Formal Languages — CSE 4083 & CSE 5210

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Answer the questions in the space provided.

Your Name, Class, & Major: ______________________________

You are not to consult other people when answering these questions.
You may consult sources (books, web pages, journals, *etc*, but cite your sources and don't simply copy and paste answers.

1 Deterministic Finite Automata

1. (10 points) Consider a DFA M = $(Q, \Sigma, \delta, s, f)$ with States $Q = \{s, q_1, q_2, f\}$, where s is the *start* and f is the *final* state; Alphabet $\Sigma = \{0, 1\}$; and transition function δ.

   Construct a state transition table for δ (or you can draw a state transition diagram) that recognizes regular expressions that are binary strings and multiples of 3, for example, the strings

   $$0, 11, 110, 1001, 1100, \ldots$$

   would be accepted strings, but

   $$1, 10, 100, 101, \ldots$$

   would not be accepted.

   (Hint: Think, if $n = 3k$ is a multiple of 3, then the next multiple of 3 is $3k + 3$. this could be accomplished by a transition from the current state to a next state by scanning 3 ones.)

2 Nondeterministic Finite Automata

2. (15 points) Explain how any NFA (with λ (or ϵ) transitions) can be converted into a DFA that accepts the same language as that accepted by the NFA. That is, the expressive power of NFAs and DFAs are equivalent. This is known as the *Rabin-Scott Theorem*.

3 Regular Expressions in the Programming World

3. (10 points) Consider a programming language that has identifiers that start with a lowercase ASCII letter

$$A = \{a...z\}$$

followed by a string of 1 or more digits

$$D = \{0...9\}$$

or 1 or more lowercase ASCII letters. Show how to write this specification as a regular expression.

4 Closure Properties of Languages

Answer these `True` (T) or `False` (F) questions. Give a brief explanation of your answer (for example, explain how to construct a machine that implements the property.)

4. (5 points) ___ Regular languages are closed under *intersection*.

5. (5 points) ___ Regular languages are closed under *Kleene-star*.

5 Decision Properties of Languages

6. (5 points) What does it mean to say that a "yes" or "no" question is *undecidable*?

7. (5 points) ___ Answer `True` (T) or `False` (F): It is decidable whether or not the language of a DFA is empty or non-empty. Give an explanation of your answer.

8. (5 points) ___ It is decidable whether or not the language of a DFA is finite or infinite.

9. (5 points) ___ It is undecidable whether or not the a string s is accepted by a DFA. Answer `True` (T) or `False` (F): Give an explanation of your answer.

10. (5 points) ___ Answer `True` (T) or `False` (F): It is decidable whether or not two regular languages L_1 and L_2 are equal.

    Give an explanation of your answer.

6 Equivalence Relations

11. (10 points) On the set $\mathbb{N}$ of natural numbers define an equivalence relation $n \equiv m$ if and only if

$$n \mod 3 = m \mod 3$$

(Hint: Recall any natural number n can be written as $n = 3q + r$ n with quotient q and remainder r. And $n \mod 3 = \{kr : k \in \mathbb{N}\}$ The set of all natural numbers that have a remainder of r when divided by 3.

The slick way of saying this is: $n \equiv m$ if and only if they both have the same reminder when divided by 3.

Prove that $\equiv$ is an equivalence relation on the set of natural numbers.

7 The Pumping Lemma for Regular Languages

12. (10 points) DFAs can't count to an arbitrary natural number! Use the pumping lemma for regular languages to show that language

$$\text{EQ} = \left\{w \in \{a,\, b\}^* : w = a^i b^i\right\}$$

is not regular. Here the number of a's in the prefix of w equals the number of b's in the suffix of w.

8 Context Free Languages

13. (5 points) Consider the CFG G defined by the productions:

$$S \rightarrow aS|Sb|a|b$$

Prove by induction that no string in L(G) has ba as a sub-string.

Hint: To show this do induction on the length of the strings.

14. (5 points) Give simple English language descriptions for the strings generated by the productions following four grammars ($G = (V,\, T,\, P,\, S)$:

1. $G1 \rightarrow S|aS|a$

2. $G2 : S!aSa|aa|a$

3. $G3 : S|SaS|a$

Question	Points	Score
1	10	
2	15	
3	10	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
11	10	
12	10	
13	5	
14	5	
Total:	100	