CE 28 | Analytical and Computational Methods in Civil Engineering III
*1st Semester | AY 2018-2019*
**Machine Problem 1 | Finite Difference Method**

# ONE-DIMENSIONAL POLLUTANT MIGRATION

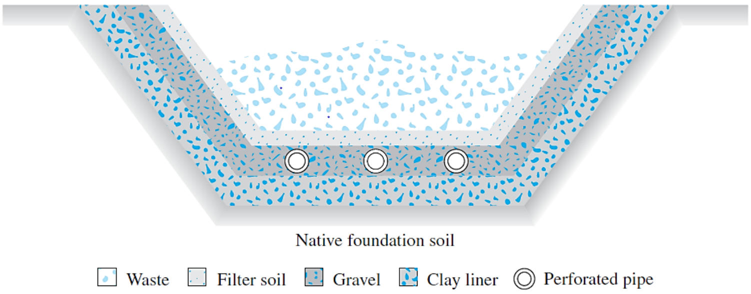


*Figure 1. Cross-section of single clay liner system for a landfill (Das,*

A single clay liner system landfill is constructed in Town A. The design parameters of the landfill are as follows:

**Settling velocity: 0.0063 meters per annum | Thickness of the clay liner: 3.0 meters**

Characterization of the leachate from the landfill show that the following contaminant species are present:

*Table 1. Contaminant Species and their Migration Parameters*

| Parameter | Acetone | DCM | Calcium | Chloride | Sodium | Magnesium |
|---|---|---|---|---|---|---|
| Dispersion Coefficient, D | 0.018 | 0.008 | 0.012 | 0.018 | 0.014 | 0.012 |
| Retardation Factor, R | 1.770 | 1.240 | 6.000 | 1.000 | 1.450 | 13.500 |

The soil layer below the clay liner can be assumed as impermeable. The landfill started empty and reached its capacity after a decade. The inflow of contaminants was constant for the succeeding eight years. The contents of the landfill were gradually removed throughout the next two years as to convert the land for residential use.

Generate a model describing the migration of these contaminants through the 3.0 m liner. Compare the solutions between the **Forward-Time-Center-Space (FTCS) Method** and the **Crank-Nicholson Method**.

## Governing Equation: 1-D Advection-Dispersion-Reaction Equation

$$R\frac{\partial C}{\partial t} = v\frac{\partial C}{\partial z} + D\frac{\partial^2 C}{\partial z^2}$$

where:
$C(z,t)$ – concentration of the pollutant at a distance $z$ from the surface at any time $t$
$D$ – dispersion coefficient in $m^2/yr$
$v$ – settling velocity in m/yr
$R$ – retardation factor

## Initial & Boundary Conditions:

- $$C(z,0) = 0$$

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- $$C(0,t) = \begin{cases} 10{,}000 - 100(x-2)^2 & , \quad 0 \le t \le 10 \\ 10{,}000 & , \quad 10 < t \le 18 \\ \text{linearly decreases from } 10{,}000 \text{ to } 0 & , \quad 18 < t \le 20 \\ 0 & , \quad t > 20 \end{cases} \left(\frac{\text{mg}}{\text{L}}\right)$$
- $$C_z(3.0,\ t) = 0$$
- $$z \in [0,\ 3.0]$$

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## ONE-DIMENSIONAL POLLUTANT MIGRATION

### REPORT PAPER

One report paper is needed for this machine problem – produce a hard copy and a pdf file – containing the following:

1. **Title Page** including the academic integrity pledge
2. **Abstract** – contains the summary of the report which includes the problem being solved, the numerical model used and the summary of results.
3. **Problem Statement** – formulation of the problem which includes (a) physical significance of the problem, and (b) derivation of the resulting partial differential equation.
4. **Numerical model** — includes (a) the discussion on the numerical methods (FTCS and Crank-Nicholson) used to solve the problem, and (b) **formulation** of the finite difference model.
5. **Implementation of Numerical Model** — includes (a) discussion on **how to use** the finite difference model, and (b) flowcharts or pseudocodes.
6. **Results and Discussion** — results (and interpretation) in the form of well-prepared figures with proper axis labels. The following points should be included in this section:
   a) **Merged** plot of the concentration profiles of **ALL CONTAMINANTS** along the depth of the liner at the following times: $\boldsymbol{t}$ **= 5, 10, 15, 20, 40, & 60 years**. Do this for both the Crank-Nicholson and FTCS methods.
   b) Tabulate the concentration (for every **0.2 m**) at $\boldsymbol{t} = 20$ **years** for both Crank Nicholson and FTCS method and compare and differentiate the two.
   c) Identify the first contaminant specie which will break through the 3.0 m-liner. (i.e., $\boldsymbol{C}$@3.0 **m** = 10 **mg/L**)
   d) At what values (and of which parameters) will your solution be unstable for the contaminant species in item (c)?
7. **Conclusions and Recommendations**
8. **Source Code**

### SUBMISSION

The hard copy of the report paper must be submitted on **April 2, 2019** during laboratory class. **NO LATE SUBMISSIONS**.

The pdf file of the report paper and the source code (including all libraries used, if any) should be uploaded to our Google Classrooms on or before **April 2, 2019, 11:59 pm.**