Zero-inflated beta random effect model (ZIBR)

For a given bacterial taxon, Y_{it} $(i = 1, 2, \ldots, N, t = 1, 2, \ldots, T)$ is the relative abundance for subject i at time t.

$$\begin{aligned} Y_{it} &\backsim 0 \text{ with probability } 1 - p_{it} \\ &\backsim \text{Beta}(\mu_{it}\phi, (1-\mu_{it})\phi) \text{ with probability } p_{it}, \end{aligned}$$

where $0 \leq Y_{it} < 1$, $0 < \mu_{it} < 1$ and $\phi > 0$. Let X_{it}, Z_{it} be the covariates.

$$\text{logit}(p_{it}) = \log\left(\frac{p_{it}}{1-p_{it}}\right) = a_i + \alpha_0 + X_{it}^T\alpha,$$

$$\text{logit}(\mu_{it}) = \log\left(\frac{\mu_{it}}{1-\mu_{it}}\right) = b_i + \beta_0 + Z_{it}^T\beta,$$

where a_i and b_i are the individual-specific random intercepts (allow across time correlations)

$$a_i \sim N(0, \sigma_1^2), \quad b_i \sim N(0, \sigma_2^2).$$