Tunnel Dynamics Through Earth and Moon

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I. Introduction

This report deals with the physics of a theoretical tunnel through the Earth along with the Moon. Through numerical simulations with Python-coded models, we study the effects that variations in gravitational influence, air drag, and Coriolis effect would have on falling time for a freely falling test mass in a vertical shaft. The models are made for both homogeneous and non-homogeneous density profiles for the Earth but modeled the Moon as being a constant-density body. The project built upon the fundamentals of physics, including Newton's Second Law, that force and acceleration are related, inverse-square relationships for gravitational force, and that the gravity inside a planet of uniform density goes inversely as the radial distance from the center.

For these calculations, we used various Python libraries, such as numpy, which provides all necessary array operations and maths operations, scipy.integrate.solve_ivp to solve differential equations, and scipy.integrate.quad to compute integrals for force and mass. All final models were created with the expected planetary constants and physical relations. Motion was solved and graphed for free-falling bodies within planetary interiors in a stepwise manner. Plots of position, velocity, density, and force as functions of radius or time were created using the matplotlib library.

II. Computing the Fall Time (With Drag and Variable Gravity)

This case has been handled in free fall through the entire length of the mine shaft (4 km), and the approximation here is that of constant gravitation. In such a case the analytical fall time was equal to 28.6 seconds.

Then, we numerically solved the second-order differential equation via the solve_ivp function. After consideration of the gravity linearly decreasing with radius, the numerical falling time was 29.3 seconds. We included a quadratic air drag term with a terminal velocity of 50 m/s and falling time became 32.8 seconds.

The results are consistent with physical expectations. The fall is decelerated due to drag, while gravity changes with altitude introduce a lesser correction. For visualization, the two were plotted against time with matplotlib, using the twinx() function to have the two on different vertical axes.

III. Feasibility of Depth Measurement (Including Coriolis Forces)

For our model of the Coriolis effect, we let a mass drop into a shaft at the equator from rest. The Coriolis force would deflect the object without drag by about 238.5 meters. Considering drag, it reduced the lateral displacement to 3.4 meters.

In both cases, however, the shaft is only 5 meters wide, so the object would strike the wall. Again, the striking is delayed, though, because of drag. With the 2.5-meter transverse displacement, it hit the wall at around 9.4 seconds; around 22.7 seconds with drag.

These result show how Coriolis forces contribute highly against measurement systems based on vertical movement. We showed the lateral movement by plotting the object's lateral movement against depth in the shaft.

IV. Crossing Times for both Homogeneous and Non-Homogeneous Earth

How variations in internal density affect fall time was then worked out. For constant-density Earth (n = 0), time to fall to the center was 1267.3 seconds, and maximum speed at the center was 7905.3 m/s. For a distribution of mass more concentrated at the center (n = 9), fall time shortened to 943.8 seconds; maximum speed increased up to 18370.6 m/s.

As a continuation of the computation, we would compare the fall time through Earth with that through a tunnel in the Moon. The Moon was modeled as a sphere of homogenous density. Its time to fall toward the center amounted to 1625.1 seconds, while the complete oscillation period was 6500.5 seconds. In the case of Earth, the total period oscillating was 5069.4 seconds.

The greater fall time indicated that the Moon had much lower average density, which is about 60.8 percent of that of Earth. Fall time and orbital period are directly proportional to the square root of inverse density. The numerical result indicated that this scaling was indeed fulfilled and our model was self-consistent.

V. Discussion and Future Work

These simulations show how sensitive time of fall is to a planet's internal structure and outside forces, such as drag and Coriolis acceleration. Future work can consist of non-equatorial tunnels, where the direction of the Coriolis force is changing, more advanced drag models, or even models like the PREM (Preliminary Reference Earth Model) with layered structure. It may be worthwhile investigating how planetary rotation and tidal forces impact transplanetary tunnels.

This project demonstrates the power of numerical modeling to study classical mechanics through the exercise of planets. By integrating known physics with computation, it becomes possible to model realistic behavior and understand hypothetical scientific and engineering scenarios.