1 Tesla

Problem: You are given a list of n charging stations, and at each station i you can charge your car to travel a distance a_i. The stations are arranged in a circle and you can travel from station i to $i+1$ (where $n+1=1$) at a distance of 10. Furthermore $\sum_{i=1}^{n} a_i = 10n$. Find a station from which you can complete a loop without running out of charge.

Solution: Start at point 1 and simulate the process of going around the circuit, keeping track of the remaining charge c and the starting station s. At each station set $c := c + a_i - 10$. If c becomes negative at station i, restart the simulation starting at the next station $s := i+1$. As soon as you have completed a loop, the result is s.

This will take at at most 2 trips around the circle, thus having a complexity of $O(2n)$.

Proof. To prove the correctness of our solution, we will prove that from any starting point s, you will either complete a full loop without reaching a negative charge, or you will restart at a valid starting point for a loop.

Let a solution be r (the proof that a solution always exists is left to the reader). Starting from any station s, going around the loop (possibly reassigning s), you will reach station r.

Case 1. *The remaining charge at r starting from s is negative.*

You restart the simulation from r, which will result in a complete loop, yielding the correct answer.

Case 2. *The remaining charge at r starting from s is zero.*

In this case either $r=s$ or both r and s are valid solutions. Note that the path from s to r is possible without reaching a negative charge. The loop from r to r is possible, so the path from r to s is possible. If s to r and r to s are possible, then the loop from s to s is possible.

Case 3. *The remaining charge at r starting from s is positive.*

The loop from r to r will result in a charge of 0, as the charge after a full loop is

$$c = \sum_{i=1}^{n}(a_i - 10) = \sum_{i=1}^{n} a_i - 10n = 10n - 10n = 0$$

So if the path from s to r results in a positive charge, then the path from r to s will be negative. But this is a contradiction, so this case will never occur.

□