NORTHWESTERN UNIVERSITY | SCHOOL OF PROFESSIONAL STUDIES

Final Exam

Points possible: 100

Description: The final exam will cover topics from sessions 1-9.

Resources: The exam is completely open book. You may use course textbooks, materials provided on Canvas, or basic graphing calculators (such as TI 83 or 84). Any more advanced calculators, Excel Solver, Web calculators, Web-graphic calculators, or simplex method calculators are not allowed. Programming languages other than Python are also not permitted.

For questions that require calculations, all calculations should be shown, not just the final answer. This will allow for partial credit for those answers that might be set up correctly but have calculation errors. For questions that specifically require Python, the code and output should be included with your answer. For questions that require graphs, only use Python.

Restrictions: All answers are to be your work only. You are not to receive assistance from any other person.

To complete the exam:

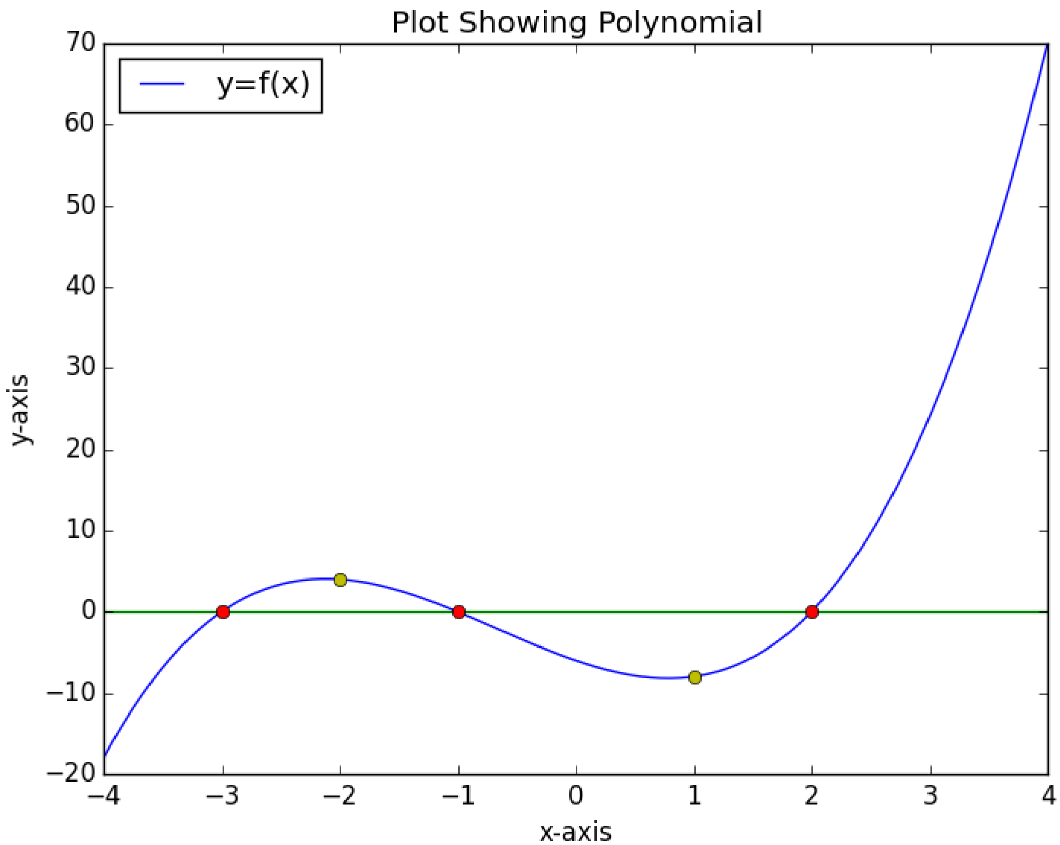
1. Answer all questions on the exam thoroughly. Create a Microsoft Word document, including the question number, the question, your typed answer, and graphs if required. You may use Word's equation editor to complete your answers.

2. Once you have completed your exam, return to the exam item where you downloaded the exam PDF, click View/Complete Assignment, and submit your document.

1. A patient takes vitamin pills daily. Each day he must have at least 240 IU of vitamin A, 5 mg of vitamin B_1, and 120 mg of vitamin C. He can choose between pill 1, which contains 120 IU of vitamin A, 1 mg of vitamin B_1, and 15 mg of vitamin C; and pill 2, which contains 30 IU of vitamin A, 1 mg of vitamin B_1, and 40 mg of vitamin C. Pill 1 costs $0.10, and pill 2 costs $0.20. **Using Python**, determine how many of each pill he should take daily in order to minimize cost and determine the minimum cost. Also, determine the amount of surplus of each type of vitamin, if any.

2. Two stores sell a certain product. Store A has 38% of the sales, 4% of which are defective. Store B has 62% of the sales, 1% of which are defective. The difference in defective items is due to different levels of pre-sales checking of the product. A person receives a defective item of this product as a gift. What is the probability it came from store B?

3. The following is a graph of a third degree polynomial with leading coefficient 1. Determine the function depicted in the graph. **Using Python**, recreate the graph of the original function, $f(x)$, as well as the graph of its derivative.



4. The average number of vehicles waiting in line to enter a parking ramp can be modeled by the function

$$f(x) = \frac{x^2}{2(1-x)}$$

where x is a quantity between 0 and 1 known as the traffic intensity. Find the rate of change of the number of vehicles in line with respect to the traffic intensity for x = 0.3.

5. Using data in a car magazine, we constructed the mathematical model

$$y = 100e^{-0.0482t}$$

for the percentage of cars of a certain type still on the road after t years. **Using Python**, determine the rate of change of the percent of cars still on the road after 5 years.

6. For the following function, determine the domain, critical points, intervals where the function is increasing or decreasing, inflection points, intervals of concavity, intercepts, and asymptotes where applicable. Use this information to graph the function.

$$f(x) = -5x - \frac{15}{x}$$

7. The population of Asian Carp in Lake Michigan is described by the logistic equation $G(t) = \dfrac{12,000}{1+19e^{-1.2t}}$ where G (t) is the population after t years. Find the point in time at which the growth rate of this population begins to decline.

8. For a certain drug, the rate of reaction in appropriate units is given by

$$R'(t) = \frac{3}{t+1} + \frac{5}{\sqrt{t+1}}$$

where t is time (in hours) after the drug is administered. Find the total reaction to the drug from 8 to 24 hours after it is administered.

9. Show that the following function is a probability density function on $(0, \infty]$.

$$f(x) = \begin{cases} x^3/12, if\ 0 \leq x \leq 2 \\ 16/x^4, if\ x > 2 \end{cases}$$

Determine $P(1 \leq x \leq 5)$.

10. The time between goals (in minutes) for a professional soccer team during a recent season can be approximated by

$$f(x) = \frac{1}{85} e^{-\frac{x}{85}}$$

Using Python determine the (separate) probabilities that the time for a goal is no more than (a) 68 minutes, and (b) 457 minutes or more.