Formal Languages — CSE 4083 & CSE 5210

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1 Deterministic Finite Automata

1. Consider a DFA $M = (Q,\Sigma,\delta,s,f)$ with States $Q = s, q_1, q_2, f$ where s is the start and f is the final state;
   Alphabet $\Sigma = 0, 1$ and transition function δ.

Construct a state transition table for δ (or you can draw a state transition diagram) that recognizes regular expressions that are binary strings and multiples of 3, for example, the strings:

$$0, 11, 110, 1001, 1100, ...$$

would be accepted strings, but

$$1, 10, 100, 101, ...$$

would not be accepted.

> *Hint: Think, if $n = 3k$ is a multiple of 3, then the next multiple of 3 is $3k + 3$. This could be accomplished by a transition from the current state to a next state by scanning 3 ones.

2 Nondeterministic Finite Automata

2. Explain how any NFA (with λ (or ϵ) transitions) can be converted into a DFA that accepts the same language as that accepted by the NFA. That is, the expressive power of NFAs and DFAs are equivalent. This is known as the Rabin-Scott Theorem

3 Regular Expressions in the Programming World

3. Consider a programming language that has identifiers that start with a lowercase ASCII letter

$$A = \{a..z\}$$

followed by a string of 1 or more digits

$$D = \{0..9\}$$

or 1 or more lowercase ASCII letters. Show how to write this specification as a regular expression.

4 Closure Properties of Languages

Answer these *True* (T) or *False* (F) questions. Give a brief explanation of your answer

> *(for example, explain how to construct a machine that implements the property.)

4. Regular languages are closed under intersection.

5. Regular languages are closed under Kleene-star.

5 Decision Properties of Languages

6. What does it mean to say that a “yes” or “no” question is *undecidable*?

***True* (T) or *False* (F):**

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7. It is decidable whether or not the language of a DFA is empty or non-empty. Give an explanation of your answer.

8. It is decidable whether or not the language of a DFA is finite or infinite.

9. It is undecidable whether or not the a strings is accepted by a DFA.

10. It is decidable whether or not two regular languages L_1 and L_2 are equal. Give an explanation of your answer.

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6 Equivalence Relations

11. On the set $\mathbb{N}$ of natural numbers define an equivalence relation $n \equiv m$ if and only if

$$n \mod 3 = m \mod 3$$

> *Hint: Recall any natural number n can be written as $n = 3q + r$ n with quotient q and remainder r.
> And $\boldsymbol{n \mod 3 = \{kr : k \in \mathbb{N}\}}$ The set of all natural numbers that have a remainder of r when divided by 3.

Prove that ≡ is an equivalence relation on the set of natural numbers.

7 The Pumping Lemma for Regular Languages

12. DFAs can't count to an arbitrary natural number! Use the pumping lemma for regular languages to show that language

$$EQ = \{w \in \{a, b\}^* : w = a^i b^i\}$$

is not regular. Here the number of a's in the prefix of w equals the number of b's in the suffix of w.

8 Context Free Languages

13. Consider the CFG G defined by the productions:

$$S \rightarrow aS|Sb|a|b$$

Prove by induction that no string in $L(G)$ has ba as a sub-string.

> Hint: To show this do induction on the length of the strings.

14. Give simple English language descriptions for the strings generated by the productions following four grammars (G=(V,T,P,S)):

    1. $G_1 \rightarrow S|aS|a$

    2. G_2 : $S!aSa|aa|a$

    3. G_3 : $S|SaS|a$

Question	Points	Score
1	10	
2	15	
3	10	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
11	10	
12	10	
13	5	
14	5	
Total:	100	