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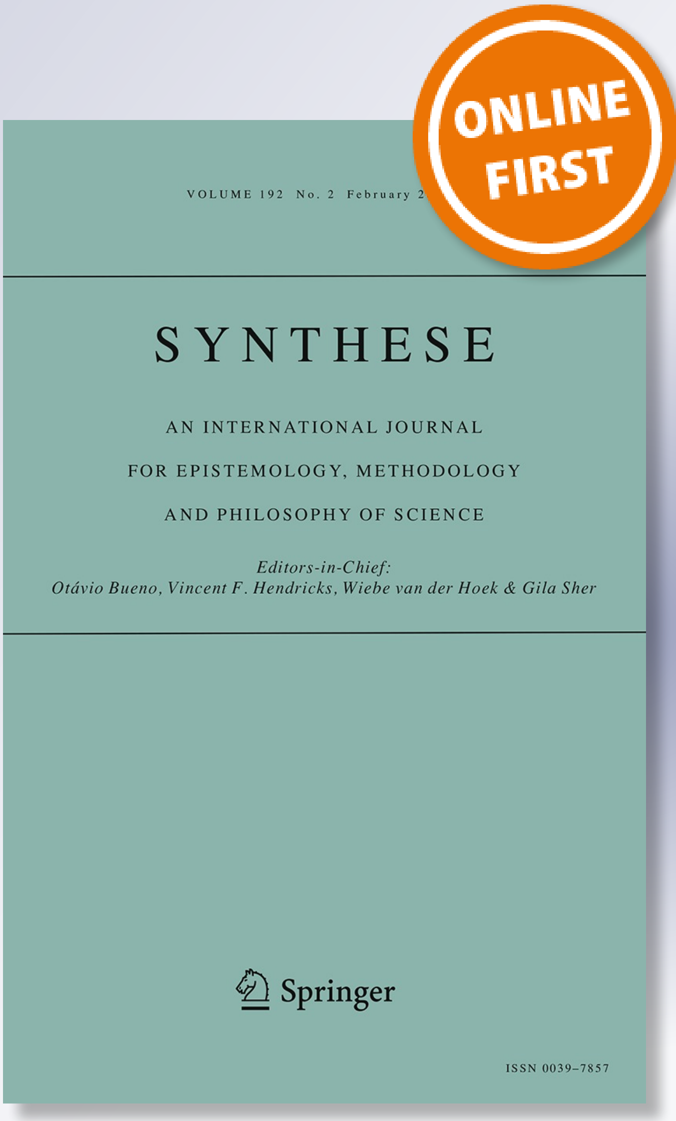
Synthese
An International Journal for Epistemology, Methodology and Philosophy of Science

ISSN 0039-7857

Synthese
DOI 10.1007/s11229-015-0682-8



Springer

Synthese
DOI 10.1007/s11229-015-0682-8

Formal reconstructions of St. Anselm's ontological argument

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Received: 2 July 2014 / Accepted: 4 February 2015

Abstract In this paper, we discuss formal reconstructions of Anselm's ontological argument. We first present a number of requirements that any successful reconstruction should meet. We then offer a detailed preparatory study of the basic concepts involved in Anselm's argument. Next, we present our own reconstructions—one in modal logic and one in classical logic—and compare them with each other and with existing reconstructions from the reviewed literature. Finally, we try to show why and how one can gain a better understanding of Anselm's argument by using modern formal logic. In particular, we try to explain why formal reconstructions of the argument, despite its apparent simplicity, tend to become quite involved.

Keywords Anselm of Canterbury · Ontological arguments · Proofs for the existence of God

1 Introduction

1.1 Aims of the paper

A variety of formal reconstructions of Anselm's ontological argument (and ontological proofs in general) has been presented in philosophical papers.[1] Though we will develop further reconstructions, the aim of this paper is not simply to add more to

[1] An extensive overview can be found in Uckelman (2012).

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Published online: 20 February 2015

the widespread literature, but to discuss and compare them with respect to some previously outlined standard. In order to do so, we will first delineate several criteria according to which formal reconstructions of informal arguments can be judged as to their adequacy.

Second, we aim to show that and how formally reconstructing the structure of Anselm's argument gives one a better understanding of the argument itself. It is apparent from earlier attempts to formalize Anselm's reasoning that an adequate formal representation of his argument can be quite intricate, which will also be the case for the reconstructions provided in this article. In particular, the second aim of this article includes explaining why the reconstruction of such a seemingly short and simple argument needs relatively complex technical devices.

1.2 Requirements of formal reconstructions

Ian Logan, in his book on Anselm's *Proslogion*, cites Geach and Strawson's claim that 'the form of a particular concrete argument is not reducible to a single logical form' and concludes that this applies in particular to Anselm's argument.[2] We think that this is true and indeed a crucial point. Yet, it seems evident that some reconstructions fit Anselm's reasoning better than others do. The question, then, is this: When confronted with different reconstructions, on what basis are we supposed to decide which one is better? In the following, we will present a list of requirements that we think a good reconstruction must meet. Of course, we do not consider this list complete and the conditions are interrelated. Nevertheless, we wanted to make transparent which criteria of adequacy we used in our own investigations. It is important to notice, however, that these are criteria for the quality of *reconstructions*, not of arguments. Hence, a good reconstruction of a bad argument has to be a bad argument; otherwise, it is an emendation, not a reconstruction.[3]

(1) The reconstruction should *locally conform* with what the author said. By the term 'local conformity' we mean that the reconstruction is in accordance with the argument or piece of text that is reconstructed. (In the case at hand, this will be the argument presented by Anselm in the second chapter of his *Proslogion*.) In particular, the basic concepts should be represented in such a way that there is a

[2] Cf. Logan (2009, p. 176). This implies that the refutation of *one* formal reconstruction of the argument is never enough to refute the argument itself. Logan accuses some commentators (e.g., Millican 2004) of exactly this fallacy (Logan 2009, p. 176f).

[3] Matthews and Baker (2010) hold that 'much of [the] literature ignores or misrepresents the elegant simplicity of the original argument'. But the argument that is offered is, first, still an informal argument; therefore, it does not offer a possibility to understand why *formal reconstructions* tend to be complex. Second, it depends on a distinction between 'mediated' and 'unmediated causal powers', a distinction that we do not see in Anselm's argument. The presented argument may be simple and elegant, but it is, in our terminology, an attempted emendation, not a reconstruction. (Oppy 2011 started a debate between Oppy and Matthews/Baker.)

one-to-one correspondence between important expressions in the language of the author and the signs used in the reconstruction—*unless* there are good reasons against it.

(2) A further requirement is that of *global conformity*: A reconstruction must be maximally compatible with what the author said elsewhere. Reconstructions that attribute a view to the author that obviously contradicts one of the author's views in one of his other writings, should be avoided. (In the case of Anselm's argument we might, for instance, have to look at the rest of *Proslogion* or the *Monologion*.)

(3) The structure of the formal reconstruction should represent the fundamental structure of the argument. It should be no more and no less detailed than is necessary to map the argument. On the one hand, this means that the core of the argument should not be packed into a single premiss as a whole. On the other hand, it means that unnecessary distinctions should be avoided; in particular, a word-by-word translation does not necessarily constitute a good reconstruction.

(4) As we have indicated already, conformity with the text overrules consistency and cogency. In a second step, improvements may be suggested. In any case, these steps should be clearly separated.

(5) If the argument and, therefore, its reconstruction are deductively valid, the premisses should contain the conclusion *in a non-obvious way*. The conclusion *has* to be contained in the premisses; otherwise, the reasoning would not be deductive.[4] But an argument can convince someone only if it is possible to accept the premisses without already recognizing that the conclusion follows from them. Thus, the desired conclusion has to be 'hidden' in the premisses.

(6) The premisses should be plausible from the standpoint of the author; the reconstruction must therefore not involve anything the author could not have meant. So this requirement forces the interpreter to take into consideration what, in the particular argument, is (or is not) likely to be *intended* by the author. In particular, we should not attribute to a philosopher premisses that are *obviously* false.

(7) Beyond that, for an ontological argument to succeed, the premisses not only have to be true, but must also be *analytically* true. Hence, we should attribute to the author only premisses that he could have held to be true for conceptual (non-empirical) reasons. The premisses should be direct consequences of conceptions presupposed by the author—i.e. they must follow from the author's understanding of a certain expression.[5]

[4] Sometimes, proofs of the existence of God are accused of being question-begging, but this critique is untenable. It is odd to ask for a deductive argument whose conclusion is not contained in the premisses. Logic cannot pull a rabbit out of the hat.

[5] Of course, it is notoriously hard to make precise what is to be counted as an *analytic truth* and it is even harder to reconstruct what a particular author would count as such. As concerns Anselm's argument, we shall, in subsequent chapters, try to make more precise what we take to be Anselm's understanding of key concepts in his argument, such as 'existence', 'being greater' etc., in order to make our assumptions plausible consequences of relations between these concepts.

We will use these requirements throughout this article in application to reconstructions of Anselm's argument for the existence of God, as set forth in *Proslogion*, Chap. II. There might be cases, however, where one requirement can be satisfied only on pain of violating another one. In such cases, the best we can do is to comply with as many requirements as possible and provide some sort of explanation as to what goes wrong. In the following section we will first discuss some general features of Anselm's argument and some of the key concepts involved in it. Sections. 3 and 4 will be devoted to a detailed discussion of various formalizations of Anselm's argument in classical and modal logic and in the final Sect. 5 we discuss what we can learn from the formalizations provided in Sects. 3 and 4.

2 Fundamentals and basic concepts

2.1 Anselm's argument in chapter II of *Proslogion*

For further reference, we quote the relevant passages of Anselm's argument here at length:

> (II.1) Ergo, Domine, qui das fidei intellectum, da mihi, ut, quantum scis expedire, intelligam, quia es sicut credimus, et hoc es credimus.
> Well then, Lord, You who give understanding to faith, grant me that I may understand, as much as You see fit, that You exist as we believe You to exist, and that You are what we believe You to be.
> (II.2) Et quidem credimus te esse aliquid quo nihil maius cogitari possit.
> Now we believe that You are something than which nothing greater can be conceived.
> (II.3) An ergo non est aliqua talis natura, quia *dixit inspiens in cordo suo: non est deus*?
> Or can it be that a thing of such nature does not exist, since *the fool has said in his heart, there is no God* [Ps. 13: 1; 52:1]?
> (II.4) Sed certe ipse idem inspiens, cum audit hoc ipsum quod dico: 'aliquid quo maius nihil cogitari potest', intelligit, quod audit;
> But surely, when this same Fool hears what I am speaking about, namely 'something than which nothing greater can be conceived', he understands what he hears,
> (II.5) et quod intelligit, in intellectu eius est, etiam si non intelligat illud esse.
> and what he understands is in his understanding, even if he does not understand that it exists [in reality].
> (II.6) Aliud enim est rem esse in intellectu, alium intelligere rem esse.
> For it is something else that a thing exists in the understanding than to understand that a thing exists [in reality].
> (II.7) Nam cum pictor praecogitat quae facturus est, habet quidem in intellectu, sed nondum intelligit esse quod nondum fecit. Cum vero iam pinxit, et habet in intellectu et intelligit esse quod iam fecit.
> Thus, when a painter plans beforehand what he is going to execute, he has [the picture] in his understanding, but he does not yet think that it actually exists because he has not yet executed it. However, when he has actually painted it, then he both has it in his understanding and understands that it exists because he has now made it.
> (II.8) Convincitur ergo etiam insipiens esse vel in intellectu aliquid quo nihil maius cogitari potest, quia hoc cum audit intelligit, et quidquid intelligitur in intellectu est.
> Even the fool, then, is forced to agree that something than which nothing greater can be conceived exists in the understanding, since he understands this when he hears it, and whatever is understood is in the understanding.

(II.9) Et certe id quo maius cogitari nequit, non potest esse in solo intellectu.
And, surely, that than which a greater cannot be conceived cannot exist in the understanding alone.
(II.10) Si enim vel in solo intellectu est, potest cogitari esse et in re; quod maius est.
For if it exists solely in the understanding, it can be conceived to exist in reality also, which is greater.
(II.11) Si ergo id quo maius cogitari non potest, est in solo intellectu: id ipsum quo maius cogitari non potest, est quo maius cogitari potest.
If, then, that than which a greater cannot be conceived exists in the understanding alone, this same than which a greater cannot be conceived is [something] than which a greater can be conceived.
(II.12) Sed certe hoc esse non potest.
But surely this cannot be.
(II.13) Existit ergo procul dubio aliquid quo maius cogitari non valet, et in intellectu et in re.
Something than which a greater cannot be conceived therefore exists without doubt, both in the understanding and in reality. (Latin in Anselm von Canterbury 1995, p. 84)[6]

Other passages in Anselm's *Proslogion* and other writings that are relevant for the reconstruction of Anselm's argument in Chap. II will be provided as we go along.

As one can see, Anselm's language is very formal. He uses only a small number of words and does not alternate between words with a similar meaning as Aquinas, for example, does. Moreover, each proposition in the argument is presented as being inferred by previously established propositions by necessity. This is what makes Anselm's argument attractive for logicians. Some have even argued that Anselm's 'unum argumentum' is itself one of the first formalisations of the ontological argument.[7]

Furthermore, Anselm apparently uses the same (or a very similar) pattern of proof throughout the *Proslogion* in order to show that God has each property which is better to have than not to have. This seems to be the case particularly in Chap. III, where Anselm wants to establish that God (or *that than which nothing greater can be conceived*) not only exists in reality, but that he does so necessarily. In Chap. V, he attributes justness, truthfulness, and blessedness to *that than which nothing greater can be conceived*, on the ground that something that would lack these properties would be less than could be conceived.[8] Accordingly, one can consider the argument in Chap. II as revealing a kind of constant proof structure, which is used by Anselm throughout the *Proslogion*.

However this may be, the question to start with is simply this: Which logical system or framework best suits Anselm's reasoning in Chap. II?[9] Before we can approach this question, we have to clarify some of the notions that are central to Anselm's argument.

6 The translation mainly follows Anselm of Canterbury (2008, p. 87f), but some minor changes are made to achieve unequivocal terminology. As we shall see in Sect. 2.4, there is good reason to translate 'maius' as 'bigger' rather than 'greater', but we will stick to the traditional translation in what follows.

7 Logan (2009, p. 176), for instance, attributes such a view to Graham Oppy.

8 Cf. Anselm of Canterbury (2008, p. 89).

9 Although it is not obvious that modern logic has the right tools for analysing Anselm's argument, throughout this article, we will stay within the limits of established logical systems as we saw no need to introduce new formalism. (Henry 1972, for instance, does.)

2.2 Existence

Having established in Chap. II that God exists in reality from the assumption that God exists at least in the understanding, Anselm proceeds in Chap. III by proving that it is inconceivable that God does not exist.[10] This suggests that at least three conceptions of existence have to be distinguished in Anselm's *Proslogion*:

(1) Existence *in the understanding* (*esse in intellectu*)
(2) Existence *in reality*
(3) *Necessary* existence (*non-existence being non-conceivable*)

It seems obvious to us that Anselm thinks of existence in reality as a substantial property in the sense that a thing can have or lack existence just as it can be green or not green.[11] Therefore, we will exclusively deal with versions that use a primitive predicate $E!$ expressing this property.[12] It is beyond the scope of this paper to discuss general objections against this conception, as developed by Kant, Frege and many others.

With regard to *existence in the understanding*—which Anselm further elucidates with his example of the painter in (II.7)—three perspectives are relevant. First of all, when we look at Anselm's wording, *existence in the understanding* is effectively treated parallel to *existence in reality*. This would suggest that *existence in the understanding* should be understood as a substantial property, expressed by a predicate U, which an object may have or lack just like *existence in reality*. However, the fact that Anselm uses *existence in the understanding* as a grammatical predicate does not in itself tell us anything about the meaning of *existence in the understanding* and which role this notion plays in the formal structure of Anselm's proof. So, second, one has to take a look at what it means to *exist in the understanding* and how this notion relates to *existence in reality*. Here, the fool plays a decisive role, since it is the (real) existence of the fool which guarantees that both concepts become intelligible in the first place. If not even the fool existed, it would not be clear what *existence in reality* could mean altogether. By contrast, *existence in the understanding* seems to be intelligible only when relativized to someone who understands: Indeed, it is again the *fool* who understands or has something in his understanding. But nothing could be said to *exist in the understanding of the fool* if the fool did not exist in reality.[13] In order to

[10] Cf. Anselm of Canterbury (2008, p. 88).

[11] Instead of distinguishing kinds of existence, in another fragment, Anselm distinguishes four ways of using 'something', concluding that 'when that which is indicated by the name and which is thought of in the mind does in fact exist', then this is the only way of using 'something' properly (Anselm of Canterbury 2008, pp. 477–479). However, we shall not pursue this line of reasoning. Alston (1965) elaborately discusses the adequacy of different conceptions of existence for Anselm's proof.

[12] Note that as our $E!$ is supposed to stand for a substantial property, it is not to be confused with Russell's existence predicate $E!$, which is defined contextually and does not express a genuine property at all.

[13] We are grateful to an anonymous referee for drawing our attention to this point.

represent this in our formalism we would thus have to construe Anselm's *being in the understanding* as a predicate true (or false) of objects *relative to a particular person* and, instead of the predicate U, would have to use a predicate U_i explicitly indicating a particular person i. But we hold that it is no coincidence that the fool is no longer mentioned from (II.9) until the end of the proof in Chap. II. Anselm here does not distinguish between *being in the understanding of a particular person* (viz., the fool) and *being in the understanding tout court*, i.e., *being understandable*. Even though the role of the fool is important in order to make the distinction between *existence in the understanding* and *existence in reality* intelligible, the fool does not matter for the formal structure of the argument given in (II.9)–(II.13). We will therefore stick with the simple predicate U, standing for *being understood* or *being understandable*.[14] Third, we have to consider Anselm's *existence in the understanding* as it figures in the formal structure of his argument. At the minimum, we can see that Anselm starts his proof with a certain being that exists in the understanding and then aims to show that this being exists in reality as well. So at least with respect to the order in the argument, *existence in the understanding* is prior to *existence in reality*. It thus seems reasonable to express *existence in the understanding* by means of the existential quantifier and to use a predicate for *existence in reality*. Since the purpose of the predicate U is merely to single out the understandable objects, we can choose the understandable objects as forming the domain of the quantifiers and so it is not necessary (though possible) to use U.[15]

Regarding the third mode of existence—i.e. Anselm's 'non cogitari potest ... non esse' ('cannot be conceived not to be')—most commentators seem to agree in interpreting this locution as somehow expressing *necessary existence*.[16] We shall postpone the discussion of how this is understood to Sect. 2.5, where the issue of Anselm's 'cogitari' will be considered at length.

2.3 God

Throughout this paper, we shall use **id quo** and **aliquid quo** as abbreviations for *that than which nothing greater can be conceived* ('id quo maius cogitari non potest') and *something than which nothing greater can be conceived* ('aliquid quo nihil maius cogitari potest'), respectively. The expression 'God' or 'is a God' will sometimes be

[14] This way of looking at Anselm's *esse in intellectu* is quite common. David Lewis, in his Lewis (1970), for instance, introduces a predicate Ux, representing the unrelativized predicate 'x is an understandable being'. A similar treatment can be found in Oppenheimer and Zalta (1991).

[15] If one insists on expressing *being in the understanding* by a predicate, it would be used in such a way that it could be 'cancelled out' in proofs ($\exists x(Ux \wedge ...)$ and then $\forall x(Ux \rightarrow ...)$). For further discussion on this issue, see Oppenheimer and Zalta (1991).

[16] See, e.g., Hartshorne (1941), Hartshorne (1965), Malcolm (1960); both authors argue that Chap. II does not offer a conclusive argument, whereas Chap. III would, if reconstructed in modal logic.

used as a shorthand of the predicative expression, 'being something than which nothing greater can be conceived'. In the formal reconstructions we shall discuss, 'is a God' will be represented by a one-place predicate 'G', defined by the formal analogue of *being something, than which nothing greater can be conceived* (and which may vary from reconstruction to reconstruction). We are well aware though that

(1) at crucial passages in Chap. II, Anselm uses the locution 'that than which nothing greater can be conceived' ('id quo maius cogitari non potest') instead of 'something than which nothing greater can be conceived' and that one might argue that this locution has to be construed as a *definite description*, and
(2) it is not clear (not in Chap. II at least) that being something than which nothing greater can be conceived is the same as being something that we would traditionally call 'God'.

We think that, as far the argument in Chap. II is concerned, and which is our main business in this article, (2) is simply irrelevant. The argument in Chap. II is, in the first place, intended to show that **aliquid quo** exists, and it is only in subsequent chapters of the *Proslogion* that Anselm attributes the usual divine attributes (justness, truthfulness, blessedness, omnipotence etc.) to the uniquely identified **id quo**, thereby establishing that **id quo** is indeed God as we usually conceive of him.[17]

From a logical point of view, the first objection seems more severe. Whether or not a reconstruction of Anselm's argument is valid may crucially depend on whether **id quo** has to be understood as a definite description. But we think that it is not just that we do not *have to* understand **id quo** as a definite description, but that we *should* not.[18] For one thing, if **id quo** had to be read as a definite description, Anselm would be committed to presupposing the *uniqueness*[19] of **aliquid quo** already in Chap. II, which seems to be in conflict with the fact that only in Chap. III does Anselm mention God's uniqueness for the first time.[20] Rather, it seems to us that Anselm is using this diction only as a device to refer back to *something* 'than which nothing greater can be conceived'. In other words, we think that Anselm's **id quo** is best understood as

[17] Compare Logan (2009, pp. 18, 91, and 114). Logan (2009, pp. 15–17, pp. 125–127) also discusses at length what Anselm means by 'unum argumentum' and, in particular, argues that its form is, to a high degree, a product of Aristotelian dialectics. Therefore, it has the form of a syllogism (Logan 2009, p. 17):

God is X; X exists ∴ God exists

where X, the middle term, is Anselm's **aliquid quo**. We will exclusively deal with the proof that **aliquid quo** exists.

[18] For diverging views, see, e.g., Oppenheimer and Zalta (1991) or Morscher (1991).

[19] Note that in our construal, the *existence* presupposition does no harm (even if **id quo** had to be construed as a definite description), for what is presupposed is only the ontologically neutral *existence in the understanding*, expressed by the quantifier.

[20] 'In fact, everything else there is, except You alone, can be thought of as non existing. You, alone then, ...' (Anselm of Canterbury 2008, p. 88).

an *auxiliary name*, which is used to prove something from an existence assumption.[21] If we want to prove some statement γ from an existential statement $\exists x\phi(x)$, we assume that some arbitrary object g is such that it satisfies $\phi(x)$ and prove γ from this assumption. Now, in Anselm's argument, γ is simply the statement that God exists in reality and $\phi(x)$ is the condition, *being something than which nothing greater can be conceived*. Anselm's locution **id quo** is simply a clumsy way of referring to one such fixed thing throughout the proof, and whose existence is affirmed by the premise $\exists x\phi(x)$, stating that **aliquid quo** exists in the understanding. Consequently, we will consider any reconstruction that translates **id quo** as a definite description as violating requirement (3) as well as (6).

2.4 Greater

Another key notion that we must consider is Anselm's 'greater' ('maius')—a concept that does not figure prominently in any other ontological proof. Here, two questions must be kept apart:

(1) What, if anything, defines the meaning of *greaterness* in its entirety in terms of more fundamental concepts?
(2) What are *necessary and/or sufficient conditions* for something to be greater than something (else)?

As to the first question, we see no answer based on Anselm's *Proslogion* (or anything Anselm has ever written for that matter). All we can reasonably expect is to give an answer to the second question, which is in one way or another 'revealing'. In what follows, whenever we talk about the *meaning* of *greater*, we have in mind such conditions. But what could such 'revealing' conditions be?

In Chap. II of *Proslogion*, all that Anselm literally tells us about the greater-relation is that existence in reality makes **id quo** greater.[22] Therefore, if we take Chap. II ((II.10), in particular) *in isolation*, it would be consistent to interpret Anselm in such a way that any property whatsoever makes something greater. Such a reading would seem to be corroborated by the fact that Anselm uses the ontologically and morally neutral term, 'maius' ('greater'), instead of the judgemental 'melius' ('better'), which is introduced only in Chap. III.[23] On the other hand, it is compatible with Anselm's

[21] In linguistics, devices with a similar function are called *E-types* (sometimes *D-types*). We thank an anonymous referee for this hint.

[22] Concerning the question of why existence apparently makes 'greater', see Millican (2004), Nagasawa (2007) and Millican (2007). Whether existence makes *everything* greater is a question that cannot be discussed here at length, but Logan convincingly argues that both options can be maintained (Logan 2009, p. 94f).

[23] Seneca is the only one of Anselm's possible sources who also uses 'maius'; all the others use the judgemental 'melius' (Logan 2009, p. 93). Whereas Descartes, Leibniz and others use 'perfections' or similar expressions and, hence, morally charged terms, Anselm uses a more neutral term. One might be

argument in Chap. II that existence in reality is the *only* property that makes something greater. As regards Chap. II, the situation is that apparently any class of properties can be consistently assumed to be the class of properties that makes a thing greater, as long as *existence in reality* is in this class. Even though it is implausible to assume that *any* property whatsoever makes something greater (or that *only* existence makes something greater), the valid core of this, and one of the main ideas that we shall follow in the course of this paper, is that something is greater than something else if it has *more properties of a certain sort*. After all, what else might make something greater if not some property? Hence, we try to incorporate a broader understanding of Anselm's reasoning, according to which, existence is only one property among many that in some way determine the greater-relation. But what sort of properties would that be? Depending on the textual sources that are taken into consideration, various candidates that are at least compatible with Anselm's *Proslogion* come to mind. If the context that is considered relevant for the reconstruction is broadened to include Chap. III, *necessary existence* comes into play as a candidate for a property that makes something greater. This would seem to imply that at least various modes of existence are such that having them makes a thing greater.[24] Going a step further, we might take into account the rest of *Proslogion*. In Chap. V, Anselm argues that God is 'just, truthful and blessed and whatever it is better to be than not to be' (Anselm of Canterbury 2008, p. 89) as a consequence of the proposition that God is *that than which nothing greater can be conceived*. In subsequent chapters, he attributes omnipotence, compassion and passionlessness to God as well. This would point to an interpretation according to which not only modes of existence make greater, but each property in a certain, more inclusive class of *positive* properties. Such an interpretation seems to be confirmed if we consider other writings by Anselm, where we find specifications to the effect that God has *only* properties that are 'universally good'.[25]

In our opinion, the moral here is that a decision as to which properties make something greater and which do not depends on a *weighting* of the available textual evidence with regard to its relevance for the argument in Chap. II—a weighting, moreover, that is inevitably arbitrary to *some* degree. Consequently, a reconstruction that tries to incorporate the intuition that it is *properties* that make greater, has to leave room for various candidates of classes of properties that determine this basic relation. In the formal reconstructions where quantification over properties is needed, the second-order quantifiers should, therefore, be understood as relativized to a 'parameter-class'

Footnote 23 continued
tempted to translate 'maius' as 'bigger' instead of 'greater'. This would be unusual and might sound a bit uncouth, but it would accommodate the neutrality of 'maius'. (Of course, we are aware that 'maius' can also be understood as judgemental, but—unlike 'melius'—it does not *have to be* understood this way.)

[24] St. Augustine had already introduced something like a 'scale of being' and we know that Anselm was well aware of Augustine (see Matthews 2004, p. 64). In *Proslogion*, however, the logical structure of the argument does not involve different degrees of properties, but differences in the kinds of properties.

[25] See Chap. 15 of *Monologion* (Anselm of Canterbury 2008, p. 15); compare Leftow (2004, pp. 137–139).

of properties $\mathcal{P}$. The only thing we require is that this class be *consistent*—i.e. that the properties in $\mathcal{P}$ should not contradict each other[26] and, in order to avoid technical inconveniences, that the properties in $\mathcal{P}$ are *primitive*. Although Anselm remains silent on this issue (as shall we), problems related to this topic become important for Leibniz, and later, Gödel in their attempts to rigorously prove the *existence in the understanding/possible existence* of God by proving that certain classes of properties are consistent.

In most of our reconstructions, the greater-relation will be represented by a two-place relation '$>$', applying to pairs of individuals. Obviously, by using the sign '$>$', we want to indicate that Anselm's greater-relation defines some kind of *ordering*. Which properties exactly this ordering should have is not clear from Anselm's text. We may plausibly assume that the relation '$>$' is *irreflexive*, i.e. that nothing is greater than itself.[27] However, this raises a problem with a faithful rendering of (II.10) and (II.11) by means of '$>$' alone. Anselm here apparently argues that if God did not actually exist, then something could be conceived to be greater than God—namely, He himself! The idea seems to be that if God did not exist, we could somehow think of this very being and simply 'add' the property of *existence* and thereby think of this being as existing. If, however, as we assume, '$>$' is irreflexive and thus no being is (in fact) greater than it (in fact) is, there seems to be no obvious way to render the contradiction that Anselm derives from the assumption that God does not exist, viz., that God is (or *would* be) greater than himself. So, in order not to violate requirement (6), we must not attribute such a trivialized version to Anselm. The general principle behind Anselm's claim seems to be that anything that lacks existence *could* be *conceived* to be greater than it in fact is and it is this principle, that somehow has to be implemented in a successful reconstruction. *How* this principle is implemented and how it is implemented *best* seem to be further questions that are open to discussion and depend on substantial questions concerning identity and individuation. In accordance with the Geach/Strawson view, we shall present various formal reconstructions of what we take to be the punch line of Anselm's argument and, in particular, in (II.10).

2.5 Cogitare/Intellegere

A further topic that is much-discussed in the literature is how Anselm's 'cogitari potest' and its relation to 'esse in intellectu' are understood properly.[28] For the sake

[26] Otherwise, God would have to have both positive properties, P and Q, in order to be **id quo**. But, on the other hand, if P and Q contradict each other, he *cannot* because, by assumption, nothing is both P and Q.

[27] A further property that seems to be mandatory is that '$>$' be *asymmetric*. Thus, '$>$' is likely to define some *partial order*. Whether any two objects can be compared as to their greatness as well—i.e. whether the ordering is *total*—cannot be decided on the basis of Anselm's writings.

[28] See, e.g., Schrimpf (1994, pp. 29–31).

of clarity, we will translate ‘cogitare’ consequently by ‘conceive’ and ‘intellegere’ by ‘understand’.

Concerning Anselm’s ‘cogitare’, two main lines of interpretation leading to radically different reconstructions can be distinguished. The first interpretation treats *cogitari potest* in Chap. II as something that is said of *objects*. This use of *being conceived* is clearly suggested in Chap. IV, for instance, where Anselm uses ‘cogitatur res’. Therefore, following this approach

(1) x is something than which nothing greater can be conceived

is equivalently expressed by

($1'$) there is no conceivable being that is greater than x

In the second interpretation, clearly suggested in Chap. III, the *cogitari potest* is interpreted as a modal *operator*, operating on *sentences*. That is, (1) is supposed to be equivalent to

($1''$) it is not conceivable that there is something greater than x.

More specifically, in the first interpretation, the story goes along the following lines: The *cogitari potest* is just a device to introduce the reader to the discourse that follows and which is, in the first instance, about *conceivable objects*. Being conceivable is introduced as an alternative to ‘esse in re’. The upshot is this: Had Anselm used the locution ‘aliquid quo nihil maius est’ instead of ‘aliquid quo nihil maius cogitari potest’, one would naturally take Anselm to be saying that among the things existing *in reality*, nothing is greater than God. Therefore, the point of introducing the *cogitari potest* in the first place in this interpretation is clear enough: In the absence of any specification or contrast, one would understand ‘esse’ as ‘existing in reality’, which is obviously orthogonal to Anselm’s intention. In this interpretation, Anselm’s *cogitari potest* is, as regards God, specified as *being in the understanding* (‘esse in intellectu’). As Anselm explains in Chap. IV, there are two ways of *being conceived*: ‘when the word signifying is conceived’ and ‘when the very object which the thing is, is understood’.[29] For example, ‘the largest prime number’ can be *conceived* by a competent English speaker, as far as the ‘word signifying’ is concerned. However, since it is mathematically impossible for there to be a largest prime number, the term ‘the largest prime number’ cannot be *understood* in the second sense. Anselm’s **id quo** can be ‘conceived’ in both ways. Yet, as we may neglect whatever can be *conceived* of in the first way *only*, the range of individuals (universe of discourse) can be determined as the things being in the understanding. Summing up, on the first approach, conceivability applies to *objects*. The conceivable objects are exactly the understandable objects (object that exist in the understanding), and those are exactly the objects the quantfiers range over.

29 Anselm of Canterbury (2008, p. 88f).

In the second approach, *conceivability* is not treated as a predicate true of objects that introduces the reader to 'non-factual discourse'. Rather, it is understood as a modality of some kind, expressed by an operator $\Diamond$, which is supposed to govern sentences (or formulas more generally). *Which* modality is expressed by *conceivability* is a matter of controversy. *Conceivability* has clear epistemic connotations, so it is not obvious—in fact, rather unlikely—that it could be identified with alethic *possibility*.[30] But the question of *which* modality is expressed by *conceivability* and whether it is simple or compound, is less important as long as certain basic principles hold for *conceivability* thus construed. For instance, the non-conceivability of the negation of some sentence should always imply the truth of this sentence. A more detailed discussion of this issue, and of the question of which system of modal logic is presupposed for the argument to go through, will be provided in Sect. 4.

It seems to us that, *prima facie*, neither interpretation can claim to be definitely correct. Both can be justified on some ground. Indeed, the Latin nomenclature itself is ambiguous here—a circumstance that is reflected by the fact that translations disagree with regard to other critical passages in Chap. III as well as Chap. II. The decision as to which interpretation should be preferred has wide-ranging consequences and divides this paper into Sects. 3 and 4.[31]

3 Reconstructions in classical logic

3.1 Propositional logic

Before we begin reconstructing Anselm's argument, let us first sketch the outer structure. The most general pattern in his argument can be rephrased as a reductio ad absurdum. Anselm in (II.8) assumes (or takes it as established by his argument in (II.4) and (II.5)) that **aliquid quo** exists at least in the understanding.[32] Referring to one such thing by the idiom **id quo**, he then concludes from the assumption that **id quo** exists *only* in the understanding that there would be something greater than **id quo** ((II.8)–(II.10)). From this, the contradiction that **id quo** would not be **aliquid**

[30] Henry (1972, p. 108f), for instance, argues that Anselm thinks of 'being inconceivable not to be' as something stronger than 'being necessary'—a view he attributes to Anselm's Boethian background. An interesting definition has been suggested in Morscher (1991), in which a *conceivability*-operator D is defined by means of the composition of an epistemic component D' ('it is conceived that') and an alethic component $\Diamond$ ('it is possible that'); thus: $Dp := \Diamond D' p$. The epistemic operator D' itself can be defined by $D' p := \exists y D''_y p$, where $D''_y p$ is supposed to stand for 'y thinks that p'. Therefore, unpacking the definitions, Morscher's notion of *conceivability* could be stated in the following way: It is conceivable that p if and only if it is possible that someone thinks that p.

[31] A similar distinction is discussed by Dale Jacquette in his Jacquette (1997). Jacquette favours—against Priest—a version that uses *conceivability* as an operator on propositions or propositional functions.

[32] Maydole (2009) tries to provide a formal reconstruction of Anselm's reasoning in (II.4)–(II.5). In the following, however, we shall exclusively deal with (versions of) Anselm's argument as it is stated in (II.8)–(II.13), since it is here that we think the very heart of Anselm's ontological argument is lying.

quo is derived in (II.11). He then concludes in (II.13) that **aliquid quo** is both, in the understanding and in reality.

Following Morscher and using the abbreviations A for the proposition that **id quo** does not exist in reality, B for the proposition that something can be conceived to be greater than **id quo**, and C for the contradiction in (II.11), the basic structure of Anselm's argument can be depicted as follows:[33]

(1) $\neg A$
(2) $\neg A \rightarrow B$
(3) $B \rightarrow C$
___
(4) C
(5) A

It should be clear that, in itself, this is not a 'reconstruction' of Anselm's argument. In order to assess the argument, we need to show, in accordance with our requirements from Sect. 1.2, how premisses (2) and (3) can be justified and what the contradiction C consists in. In order to do so, we must go a step further and take into account the more fine-grained, quantificational structure of Anselm's argument.

3.2 First-order logic

In accordance with our first line of interpretation of the idiom 'cogitari potest' (cf. Sect. 2.5), we shall first take care of Anselm's **aliquid quo**. Recall that, according to this line of interpretation, the locution 'x is **aliquid quo**' is explicated by 'there is no conceivable being greater than x', where the conceivable beings are precisely those that are in the understanding—i.e. those beings that are embraced by the first-order quantifiers (cf. Sect. 2.2). Therefore, being a God (being **aliquid quo**) can be defined in the following way:

Def C-God: $\quad Gx :\leftrightarrow \neg\exists y(y > x)$

where $x > y$ stands for 'x is greater than y'.

What is to be proved then is the following statement expressing that **aliquid quo** exists in reality:

God!: $\quad \exists x(Gx \wedge E!x)$

Throughout this paper, whenever we shall be trying to prove that a God (or **aliquid quo**) exists, we shall be concerned with the statement **God!**.

According to (II.8), an x such that Gx exists at least in the understanding. Therefore, our first premiss will be:

ExUnd: $\quad \exists x Gx$

This will be a premiss for each of the reconstructions that we shall consider.

[33] See Morscher (1991, p. 65).

A first idea to formalise the punch line of the argument could be achieved by the following premiss:

Greater 1: $\forall x(\neg E!x \rightarrow \exists y(y > x))$

This seems to be a good approximation,[34] but offers no clues regarding why we should consider it to be true. The problem with this version seems to be that it is too rough and, therefore, in violation of requirement (3).[35] The axiom does not tell us *anything* about *being greater*, besides the fact that for everything that does not exist there exists something greater. We are well aware of many conceptions of *being greater* and *existence* for which this is not the case. For example, we might say that there is no round square without implying that there is something greater than a round square. Therefore, in order to fulfil requirement (7), it must be spelt out, at least to some degree, what 'being greater' *means*; instead of being plausible (from Anselm's point of view and in general), **Greater 1** seems to be a rather strong—and apparently unjustified—claim. The least that we would require of an explanation of *being greater* is that it should *compare* something; it should give us conditions under which a certain relation holds. But, in **Greater 1**, the antecedent is a simple existence claim; no comparison at all is involved. Hence, a first improvement would be:

Greater 2: $\forall x \forall y(E!x \wedge \neg E!y \rightarrow x > y)$

Whenever one of two things exists and the other one does not, the existing one is greater. However, it is evident that, based on the premisses given so far **God!** does not follow, because it might be the case that nothing at all exists in reality. If so, then each of the premisses stated so far is satisfied, but it is not the case that **aliquid quo** exists in reality. Therefore, in order to make the argument valid, we would have to add as a further premiss that

E!: $\exists x E!x$

Although we can prove **God!** based on the premisses presented thus far, we do not think that this provides an argument cogent to Anselm's reasoning. Whatever merits this reconstruction may have in its own right, it seems to us that it fails to be true to Anselm's actual reasoning in many respects. We mention only some of these failings. Obviously, nowhere in the relevant passages (II.8)–(II.13) does Anselm explicitly

[34] In particular, even on this rough reconstruction, Anselm's argument is immune against Gaunilo's famous 'island-objection', claiming that, were Anselm's argument sound, we could, by analogy, also prove the existence of a 'most perfect island'. Now, to see that this is not the case, let Gaunilo's island g be defined by $Ig \wedge \neg \exists y(Iy \wedge y > g)$, expressing that g is an island such that no conceivable island is greater than g. It can be seen quite easily that we cannot prove a contradiction from **Greater 1** and **ExUnd** from the assumption that g does not exist unless we strengthen **Greater 1**—rather implausibly—to $\forall x(\neg E!x \rightarrow \exists y(Iy \wedge y > x))$, expressing that whenever some being does not exist in reality, there is some *island* which is greater than this being. Similar remarks can be made concerning all the reconstructions that will follow.

[35] Sobel (2004, pp. 60–65), for instance, presents the argument in this form, though his aim is not to present a faithful reconstruction.

assume that something actually exists, nor does it seem to play a decisive role in his argument. Hence, requirement (1) is not satisfied. Two further points must be kept in mind. First, the reconstruction still does not provide an understanding of *being greater* that makes the premisses analytic truths, thus again violating requirement (7). Second, **Greater 2** compares *two* things. Yet, as we have already seen in Sect. 2.4, Anselm seems to compare only *one* thing with itself (namely, **id quo**). This would not be a problem if the comparison of one object with itself could simply be seen as a special case of comparing two objects, but **Greater 2** does not allow for this. **Greater 2** only allows the inference that an object which both exists and does not exist is greater than itself. But this is hardly what Anselm is arguing for. What Anselm *is* arguing is that if something does not exist, it could be *conceived* to be greater by conceiving of this object as existing (cf. Sect. 2.4).

Therefore, the task for the next section will be (a) to find a sufficient understanding of *being greater* and (b) to find a way to compare a thing with itself.

3.3 Higher-order logic

In the following, we will use second-order quantifiers $\forall_{\mathcal{P}} F$ and $\exists_{\mathcal{P}} F$, where the subscripts indicate that we are quantifying over some restricted class of properties.[36] $\forall_{\mathcal{P}} F \phi(F)$ and $\exists_{\mathcal{P}} F \phi(F)$ should be understood as abbreviations for $\forall F(\mathcal{P}(F) \rightarrow \phi(F))$ and $\exists F(\mathcal{P}(F) \wedge \phi(F))$, where $\mathcal{P}$ stands for some class of primitive properties. As explained in Sect. 2.4, the primitive properties can be chosen in such a way that they are, e.g., the 'positive' properties, leaving room for possible explications of 'positiveness' and, hence, of which properties exactly make greater.[37] Therefore, $\mathcal{P}$ might be conceived of as a 'parameter class' and all the proofs in this section must be understood as proof schemes that become actual proofs only once we are given an explication of what exactly belongs to $\mathcal{P}$. Again, the only thing we shall assume is that $\mathcal{P}$ is consistent.

A first possible reconstruction is based on a reformulation of Anselm's definition of God, according to which a being is **aliquid quo** if it has *every (primitive, positive) property*. Hence, we define the predicate, *being God*, by the stipulation that x is a God if $\forall_{\mathcal{P}} F F x$. Evidently, this implies **God!** if we assume that such a thing exists in the understanding and existence in reality is a primitive (positive) property. But it should be obvious that hardly any of the requirements for good reconstructions is satisfied.

36 Throughout the following sections, we shall assume, as usual, that the second-order comprehension axioms are satisfied—i.e. for every formula $\phi(x)$, it holds that (CA) $\exists F \forall x(Fx \leftrightarrow \phi(x))$. In other words, every simple or complex formula defines a property. However, most of the time, we shall be concerned with quantification over the properties in the restricted class $\mathcal{P}$ only. If there are only finitely many properties in $\mathcal{P}$, then the quantifiers can be replaced by finite conjunctions.

37 Maydole (2009), who also uses a second-order predicate P, is simply speaking of 'great-making properties'.

In particular, the conclusion **God!** is not hidden at all; it is on the table from the very beginning (requirement (5)).

Although this trivial version is not convincing, one can see the major advantage of the use of second-order logic in comparison with its first-order analogues: *Other* properties than existence are involved in the determination of *greater*. The idea here is that *being greater* consists in having more properties of a certain kind:[38]

Greater 3: $$x > y \leftrightarrow \forall_{\mathcal{P}} F(Fy \rightarrow Fx) \wedge \exists_{\mathcal{P}} F(Fx \wedge \neg Fy)$$

Anselm must have had an idea of the meaning of *greater* which implies that existence makes a thing greater (requirement (7)). Now, **Greater 3** captures this feature by stating more generally that x is greater than y just in case x has all the properties in the class $\mathcal{P}$ that y has, plus an additional property in $\mathcal{P}$ that y has *not*. As we said earlier, there is nothing in Anselm's writings that would fix, once and for all, which properties exactly are members of $\mathcal{P}$ (modes of existence, positive properties like truthfulness, being just, etc.—see Sect. 2.4).[39] The only thing which *is* definite in Chap. II is that Anselm considers existence in reality to be one of these properties. Indeed, that is the only thing we shall presuppose about $\mathcal{P}$ in what follows.

Greater 3 is quite plausible, but we have yet to incorporate the idea that Anselm apparently does not want to compare one thing with *another* thing as to its greatness, but rather one thing with *itself*. It is a hypothetically non-existing God that is compared to itself, conceived of as existing. To accommodate this intuition, let us call two things *quasi-identical* if they have exactly the same properties up to a certain set of non-essential properties $\mathcal{D} \subseteq \mathcal{P}$. Given such a set of non-essential properties $\mathcal{D}$, we may define x to be quasi-identical to y *modulo* the properties in $\mathcal{D}$; in short, $x \equiv_{\mathcal{D}} y$, by the stipulation

Quasi-Id: $$x \equiv_{\mathcal{D}} y :\leftrightarrow \forall_{\mathcal{P}} F(\neg\mathcal{D}(F) \rightarrow (Fx \leftrightarrow Fy))$$

stating that two objects are quasi-identical (*modulo* the properties in $\mathcal{D}$), just in case they share all properties except for, possibly, those in $\mathcal{D}$. The idea here is that if an object has all the properties that another thing has, except for some inessential property, then both should be considered equal. In other words, by an 'essential property' we

[38] There are earlier reconstructions, which have followed a similar line of understanding Anselm's argument. Alvin Plantinga, for instance, suggests that Anselm's reasoning might be based on the following premiss:

> (2a) If A has every property (except for *nonexistence* and any property entailing it) that B has and A exists but B does not, then A is greater than B (Plantinga 1967, p. 67)

Plantinga later (cf. Plantinga 1974, p. 200) comes to question his earlier attempt to reconstruct Anselm's argument by means of (2a). In any case, our problem here is how such an informal premiss might be spelt out in formal terms.

[39] Given an explication of the properties in $\mathcal{P}$, we may regard **Greater 3** even as a *definition* of the notion of 'being greater than'. As we shall see though, the right-to-left direction suffices for the purpose of proving that **aliquid quo** exists. Also, with **Greater 3** at hand, $>$ defines a partial order, a fact that seems to be welcome. As mentioned earlier, Anselm's greater relation should clearly determine an order of some kind.

simply mean a property such that if you 'take away' this property from an individual, it is no longer the same individual. If, in particular, $x \equiv_{E!} y$, then x and y have exactly the same (primitive) properties except for existence.[40]

With the auxiliary notion of quasi-identity at hand, we have an opportunity to come to grips with Anselm's reasoning in Chap. II. Instead of comparing the hypothetically non-existing **id quo** with the *same* thing, conceived of as existing, we can now compare a hypothetically non-existing **id quo** with the *quasi-same* thing. **Greater 3** then implies that if $x \equiv_{E!} y$, $E!x$ and $\neg E!y$, then $x > y$. So we can now compare God with himself and we have incorporated a broader understanding of what, according to Anselm, makes a thing 'greater'. Yet, the problem is that we no longer have a conclusive argument. From the premisses provided so far, **God!** cannot be proved.

In order to see what is missing, recall our assumption that **aliquid quo** exists in the understanding. Suppose **id quo** is one of those beings, and suppose further that **id quo** does not exist in reality. When we think of **id quo** as being constituted by its properties, we can think of another object, which has all the properties **id quo** has, but also the further property of existence in reality. However, in the present setting, we have no means to establish that such an object *exists*, not even in the *understanding*. The following axiom, formulated in *third*-order logic, precisely allows for this: It says that every set of properties is realised in the intellect and for every reasonable set of properties there is at least one object in the understanding having these properties:

Realization: $\quad \forall_{\mathcal{P}}\mathcal{F}\exists x\forall_{\mathcal{P}}F(\mathcal{F}(F) \leftrightarrow Fx)$

Here, the third-order quantifier $\forall_{\mathcal{P}}\mathcal{F}$ ranges over all properties of properties in $\mathcal{P}$, or all sets of $\mathcal{P}$-properties $\mathcal{F}$. Since we assume that $\mathcal{P}$ is consistent, any subset of $\mathcal{P}$ will be consistent as well. So, bearing in mind that first-order quantifiers are ranging over objects existing in the understanding, **Realization** seems plausible. It appears to be an analytic truth that any consistent set of (primitive, positive) conditions is realized by some object in the understanding. This seems to be confirmed by passages like (II.8), where Anselm claims that 'whatever is understood is in the understanding'.[41] Bearing in mind that by 'understanding something' Anselm means understanding what its properties are, we can see that whenever we conceive of a certain set of (non-contradictory) properties, this set gives rise to an object that exists in the understanding—and this

[40] Here, the restriction to primitive properties is essential. Suppose we were to use an unrestricted universal quantifier $\forall F$ in the definition of quasi-identity. Then, one instance of the comprehension scheme for second-order logic will be $\exists F\forall x(Fx \leftrightarrow \neg E!x)$ and, hence, the property of *non-existence* $\neg E!$ will be among the values of the variable F in the definition of quasi-identity. Suppose, then, $x \equiv_{E!} y$. Since $E! \neq \neg E!$, we would then have $\neg E!x \leftrightarrow \neg E!y$—i.e. $E!x \leftrightarrow E!y$. Therefore, we could no longer assume, as needed in Anselm's argument, that one of the 'two' compared things exists and the other does not. Notice also that a full account of quasi-identity would have to add some characterisation of which properties are essential for an object and which properties belong to an object only accidentally. Of course, this problem has already been discussed by ancient philosophers and cannot be discussed here in detail.

[41] See also Anselm's second reply to Gaunilo (Anselm of Canterbury 2008, p. 113).

is just what **Realization** says. So even though Anselm does not state **Realization** explicitly, we think that it is implicit in how Anselm thinks about objects.

Now that everything is in place, we are in a position to prove **God!** as follows:

Proof. First, by **ExUnd** and **Def C-God**, let g be such that $\neg\exists x(x > g)$ and suppose for reductio that $\neg E!g$. Furthermore, we define the higher-order predicate $\mathcal{F}_{E!}$ as follows:[42]

$$\mathcal{F}_{E!}(F) :\leftrightarrow Fg \vee F = E!$$

By **Realization**, we may infer that $\mathcal{F}_{E!}$ is realized by some object. Let a be such an object—i.e. $\forall_{\mathcal{P}} F((Fg \vee F = E!) \leftrightarrow Fa)$. By propositional calculus, we then have

$$(*) \qquad \forall_{\mathcal{P}} F((Fg \rightarrow Fa) \wedge (F = E! \rightarrow Fa))$$

and

$$(**) \quad \forall_{\mathcal{P}} F(Fa \rightarrow (Fg \vee F = E!))$$

From the second conjunct of $(*)$ together with the assumption that $E!$ is positive and primitive, we get $E!a$. Since, by assumption $\neg E!g$, we have $\exists_{\mathcal{P}} F(Fa \wedge \neg Fg)$. But, by the first conjunct, we also have $\forall_{\mathcal{P}} F(Fg \rightarrow Fa)$. So, from the right-to-left direction of **Greater 3**, we have $a > g$ and, hence, $\exists x(x > g)$, contradicting the assumption that there is no conceivable being greater than g. $\square$

The mindful reader will have noticed that nothing in the proof depends on the fact that a and g are identical—or quasi-identical. However, given the particular choice of the predicate to which **Realization** is applied, we can *prove* that $a \equiv_{E!} g$ and this accommodates the intuition where we do not compare *two* things as to their greatness, but one and the (quasi-)same thing.[43] Of course, strictly speaking, a and g are distinct per Leibniz' law. Yet, the only thing that distinguishes a from g is that g is assumed not to exist, whereas a is, by definition, assumed to exist. Notice that we could still prove **God!** had **Realization** been weakened to guarantee only the existence (in the understanding) of an object exemplifying all properties in a certain class of properties, but not necessarily *only* such properties.[44] Yet, in this case, we could not account for the fact that the object that would be greater than **id quo** (were we to assume that **id quo** does not exist in reality) would be the *(quasi-)same thing*.

Anselm's proof in Chap. III of the inconceivability of the non-existence of **id quo** works analogously to the proof just given. Instead of using the higher-order property $\mathcal{F}_{E!}$, we define a higher-order predicate $\mathcal{F}_{\Box E!}$ by the stipulation

[42] In what follows, we use $X = Y$ as shorthand for $\forall x(Xx \leftrightarrow Yx)$.

[43] From $(**)$, it follows that $\forall_{\mathcal{P}} F(F \neq E! \rightarrow (Fa \rightarrow Fg))$. From the first conjunct of $(*)$, it follows by weakening that $\forall_{\mathcal{P}} F(F \neq E! \rightarrow (Fg \rightarrow Fa))$ and, hence, $\forall_{\mathcal{P}} F(F \neq E! \rightarrow (Fg \leftrightarrow Fa))$—i.e. $a \equiv_{E!} g$.

[44] Therefore, instead of **Realization**, we would adopt $\forall_{\mathcal{P}} \mathcal{F} \exists x \forall_{\mathcal{P}} F(\mathcal{F}(F) \rightarrow Fx)$.

$$\mathcal{F}_{\Box E!}(F) :\leftrightarrow Fg \lor F = \Box E!$$

and prove that $\exists x(Gx \land \Box E!x)$ analogously to the proof of **God!**, given the assumption that $\Box E!$ is in $\mathcal{P}$. This can obviously be generalized. That is, for *any* property P in the parameter class $\mathcal{P}$, we can repeat the same proof by stipulating that a property F is in the class $\mathcal{F}_P$ if and only if g has the property F or F is P. Therefore, what is provided is a proof scheme that allows us to attribute *any* property to **id quo** as long as it is in the class $\mathcal{P}$, which many think Anselm is indeed doing in the subsequent chapters of *Proslogion*. Garry Mathews, for instance, states: 'In each case he concludes that whatever lacks the attribute in question would be ''less than what could be conceived'' (*minus est quam quod cogitari potest*) and so the attribute does indeed belong to God. Anselm's formula thus gives him a decision procedure for determining which are the divine attributes.'[45]

4 Reconstructions in modal logic

In this section, we shall present and review some of the more plausible reconstructions of Anselm's argument within *modal logic*. As in the previous sections, we present the reconstructions in order of increasing strength of the underlying logic. Details as to what is presupposed in each reconstruction will be discussed when the reconstructions are presented. What can be said at the outset is that we shall adopt a standard view as to the basics of modal logic. In other words, we shall express modal statements within some extension of classical logic, including the propositional operators $\Diamond$ and $\Box$. In the context of Anselm's proofs, these should be understood as formal counterparts of Anselm's 'it is conceivable that' and 'it is not conceivable that not', respectively. In order to facilitate discussion, we shall use 'possibly' and 'necessarily' interchangeably with 'conceivably' and 'not conceivably not', respectively. But this should not be understood as committing us to the view that Anselm was employing alethic modalities (see Sect. 2.5). The only laws concerning conceivability we shall assume are the usual *rule of necessitation* $\vdash \phi \Rightarrow \vdash \Box\phi$ and the axiom schemes

(K) $\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$ and
(T) $\Box\phi \to \phi$ (equivalently $\phi \to \Diamond\phi$)

which seem to be mandatory on any modal conception of conceivability which can claim to be faithful to Anselm's reasoning. In particular we must not construe conceivability in such a way that it might be possible for ϕ to be true, yet—e.g., due to limitations of imagination—ϕ to be inconceivable. That is, we must not think of *conceivability* as *conceivable by a particular person at a particular time*. Such a conception of *conceivability* would, on our view, be too narrow to be useful for an ontological argument, and in particular it would fail to be faithful to Anselm's reason-

[45] Matthews (2004, p. 72).

ing.[46] Of course, precise justifications of (Nec), (T) and (K) will depend on the exact conception of *conceivability* being employed. If, for instance, we follow Morscher and think of 'it is conceivable that ϕ' as expressing the complex modality 'it is possible that someone thinks that ϕ is true', we might justify (T) by pointing to the fact that, whenever ϕ is true, we may plausibly assume that there must be an alethically possible world in which a certain person thinks that ϕ is true.[47] Therefore, on this conception, (T) is indeed satisfied. In fact, we think that any plausible modal conception of conceivability must imply the validity of the rules and laws (Nec), (K) and (T). If further modal axioms are required, we shall explicitly indicate which ones.

4.1 Propositional modal logic

There is a certain tradition in the literature about ontological arguments, claiming that Anselm's argument is in some way bound to modal logic. More specifically, according to this tradition, the argument in Chap. II fails, whereas the argument in Chap. III succeeds or can be improved so that it does, and that, furthermore, this argument is clearly 'modal'.[48] The idea of reconstructing Anselm's arguments within modal logic therefore originates from discussions of Chap. III and many important issues about Anselm and modality are treated in this context. So we will start this section with an influential modal-logical reconstruction of Anselm's argument in Chap. III, due to Charles Hartshorne.[49]

Hartshorne's argument proceeds within the framework just presented along with the characteristic modal axiom scheme for the system $S5$:

(5) $\Diamond\phi \rightarrow \Box\Diamond\phi$

The idea is quite simple: Let p abbreviate our canonical existence-of-God statement **God!**. Hartshorne then suggests that we should adopt two premisses, which he attributes to Anselm. The first one, which he calls *Anselm's principle*, states that if p then $\Box p$, that is: if God exists, then he exists necessarily (or, more precisely, his non-existence is inconceivable). The second is the assumption that it is at least *conceivable* that God exists. Hartshorne then gives the following argument for **God!**—i.e. p.

Proof. By standard propositional logic, we have (i) $\Box p \vee \neg\Box p$. Applying the characteristic $S5$ axiom scheme (5) to the formula $\neg p$, we get $\neg\Box p \rightarrow \Box\neg\Box p$. Hence, (i) implies (ii): $\Box p \vee \Box\neg\Box p$. By contraposing Anselm's principle

[46] Anselm, for instance, argues that **id quo** is in the understanding, and therefore it is conceivable that **id quo** exists in reality. If, by conceivability, Anselm here would mean *conceivable by the fool*, there would be no reason to accept this inference. After all, the restricted mental capacities of the fool might detain him from being able to conceive of the existence (in reality) of **id quo**, even though **id quo** is in his understanding.

[47] Cf. footnote 30.

[48] See, e.g., Hartshorne (1941), Kane (1984), Malcolm (1960).

[49] Hartshome (1962, pp. 49–51).

$p \to \Box p$ and instantiating (K) with $\phi = \neg\Box p$ and $\psi = \neg p$, we arrive at (iii) $\Box\neg\Box p \to \Box\neg p$. Now, (ii) and (iii) together imply $\Box p \vee \Box\neg p$. But, by assumption, God's existence is at least *conceivable*; so, we have to reject the right disjunct and conclude that $\Box p$. From $\Box p$, we can infer p by axiom scheme (T). □

It seems to us that Hartshorne's modal proof is a good improvement of Anselm's argument in Chap. III, though one might have doubts whether it is to be counted as a good reconstruction in the sense of our requirements. In any case, it can clearly not be counted as a good reconstruction of Chap. II, which is our main concern here. Still, having a closer look at Chap. III is also relevant for Anselm's argument in Chap. II. To reiterate, Anselm argues in Chap. III for the claim that it is inconceivable that **id quo** does not exist.[50] Naturally, conceivability is here construed as a modal operator of some sort, acting on sentences. Now, one strong reason for a modal reconstruction of Chap. II is *uniformity*. If Anselm's *cogitari potest* should be construed modally in Chap. III, then our requirement (2) tells us to do so in Chap. II as well. However, there seems to be no way to deal with Anselm's argument in Chap. II within *propositional* modal logic. We shall therefore look at the prospects of reconstructing Chap. II within *first-order* modal logic, taking seriously the quantificational structure of Anselm's argument.

4.2 First-order modal logic

In order to be able to reconstruct Anselm's argument in Chap. II within first-order modal logic, some preliminary remarks on the intended semantics of the background logic and, in particular, how quantifiers are to be understood here, may be in order.

It seems reasonable to us to adopt as background logic something like the *simplest quantified modal logic* (**SQML**), a natural possibilist system of quantified modal logic.[51] Semantically, the distinct features of **SQML** are that quantifiers are construed so as to range over all *possibilia* and that *actuality* or *existence* is expressed by a primitive predicate $E!$ just as in the approach in classical logic from Sect. 3. As we have already discussed, this seems to be largely in accord with Anselm, who clearly does not want quantificational phrases to have any existential import. As a consequence of this approach, both, the *Barcan formulas* as well as the *converse Barcan formulas*

(B) $\forall x \Box \phi(x) \to \Box \forall x \phi(x)$
(CB) $\Box \forall x \phi(x) \to \forall x \Box \phi(x)$ and

are valid schemes of **SQML**. Together, (B) and (CB) imply that the universal quantifier $\forall$ and the existential quantifier $\exists$ commute with the necessity operator $\Box$ and the

[50] See Anselm of Canterbury (2008, p. 88).

[51] See, for instance, Linsky and Zalta (1994). An overview over various systems of modal logic can be found in Fitting and Mendelsohn (1998).

possibility operator $\Diamond$, respectively. We shall use these equivalences freely without any further comment. If quantifiers are construed as being possibilistic (as it seems they must be if Anselm's reasoning is to be meaningful), these equivalences are inevitable. Hence, we assume that Anselm is committed to them. Following the policy outlined in Sect. 2.2, Anselm's 'is in the understanding' will be expressed by quantifiers.

The first definition of the predicate G within modal logic that we shall consider is straightforward:

Def M-God 1: $Gx :\leftrightarrow \neg\exists y \Diamond (y > x)$

Def M-God 1 says that an object x is God if there is nothing of which it is conceivable that it is greater than x. Note that, due to the Barcan formula and its converse, $\neg\exists y \Diamond (y > x)$ is equivalent to $\neg\Diamond\exists y (y > x)$ and $\Box\forall y \neg(y > x)$. Therefore, whatever connotations any of those formulas has which another has not, from the point of view adopted here, all of them amount to the same thing.[52] The goal of each of the following reconstructions is to prove, as before, **God!**—i.e. $\exists x (Gx \wedge E!x)$.

The first variant that we shall discuss makes three assumptions. The first is again **ExUnd** from earlier—i.e.

ExUnd: $\exists x Gx$

and the second can be expressed by

PossEx: $\forall x \Diamond E!x$

The first premiss states that something which is a God is at least in the understanding and the second that everything which is in the understanding can be conceived to exist.[53] Though one could think that such a principle might be too strong, it accords quite well with the first part of (II.10). Furthermore, on a possibilist understanding of quantification, **PossEx** seems to be an analytic truth in a robust sense. Whatever 'possibilia'—or 'conceivabilia'—, i.e. the objects in the range of possibilist quantifiers, are supposed to be, they clearly ought to exist in *some* possible world.[54]

[52] We can see that **Def M-God 1** is just a modalized version of **Def C-God** of Sect. 3. Essentially, the same definitions have been adopted in Morscher (1991), Nowicki (2006) and (partly) in Lewis (1970).

[53] A similar premiss is mentioned in Maydole (2009) as well as in Lewis (1970). Logan, on the other hand, thinks that such a principle is problematic, 'since a chimera can be understood, although it cannot exist, since it is by definition a mythical beast. Nor does Anselm mean that understanding the term 'a square circle' involves the possibility of its actual existence' (Logan 2009, p. 94). Here it is important to keep in mind the distinction between 'being in the understanding' insofar as the 'word signifying' is understood and 'being in the understanding' in the sense that the signified object itself is understood. (See our discussion in Sect. 2.5.) Logan's counterexamples against the assumption that every understandable being conceivably exists make it reasonably clear that he takes the locution 'can be understood' in the *first* sense. However, as our earlier discussion should have made clear, this is not what **PossEx** is supposed to express (and it is, of course, not what Anselm is saying). So there is no need to disagree with Logan since the round square, though understandable in the *first* sense, cannot be understood in the *second* sense.

[54] The informal discussion will draw on intuitions concerning possible world semantics; therefore, it should be read with a grain of salt. In particular, it does not commit us to attribute a view to Anselm, according to which—in *Proslogion*—he was arguing in terms of possible worlds.

The third principle is a modified version of **Greater 2**, respecting the fact that Anselm wants to compare a thing with itself:[55]

Greater 4: $\forall x \forall y(\neg E!x \wedge \Diamond E!y \rightarrow \exists z \Diamond(z > x))$

In particular, by instantiating x and y with the same value, we can infer that if something does not actually exist, but it is at least conceivable that it exists, then there is something which is conceivably greater than this object. (And one is tempted to add: namely, that very object! By conceiving this very object as existing, we conceive of it as being greater than it actually is.) Given the assumptions **Greater 4**, **ExUnd**, and **PossEx**, the argument turns out to be correct.[56]

Although this reconstruction seems to be a step forward, we still think that it does not fully capture the intuition mentioned earlier—viz., that Anselm apparently wants to compare the greatness of a hypothetically non-existent being with the greatness of this very being (conceived of as actually existing) and that this being itself is the witness for the existential claim in the consequent of the conditional **Greater 4**. That is, God himself would be greater than himself, were we to assume that he did not exist. Although this intuition seems to be quite common (Lewis (1970) has emphasized this point as well as Nowicki (2006)) it is less clear how we should deal with it in terms of standard modal reasoning.[57]

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[55] The comparison principle from Sect. 3 **Greater 2** $\forall x \forall y(E!x \wedge \neg E!y \rightarrow x > y)$ does not suffice to imply **God!**: Although **Greater 2** is sufficient to produce a contradiction from the assumption of the non-existence of a God in the context of classical logic, it is not sufficient in modal logic in conjunction with **Def M-God 1**, **ExUnd** and **PossEx**. The reason for this is that **ExUnd** and **PossEx** assure us only of the *possible* existence of a God. Hence, it is consistent with **Greater 2** that there is a God g, which conceivably exists without existing *actually*. **Greater 2** simply does not say anything about *possibly existing* objects. Moreover, there seems to be no reasonable modification of **Greater 2** that *does*. Now, this version is not convincing for another reason that we have already discussed in earlier sections. Recall that, in (II.10), Anselm does not want to compare something with something *else* as to its greatness.

[56] By **ExUnd** , we are given some g in such a way that (i) $\neg\exists z \Diamond(z > g)$. By **PossEx** , then, $\Diamond E!g$. Suppose for *reductio* that $\neg E!g$. Then, by **Greater 4**, it follows from $\Diamond E!g$ and $\neg E!g$ that $\exists z \Diamond(z > x))$, thereby contradicting (i). Thus, we may conclude that $E!g$; hence, $\exists x(Gx \wedge E!x)$. A version which is similar to this reconstruction and which uses a premiss that is similar to **Greater 4**, can be found in Maydole (2009, p. 556).

[57] The problem with Lewis (1970) is that his proposed reconstructions are framed in a non-modal language, where modal claims are reformulated by means of explicit quantification over possible worlds. However, it seems to us that Anselm's argument should be rephrased in terms of a modal operator $\Diamond$, corresponding to Anselm's 'it is conceivable that', for it is only such an operator that shows up in Anselm's actual argument. Lewis' reconstructions, therefore, violate requirements (1) and (3).

In reaction to Viger (2002), who claims that Anselm's argument would fall prey to Russell's paradox, Nowicki (2006) presents another modal reconstruction of Anselm's argument. Yet, Nowicki's formulation does not make transparent what Anselm seems to be arguing for—namely, that God *himself* could be conceived to be greater, if we would assume that he does not exist. The same is true of the reconstruction provided in Morscher (1991), p. 68. In addition, Morscher's reconstruction heavily relies on his conviction that Anselm's **id quo** has to be rendered as a definite description, a view which we have already discarded earlier. A version which is similar to Morscher's can be found in Jacquette (1997). Jacquette (1997) also contains a discussion of general aspects concerning intensionality in Anselm's argument.

In the following, we shall review one particular way of doing so without leaving the familiar ground of standard quantified modal logic. To get an idea of what we have in mind here, remember Russell's well-known joke about the yacht:

> I have heard a touchy owner of a yacht to whom a guest, on first seeing it, remarked: 'I thought your yacht was larger than it is'; and the owner replied, 'No, my yacht is not larger than it is'.[58]

Analogous to Russell's analysis of the point of this joke with respect to the beliefs expressed by the guest and the owner of the yacht, we can distinguish two readings of the modal sentence

(1) The yacht could have been larger than it is.

by means of a scope distinction. On the first reading, corresponding to how the owner misinterprets the guest, it states the mathematical falsehood that a certain real number could have been greater than it is. On the second reading, corresponding to the guest's intention, it states that the size of the yacht, a certain real number, is exceeded in some possible world by another real number—viz., the size of the yacht in *that* possible world. If we let $\mathfrak{s}(x)$ stand for the function that assigns a real number r to each object x, we can then formulate the first reading by

(1′) $\Diamond(\mathfrak{s}(y) > \mathfrak{s}(y))$

and the second by

(1″) $\exists x(x = \mathfrak{s}(y) \wedge \Diamond(\mathfrak{s}(y) > x))$[59]

where the relation $>$ is supposed to designate the usual *greater than* relation between real numbers.

The same idea works in reconstructing the punch line of (II.10) by substantializing *greatnesses*. So, in complete analogy with Russell's yacht example, we will introduce a function $\mathfrak{g}(x)$, assigning to each object another object—viz., its *greatness*. We will leave it open as to what the greatness of an object *really is*, just as we had to leave it open what *being greater really means* in Sect. 3. We merely assume that *greatnesses* can be ordered in some reasonable way by a relation $\succ$. Therefore, instead of $x > y$, as in earlier reconstructions, as the formal counterpart of 'x is greater than y', we will now translate this basic idiom by $\mathfrak{g}(x) \succ \mathfrak{g}(y)$, to be read: *the greatness of x exceeds the greatness of y*. In particular, note that

(2) $\exists x(x = \mathfrak{g}(g) \wedge \Diamond(\mathfrak{g}(g) \succ x))$

58 Russell (1905, p. 489).

59 Equivalently, $\forall x(x = \mathfrak{s}(y) \rightarrow \Diamond(\mathfrak{s}(y) > x))$.

expresses that g *could have been greater than it actually is*. In the following, we will explore some of the possibilities provided by this new analysis. The first version that we shall discuss uses the following modified definition of God:

Def M-God 2: $Gx :\leftrightarrow \neg\Diamond\exists y(\mathfrak{g}(y) \succ \mathfrak{g}(x))$

Semantically, **Def M-God 2** says that x is God if there is no possible world and no being y such that the greatness of y in that world exceeds the greatness of x in that world.[60] Besides **ExUnd** and **PossEx** , we also adopt a new 'comparison axiom':

Greater 5: $\forall x\forall y(\neg E!x \wedge \Diamond E!y \rightarrow \exists z(z = \mathfrak{g}(x) \wedge \Diamond(\mathfrak{g}(y) \succ z)))$

Greater 5 states that whenever x does not exist and y exists at least conceivably, then it is conceivable that y's greatness exceeds x's *actual* greatness. In addition, **Greater 5** also allows for comparison between an object and *itself* by taking the same value for the variables x and y. **Greater 5** thus has as a particular consequence that if an object exists *conceivably*, but not *actually*, then this object can be conceived to be greater than it actually is. Indeed, when applied to **aliquid quo**, this seems to be exactly what Anselm is saying. However, as it turns out, we cannot derive **God!** from the premisses **ExUnd**, **PossEx** and **Greater 5** together with **Def M-God 2**.[61]

We can, however, modify **Def M-God 2** as follows so as to make the proof work:[62]

Def M-God 3: $Gx :\leftrightarrow \exists z(z = \mathfrak{g}(x) \wedge \neg\Diamond\exists y(\mathfrak{g}(y) \succ z))$

Def M-God 3 stipulates that a being x is a God if there is no possible world and no being y in this world such that y's greatness exceeds x's *actual* greatness. Leaving everything else as it stands, the proof is then straightforward:

Proof. By **ExUnd** we are given some g such that:

$$(*) \qquad \exists z(z = \mathfrak{g}(g) \wedge \neg\Diamond\exists y(\mathfrak{g}(y) \succ z))$$

Now, assume for *reductio* that $\neg E!g$. From **PossEx**, we know that $\Diamond E!g$ and, so, by **Greater 5**, we have $\exists y(y = \mathfrak{g}(g) \wedge \Diamond(\mathfrak{g}(g) \succ y))$. Hence, for some a:

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60 This definition essentially corresponds to premiss 3D of Lewis (1970, p. 180).

61 Consider the following constant-domain counter model $\mathfrak{M}$: Let $\mathfrak{M}$ consist of three possible worlds a (the actual world), v and w. The domain of the model consists of three objects, 1, 2 and 3. We stipulate that 1 does not exist in the actual world a, whereas 2 and 3 do, and furthermore, that 1, 2 and 3 exist in each of the other worlds. We further stipulate that $\mathfrak{g}_a(1) = 1$; $\mathfrak{g}_a(2) = 2$; $\mathfrak{g}_a(3) = 3$; $\mathfrak{g}_w(1) = 2$; $\mathfrak{g}_w(2) = \mathfrak{g}_w(3) = 1$; $\mathfrak{g}_v(1) = 2$; $\mathfrak{g}_v(2) = \mathfrak{g}_v(3) = 3$ and that $\succ_a = \{\}$; $\succ_w = \{(2, 1)\}$ and $\succ_v = \{(3, 1)\}$. Clearly, 1 is **aliquid quo** in the sense of **Def M-God 2** (whereas 2 and 3 are not), because for each world u, $\mathfrak{g}_u(1)$ is not exceeded (in u) by the greatness (in u) of any other object. Therefore, **ExUnd** is satisfied. Furthermore, since each object exists in some world, **PossEx** is satisfied as well. With respect to **Greater 5**, note that the actual greatness of 1 (the only non-existing object in a), viz., 1 *itself*, is exceeded (in w) by the greatness of 1 in w (=2) and the actual greatness of 1 is in v exceeded by the greatness of 2 and 3 in v—namely, 3. Hence, **Greater 5** is satisfied as well, for each conceivably existing object can be conceived to be greater than the only non-existing object 1. Thus, each of the premisses is satisfied; yet, by the definition of the model, the only *being than which nothing greater can be conceived* (i.e. 1), does not exist in the actual world a.

62 The new definition corresponds to premiss 3A of Lewis (1970) and to 'Assumption 8' of Oppy (2006, p. 76).

$$(**) \quad a = \mathfrak{g}(g) \wedge \Diamond(\mathfrak{g}(g) \succ a)$$

From $(*)$, for some b:

$$(***) b = \mathfrak{g}(g) \wedge \neg\Diamond\exists y(\mathfrak{g}(y) \succ b)$$

From $(**)$ and $(***)$, it follows, in particular, that $a = b$. By $(**)$, we know that in some possible world w, we have $\mathfrak{g}(g) \succ a$. But it follows from the second conjunct of $(***)$ that, in w, we also have $\neg\exists y(\mathfrak{g}(y) \succ b)$. Therefore, in particular, $\neg(\mathfrak{g}(g) \succ b)$. Now, since $a = b$, we can infer that $\neg(\mathfrak{g}(g) \succ a)$. Contradiction. □

Note that the auxiliary names a and b are supposed to designate objects (viz., *greatnesses*) *rigidly* throughout the proof. Since we have $a = b \rightarrow \Box(a = b)$ for rigid designators, the substitution in the last step is permitted.

We think that this reconstruction comes quite close to Anselm's reasoning in *Proslogion*. In particular, the reconstruction arguably captures the point of passage (II.10). Crucially, **Greater 5** implies that God's greatness *would* be exceeded by his *own* greatness, were we to assume that he did not exist. Therefore, **Greater 5** accounts for a correct rendering of Anselm's reasoning, according to which God would be greater in a conceivable world where he exists than he would be in the actual world, if he did not exist. Though it seems to us that in terms of our constraints from Sect. 1.2, the last two reconstructions accord with most of them, we see no way to decide, based on Anselm's reasoning in Chap. II alone, if he would want to adopt definition **Def M-God 2** or rather **Def M-God 3**. Both seem to have some antecedent plausibility as explicanda of Anselm's **aliquid quo**.[63] As we have found no passage in Anselm's writings that would enable us to decide between **Def M-God 2** and **Def M-God 3**, we have to leave the question open as to whether or not Anselm's argument is indeed valid. However, we think that, by the basic rhetorical maxim of making an argument as strong as possible, we should attribute the valid version to Anselm. Thus, we have a valid argument; yet, we seem to still be left with the problem of how to motivate the crucial axiom **Greater 5**. Without further explication, we seem to have no reason to accept **Greater 5** as analytic, and neither does Anselm.

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[63] In his Lewis (1970), Lewis argues that **Def M-God 2** was no plausible explication of **aliquid quo**. According to Lewis, **Def M-God 3** (or, rather, the premiss corresponding to our **Def M-God 3**), should be seen as the correct translation of **aliquid quo**. Recall that **Def M-God 3** says that a being x is a God if x's actual greatness is not exceeded by the greatness of any being in any possible world. The problem Lewis has with this is that it would give undue preference to the *actual* world over other possible worlds. Consequently, he thinks that, although the argument based on **Def M-God 3** is *valid*, it does not establish the existence of a being reasonably to be called 'God', on the ground that there is no reason to prefer a certain possible world to some other. However, Lewis cites no reasons in Anselm's writings in support of this view.

4.3 Higher-order modal logic

In order to accommodate the second basic intuition stated in Sect. 2.4—viz., that not just *existence*, but various properties might be responsible for something being greater than something else—we propose a generalized version of **Greater 5**, which achieves exactly this. The crucial idea is analogous to the idea behind **Greater 3**, which was that a being x is greater than y if it has more 'positive' properties—i.e. properties in a certain class $\mathcal{P}$. Similarly, we now let the functions $\mathfrak{g}$ and $\succ$ be constrained by the following premiss:

$$\textbf{Greater 6:} \qquad \forall x \forall y (\exists_{\mathcal{P}} F(\neg Fx \wedge \Diamond Fy) \leftrightarrow \exists z(z = \mathfrak{g}(x) \wedge \Diamond(\mathfrak{g}(y) \succ z)))$$

Greater 6 states that the actual greatness of a being x is conceivably exceeded by the greatness of y if and only if there is a positive property which x lacks, but that y conceivably possesses. Thus, **Greater 6** generalizes **Greater 5** with respect to the properties that might be responsible for something being conceivably greater than something else. But **Greater 6** also gives us a *necessary* condition for being conceivably greater. Moreover, it seems to us that **Greater 6** is indeed an analytic truth, given any reasonable explication of 'positive property'. On the one hand, if x lacks a positive property, say P, which y has at least *conceivably*, then it seems that we must be able to conceive of y's greatness, at least in *some* conceivable world, as exceeding the actual greatness of x. Just conceive of y as having P along with all the positive properties that x has. If, on the other hand, we *can* conceive of x's actual greatness as being exceeded by the greatness of y, then it seems that there must be *some* positive property, which accounts for this possibility.

Clearly, with the additional premiss that $E!$ is among the positive properties $\mathcal{P}$, **Greater 5** will immediately follow from **Greater 6**. Thus, just like before, **God!** may or may not be derivable, depending on which definition of G we adopt. As in the case of the non-modal reconstruction from Sect. 3, we can also account for the constant proof structure throughout *Proslogion*, since any property in $\mathcal{P}$ can be used instead of $E!$. That is, in conjunction with the rest of the premisses, **Greater 6** enables us to establish that **id quo** has all the properties which are better for an object to have than not to have. If, in particular, we take $\Box E!$ (again assuming its positiveness), we get a reconstruction of Anselm's argument in Chap. III that establishes (or does not establish, depending on the definition adopted) that **id quo** not only exists, but does so *necessarily*.

5 Conclusion

As regards the first aim of the paper, our reconstructions and the discussions of other reconstructions provided examples of how to apply the requirements given in Sect. 1.2. We ended up with two main reconstructions, one modal and one modal-free (the modal one having two variants, differing by the definition of God). Both approaches can cope with two important specifications that are implicit in Anselm's argument (albeit, in radically different ways):

(a) Being greater means having more properties of a certain kind
(b) A hypothetically non-existing God is compared with *itself* conceived of as existing

The modal-free reconstruction needs higher-order logic to adapt to both demands, whereas within a modal framework, first-order logic suffices to fulfil the second demand, but higher-order logic is needed to fulfil the first.

Now, to the second aim: What can be learned from the reconstructions of Anselm's argument?

First, the two different approaches accommodate the fact that one can understand Anselm's 'can be conceived' quite differently. In the non-modal approach, we can think of Anselm's 'cogitare potest' as introducing the reader to discourse about non-factual entities, whereas in the modal approach, we think of the 'cogitare potest' as an operator. Both approaches can be justified, but also rejected, on the basis of *some* textual evidence. Chap. III favours the operator-reading, whereas Chap. IV favours the reading, according to which 'cogitare potest' applies to objects. Therefore, neither of the two readings conforms with everything Anselm says in *Proslogion*. This, however, does not seem to be a problem with our reconstructions, but points to an important ambiguity in the Latin nomenclature itself. So if one accepts our reconstructions, no *unique* logical structure can be identified in Anselm's argument.

Second, we think that our reconstructions provide a hint to a certain dilemma resulting from Anselm's argument itself: If one takes into consideration only Chap. II and accordingly replaces

Greater 3: $\forall_{\mathcal{P}} F(Fy \rightarrow Fx) \wedge \exists_{\mathcal{P}} F(Fx \wedge \neg Fy) \leftrightarrow x > y$

with

Greater 3E!: $\forall_{\mathcal{P}} F(Fy \rightarrow Fx) \wedge (E!x \wedge \neg E!y) \rightarrow x > y$

(instantiation for the existence predicate) and does not use

Greater 6: $\forall x \forall y(\exists_{\mathcal{P}} F(\neg Fx \wedge \Diamond Fy) \leftrightarrow \exists z(z = g(x) \wedge \Diamond(g(y) \succ z)))$,

but

Greater 5: $\forall x \forall y(\neg E!x \wedge \Diamond Ey \rightarrow \exists z(z = g(x) \wedge \Diamond(g(y) \succ z)))$,

respectively, one gets reconstructions that are locally conform with the text in Chap. II, and, in particular, in accordance with requirement (3). However, one does not see why the axioms for *greater* should be considered analytic (violating requirement (7)). Only including information from later chapters gives the reader clues about reasons for accepting these axioms (by gaining at least some sort of understanding of *greater*). Requirement (4) says that one should prefer the non-convincing version as a reconstruction for Chap. II, therefore producing a clash with requirement (7). This is not a fault of our reconstructions. On the contrary, it is a benefit, as it shows a critical point of Anselm's argument—to be more precise, of the composition of *Proslogion*. In a sense, Chap. II is not comprehensible unless one has read the later chapters. Our reconstructions offer the best that one can expect in such a situation: The reconstructions in Sects. 3.3 and 4.3 globally conform with *Proslogion* in the sense that they use the information from all over the book and develop a reconstruction that is satisfying overall; in particular, the premisses can be considered analytic. A reconstruction that locally conforms *exactly with* Chap. II can be gained by *restriction* (instantiation) of the axioms as explained above.

Third, something can be learnt about what Anselm's argument actually achieves. What is the task of the argument? It has to hide (and then reveal) the connection between G and $E!$—one must not see in the beginning that having property G implies existence in reality. This is done by introducing the mediating term 'greater'. Anselm hides the relevant connection by not spelling out what 'greater' means. But to recognise it as a valid formal argument with analytic premisses one has to make the connection explicit.

Fourth, from our reconstructions and what has already been said about them, it becomes clear why Anselm's argument looks so simple at first glance but becomes quite complicated when formalised: First, Anselm does not reveal what he means by greater (at least not in the argument itself in Chap. II of *Proslogion*); second, there is no simple way to express, in usual logical notation, how an object can be compared *with itself* in a different mode; and third, Anselm is vague about how *things in the understanding* are related to *conceiving*. If one has to spell out these conceptions and relations in detail, the argument structure obviously becomes much richer.

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