 (2008), pp. 3003–3019.

[7] R.C. Hibbeler. Engineering Mechanics: Statics. New Jersey,USA: Pearson Prentice Hall, 2004.

[8] Richard Kirby et al. Engineering in History. McGraw-Hill Book Company, 1956.

[9] R.S narayanan and A.W Beeby. Introduction to Design for Civil Engineers. Spoon Press, 2001.

[10] Arthur Nilson, David Darwin, and Charles Dolan. Design of Concrete Structures. McGraw-Hill Book Company, 2009.

[11] B.O. Adewuyi and I.O. Oladele O.O. Daramola. "Effects of Heat Treatment on the Mechanical Properties of Rolled Medium Carbon Steel". In: Journal of Minerals & Materials Characterization & Engineering 9.8 (2010).