# PX3017 Project: Vicsek Model of Collective Motion

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23. November 2019

## 1 Introduction

In this project, a version of Vicsek model of self-driven particles was implemented. There is not a significant difference between algorithm used in this project and the method published by Tamás Vicsek et al. in 1995 as "Novel Type of Phase Transition in a System of Self-Driven Particles".[1]

This mathematical model aims to explain systems that 'exhibit complex cooperative behaviour'. Important property of those systems is that they have a state transition, during which we can observe unaligned/chaotic movement changing into a collective motion . My report covers methods which were used to simulate such cooperative system of self-driven particles, provides visualisations of this model and discusses general trends and also correlations with the original research paper.[2]

There are many motivations for similar kinds of models - most importantly in biological simulations. Phenomena from swarming behaviour of birds, coordinated motion of tiny organisms such as bacteria to movements of cars and people in the street - can be in some cases very well modelled with this algorithm. Despite such a wide range of applications, surprisingly, the model itself is very simple and follows only one simple rule, further described in the section 'Method'.

## 2 Method

The only rule which is guiding the system is: *" at each time step a given particle driven with a constant absolute velocity assumes the average direction of motion of the particles in its neighbourhood of radius $r$ with some random perturbation added."*[3]

[1] Tamás Vicsek et al. "Novel Type of Phase Transition in a System of Self-Driven Particles". In: *Physical Review Letters. 75(6) 1226–1229* (1995), pp. 1226–1229.

[2] Ibid.

[3] Ibid., p. 1226.

Let's learn about birds now!

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### 2.1 The algorithm in detail

There are $N$ particles in the system. Every particle has a precise position $\vec{x_i} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and direction $\vec{d_i} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$. The velocity of the particle $v \neq 0$ is set to be constant and same for all particles.

- In the initial step we uniformly distribute all the particles around container with size $L_x \times L_y$. We assign a unit vector in random direction to every particle. The container has periodic boundary conditions.

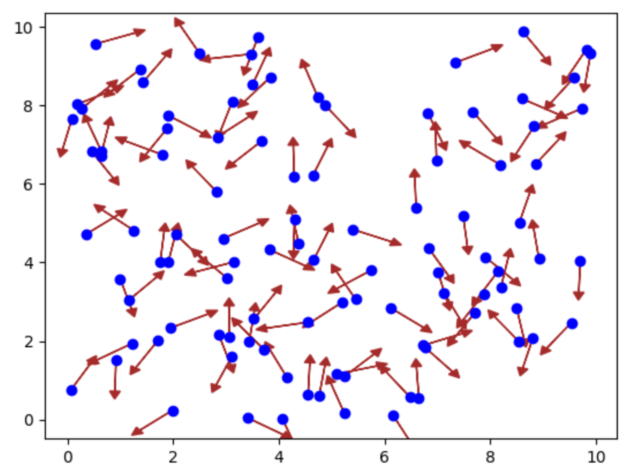



Figure 1: Initial position of the system: 100 particles with random orientation

- Each time step the particle adjust its direction and updates its position (given its velocity and direction)

$$\vec{x}_{new} = \vec{x}_{old} + \vec{v}\Delta t$$

$$\vec{d}_{new} = \begin{pmatrix} cos(\theta) \\ sin(\theta) \end{pmatrix}$$

$$\text{where } \theta = \text{angle}[\sum_{r_i < R} \vec{d_i}] + \theta_{random}$$

$$\text{and angle of any two component vector } \vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \text{ is } \alpha(\vec{x}) = arctan(\frac{x_2}{x_1})$$

$\theta_{random}$ is a random sample from the normal distribution with mean $\mu = 0$ and standard deviation $\sigma > 0$.

- Also, the periodic boundary conditions are imposed: No particle leaves the container, $0 < x_i < L_x$ and $0 < y_i < L_y$ for every $i < N$. Should any particle leave the specified boundaries, it must reappear on the opposite side of the container. For example, if $x_i > L_x$, we set $x_i^{new} = x_i - L_x$ and the particle appears on the opposite side (provided that $x_i - L_x < L_x$ which is a reasonable assumption.)

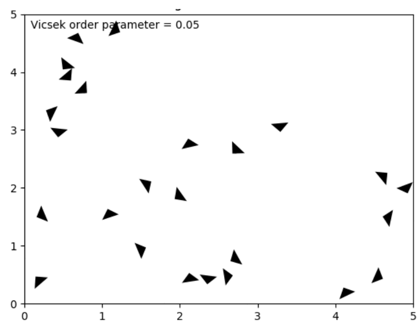
- By following this algorithm it is possible to simulate the system for any specified number of steps

## 2.2 Order parameter

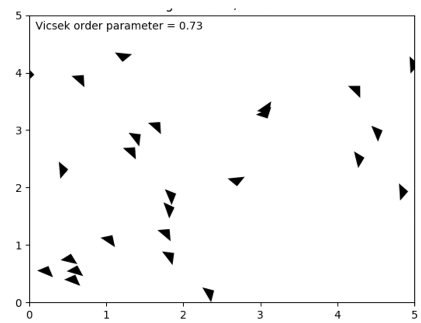
To analyze and compare how much order there is in a system, we define quantity 'order parameter':

$$n = \frac{|\sum_1^N \vec{d}_i|}{N},$$
$$\text{consequently } 0 < n < 1$$

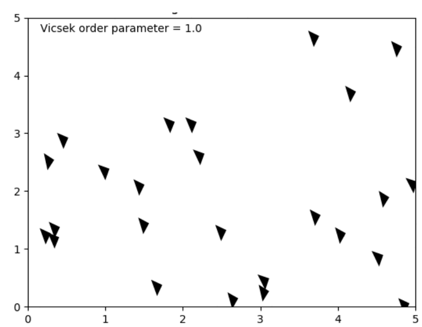
This quantity measures how ordered the system is, on scale between 0 and 1. 0 corresponds to completely unordered system, 1 is completely aligned system (fig. 2).




(a) Chaotic system, $n = 0.05$




(b) Partly aligned system $n = 0.73$




(c) Ideally coordinated collective motion, $n = 1$

Figure 2: a system of $N = 25$ birds/particles in three states with different values of order parameter

### 2.3 Finding mean order parameter

To understand the phase transition in the systems we study, it is necessary to find the average value of order parameter corresponding to each value of $\sigma$ (standard deviation of the random-generated noise). Because at each step the system has updated its state and therefore its order parameter, we have order parameter for each step. If we want to find their mean value, there is one complication - in the beginning they are completely unaligned and it is only after number of steps when the system reaches its average/mean values of order parameter.

Therefore, it is necessary to run test simulations with the similar initial conditions and by visual inspection determine the number of order parameters which will be excluded from the mean calculation (fig. 3).


Figure 3: This plot records the value of order parameter for 5 systems with different values of standard deviation $\sigma$ for each step up to 100 steps. We can see that each system reaches a mean value of $n$ after around 10-20 steps. This number determined by this visual inspection is later used for computations of the mean order parameter corresponding to specific standard deviation $\sigma$.

### 2.4 Implementation in code

There are several important points about the specific solution used in this simulation. First, in the header of the python code, there is a list of parameters which describe the system. Those can be changed, and this will determine the characteristics of the system (fig. 4)

```
#parameters
N = 25          # number of birds
L_x = 5         # boundary in x
L_y = 5         # boundary in y
v = 0.3         # speed of each particle/bird
sigma = 0.1     # st. deviation of random angle
steps = 25      # steps to complete
R = 0.5         # radius of birds' control
```

Figure 4: Variable parameters in the python code

The crucial part of the code is the function step(), which completes one step for i-th bird, i is passed as argument of the function. This function completes the algorithm described above, using other functions e.g. direction_sum() and periodic_check(). It returns new position and new direction vectors (fig. 5)

```
# computes next position and direction of a particle i
# returns two vectors - one with new position and 2nd with new direction
def step(i):
    #update position vector
    position = positions[:,i]
    direction = directions[:,i]

    #update position
    new_position =  position + v*direction
    new_position = periodic_check(new_position,L_x, L_y)

    #update direction
    new_direction = direction_sum(directions, positions, i)
    random = random_angle(sigma)
    angle = np.arctan(new_direction[1]/new_direction[0])
    new_direction = [np.cos(angle + random),np.sin(angle + random)]

    return new_position, new_direction
```

Figure 5: Function step(), used in the simulation. It follows the algorithm described above

# 3 Results

The resulting simulation is consistent with our intuition in the extreme cases, as well as in normal cases. It is demonstrated in several animations produced for this project, which are enclosed in a .ppt file as part of the assignment.

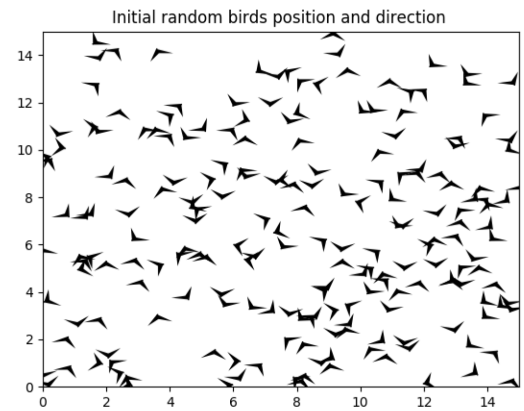
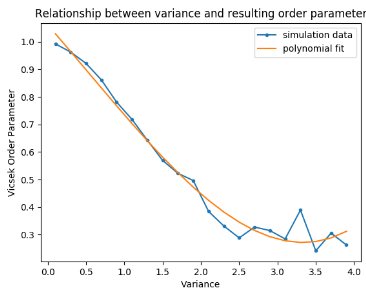



Figure 6: Visualisation of a flock of birds

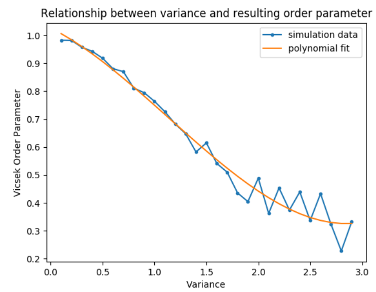
More importantly, the results seems to be consistent with the original paper published on this topic by Tamás Vicsek et al. in 1995. This similarity can be seen by comparing the phase transition plots for similar cases.

### 3.1 Phase transition plot

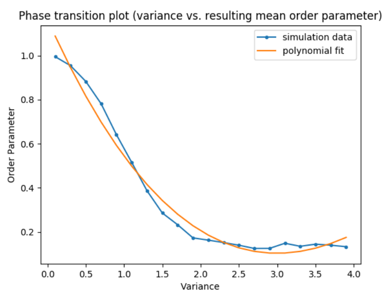
This is a very important characteristics of a system because it shows how different systems react to different values of $\sigma$ (Fig. 7).



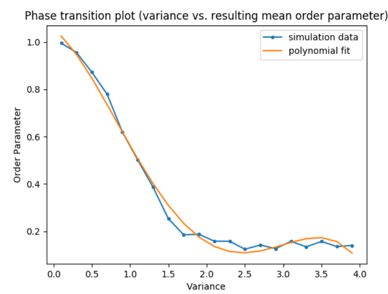

(a)




(b)




(c)




(d)

Figure 7: Four phase transition plots for different systems, all showing similar kind of trends which are consistent with the research of Tamás Vicsek et al. We can obviously observe a decreasing function because by increasing noise, we are losing the order in the system. The smoothness of the plot depends on number of steps used for each $\sigma$ to determine the mean order parameter and on number of different $\sigma$ we run the simulation for (in the cases above only 50 steps were used, while for the two plots below this was around 200 steps )

## 3.2 Phase transition plot - comparison

Better resolution phase transition was generated and compared with Vicsek's results for similar systems (Fig. 8 and Fig. 9).

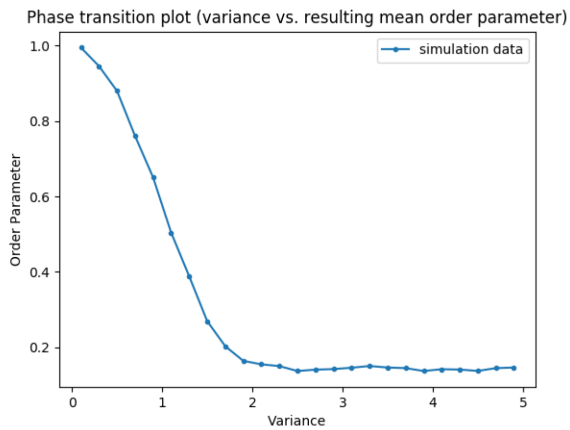



Figure 8: Phase transition plot for a simulation from this project, system of $N = 40$, $0 < \sigma < 5$, $v = 0.5$,$L_x = L_y = 3.1$

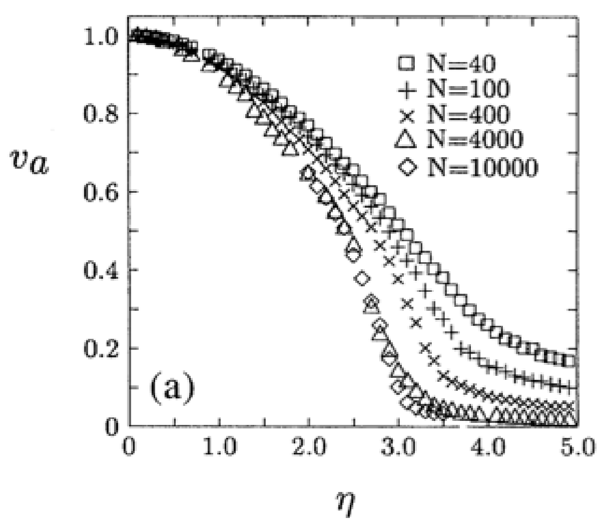



Figure 9: Vicsek's phase transition plot, Source: Tamás Vicsek et al. "Novel Type of Phase Transition in a System of Self-Driven Particles". In: *Physical Review Letters. 75(6) 1226–1229* (1995), p. 1228 , available online: https://journals.aps.org/prl/pdf/10.1103/PhysRevLett.75.1226

We can notice that although the plots exhibit similar trends, they are not the same, especially for the corresponding $N = 40$ case. There are two probable reasons for this discrepancy. First, slightly different parameters, specifically in the velocity of particles. While Vicsek et al. used very low speed, $v = 0.03$

while in fig. 8 a value of $v = 0.5$ was used (this velocity was chosen for practical reasons, because lower velocity requires longer time for the order parameter to settle - because the time between each interaction is longer. As a result it is necessary to run the simulation for higher number of steps, which significantly increases the time needed). Second, and more important is the fact that Vicsek et al. are not using a normally distributed noise with standard deviation $\sigma$ but they are using a 'uniform probability from the interval $(\frac{-\eta}{2}, \frac{\eta}{2})$.[4] This gives an explanation of slightly steeper curve in fig. 8 because the random angle can be any value from interval $(-\infty, +\infty)$, while in the fig. 9 the angle can be only from the interval $(\frac{-\eta}{2}, \frac{\eta}{2})$. As a result, larger $\sigma$ results in bigger disordering effect in the fig. 8.

# 4 Conclusions

Overall, the simulation in this project exhibits reasonable results in terms of visual aspects as well as in its qualitative properties; the former is shown in the animations produced (and attached in the .ppt file) while the latter is demonstrated by comparison of plots produced in this project with the plot from Vicsek's et al. research paper. Results are intuitive because they suggest that the order in a group of self-driven particles will decrease with introduction of an increasing noise. When the noise is decreasing, there is an increasing collective behaviour and the flock start spontaneously moving in a specific direction.

Very interesting is the fact that we can indeed see the resemblance of this elementary model to some biological phenomena that we know from our experience (motion of groups of fish, flocks of birds and in some cases even people in larger groups). This might be particularly useful in biology, ecology and various other disciplines, where collective behaviour is studied.

Regarding possible improvements of this project, I can imagine the simulation to run faster, so refactoring the code in some more efficient way will be surely helpful with this point. Also, the code could be rewritten in another language, for faster run probably in C/C++.

There are numerous interesting modifications and additional features which could be introduced in order to model more specific/realistic system. Among those, e.g. introduction of a repulsion force - accounting for the finite size of the particles, introduction of variety - particles of different characteristics: different velocities, different sizes, different radius of control and so on. Another modification might be local and non-local disturbance, e.g. predator birds, variable wind conditions. The most impressive feature would be the account of fluid dynamics (drag, turbulent and laminar flow) with an obvious question: "Will birds fly in the classical V shape formation in such model?"

[4] Ibid., p. 1227.

# References

al., Tamás Vicsek et. "Novel Type of Phase Transition in a System of Self-Driven Particles". In: *Physical Review Letters. 75(6) 1226–1229* (1995).