

# INFORMED SEARCH ALGORITHMS

## CHAPTER 4, SECTIONS 1–2

# Outline

- ◇ Best-first search
- ◇  $A^*$  search
- ◇ Heuristics

## Review: Tree search

```
function TREE-SEARCH(problem, fringe) returns a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST[problem] applied to STATE(node) succeeds return node
    fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

A strategy is defined by picking the **order of node expansion**

## Best-first search

**Idea:** use an **evaluation function** for each node  
– estimate of “desirability”

⇒ Expand most desirable unexpanded node

**Implementation:**

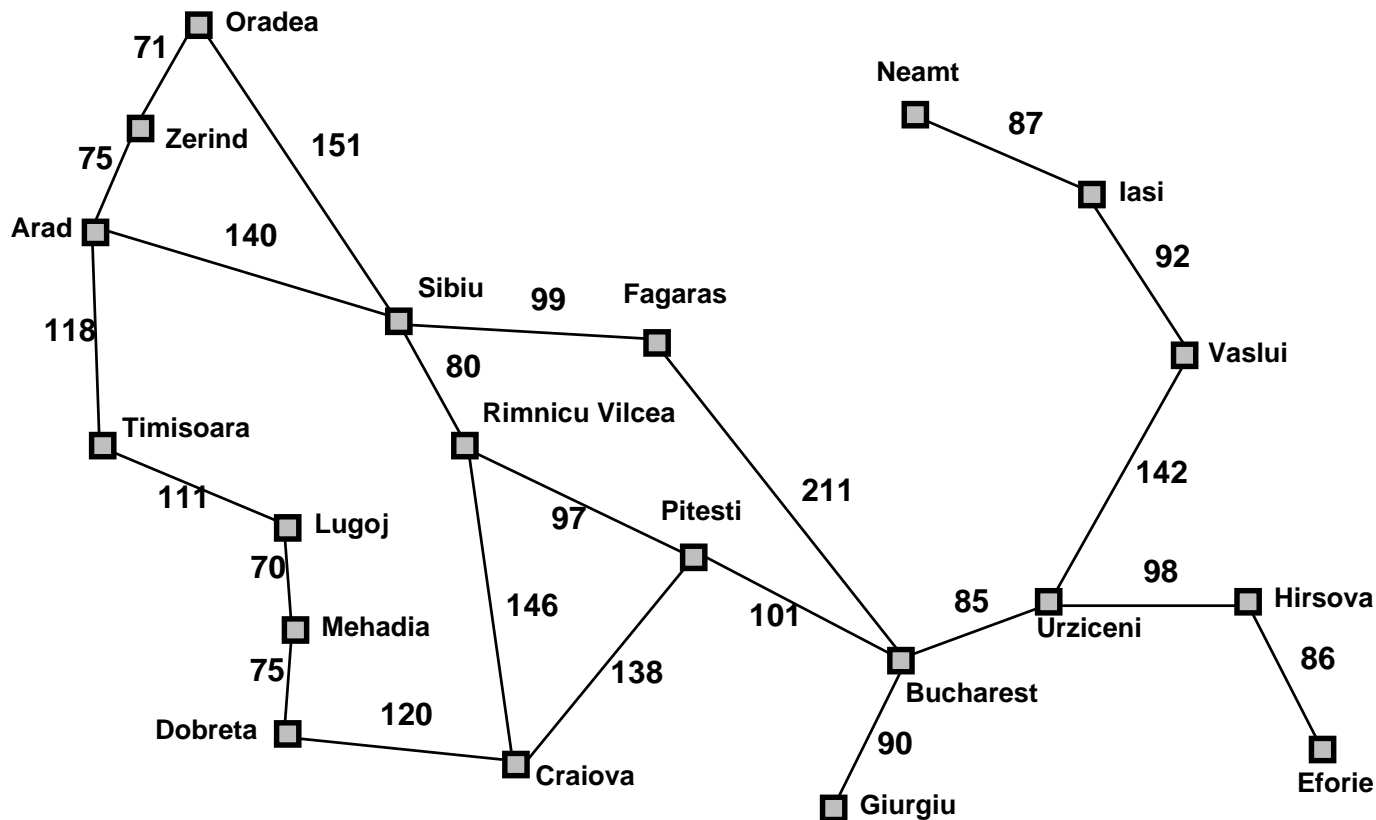
*fringe* is a queue sorted in decreasing order of desirability

Special cases:

greedy search

A\* search

# Romania with step costs in km



Straight-line distance  
to Bucharest

<b>Arad</b>	366
<b>Bucharest</b>	0
<b>Craiova</b>	160
<b>Dobreta</b>	242
<b>Eforie</b>	161
<b>Fagaras</b>	178
<b>Giurgiu</b>	77
<b>Hirsova</b>	151
<b>Iasi</b>	226
<b>Lugoj</b>	244
<b>Mehadia</b>	241
<b>Neamt</b>	234
<b>Oradea</b>	380
<b>Pitesti</b>	98
<b>Rimnicu Vilcea</b>	193
<b>Sibiu</b>	253
<b>Timisoara</b>	329
<b>Urziceni</b>	80
<b>Vaslui</b>	199
<b>Zerind</b>	374

## Greedy search

Evaluation function  $h(n)$  (**h**euristic)

= estimate of cost from  $n$  to the closest goal

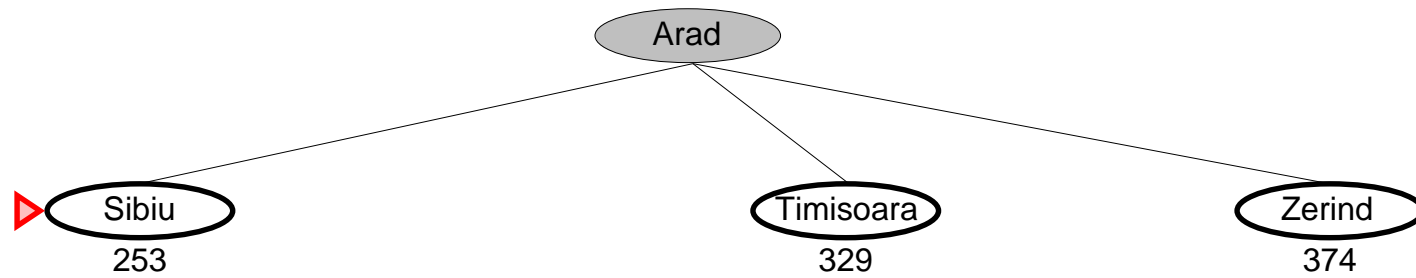
E.g.,  $h_{\text{SLD}}(n)$  = straight-line distance from  $n$  to Bucharest

Greedy search expands the node that **appears** to be closest to goal

## Greedy search example

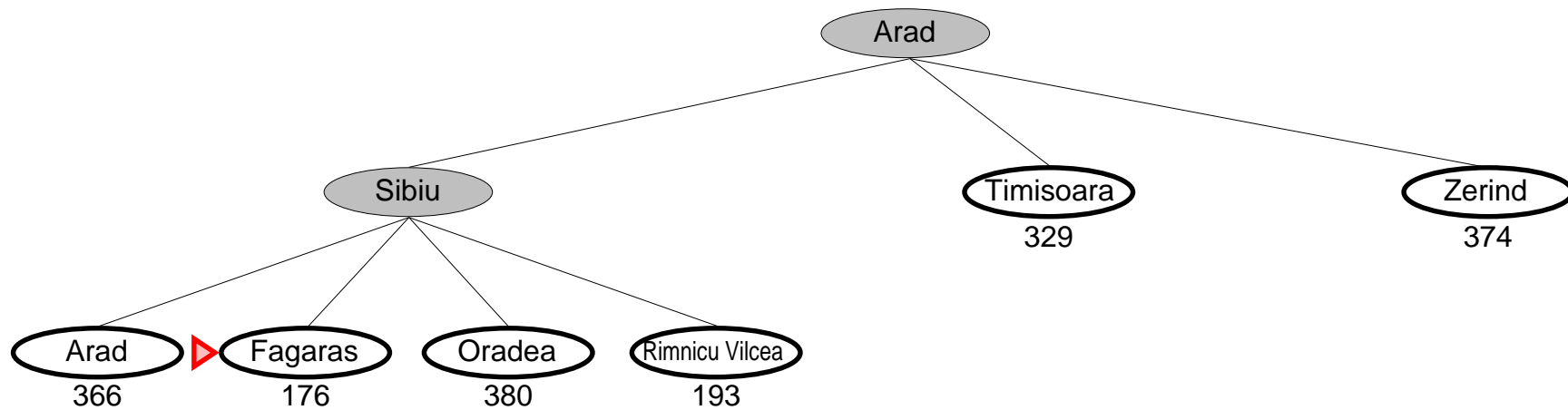
▶ Arad  
366

## Greedy search example

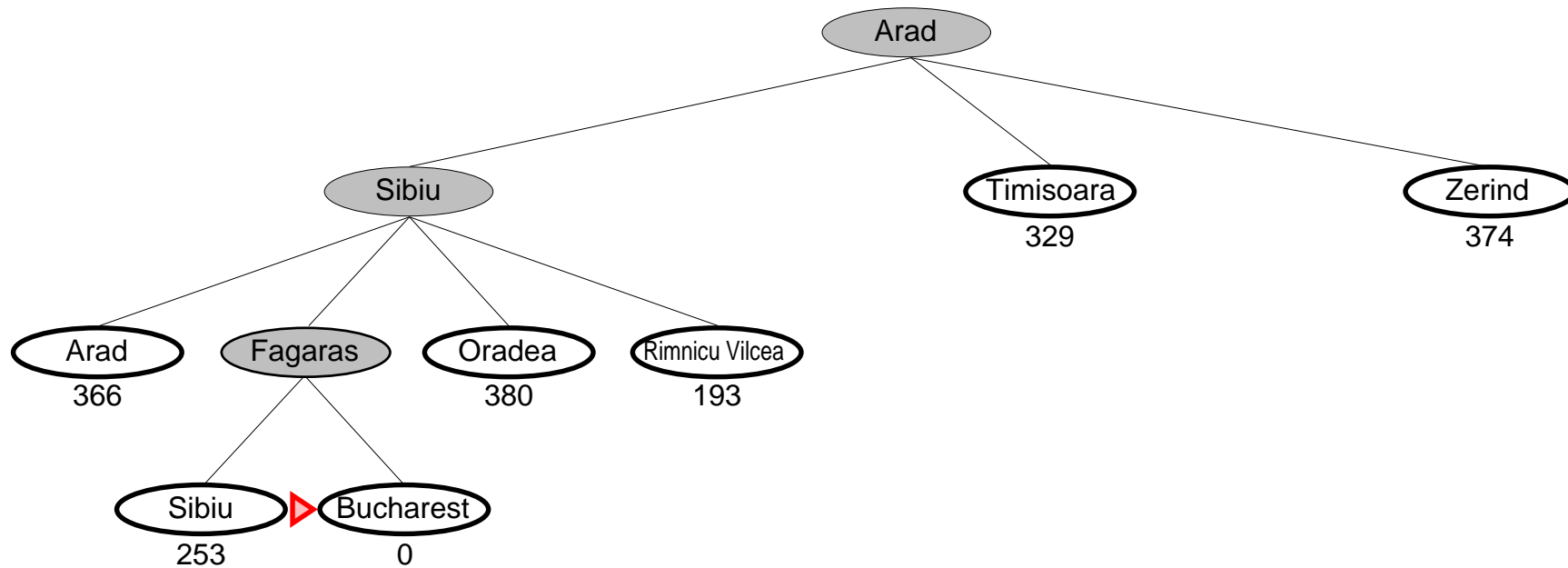




# Greedy search example



# Greedy search example



# Properties of greedy search

Complete??

## Properties of greedy search

Complete?? No—can get stuck in loops, e.g., with Oradea as goal,

Iasi  $\rightarrow$  Neamt  $\rightarrow$  Iasi  $\rightarrow$  Neamt  $\rightarrow$

Complete in finite space with repeated-state checking

Time??

## Properties of greedy search

Complete?? No—can get stuck in loops, e.g.,

lasi  $\rightarrow$  Neamt  $\rightarrow$  lasi  $\rightarrow$  Neamt  $\rightarrow$

Complete in finite space with repeated-state checking

Time??  $O(b^m)$ , but a good heuristic can give dramatic improvement

Space??

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Optimal??

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Time??  $O(b^m)$ , but a good heuristic can give dramatic improvement

Space??  $O(b^m)$ —keeps all nodes in memory

Optimal?? No

## A\* search

Idea: avoid expanding paths that are already expensive

Evaluation function  $f(n) = g(n) + h(n)$

$g(n)$  = cost so far to reach  $n$

$h(n)$  = estimated cost to goal from  $n$

$f(n)$  = estimated total cost of path through  $n$  to goal

A\* search uses an **admissible** heuristic

i.e.,  $h(n) \leq h^*(n)$  where  $h^*(n)$  is the **true** cost from  $n$ .

(Also require  $h(n) \geq 0$ , so  $h(G) = 0$  for any goal  $G$ .)

E.g.,  $h_{\text{SLD}}(n)$  never overestimates the actual road distance

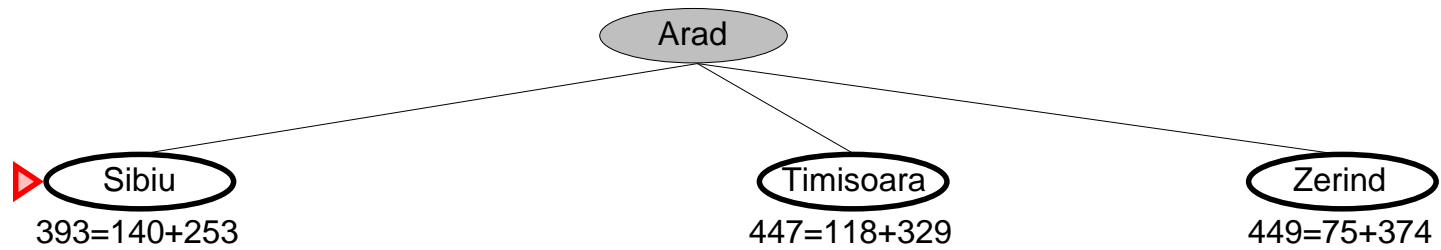
**Theorem:** A\* search is optimal



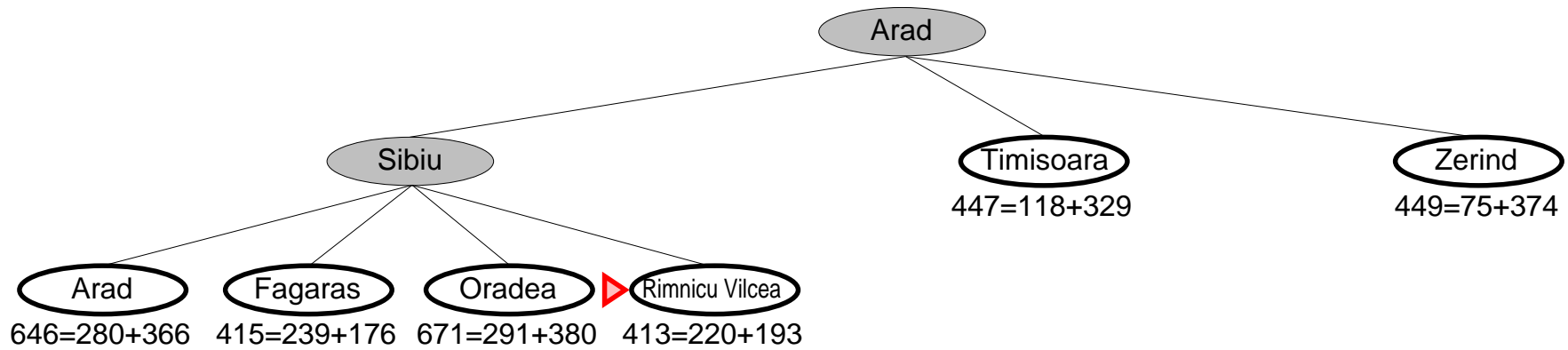
## A\* search example

▶ Arad  
 $366 = 0 + 366$

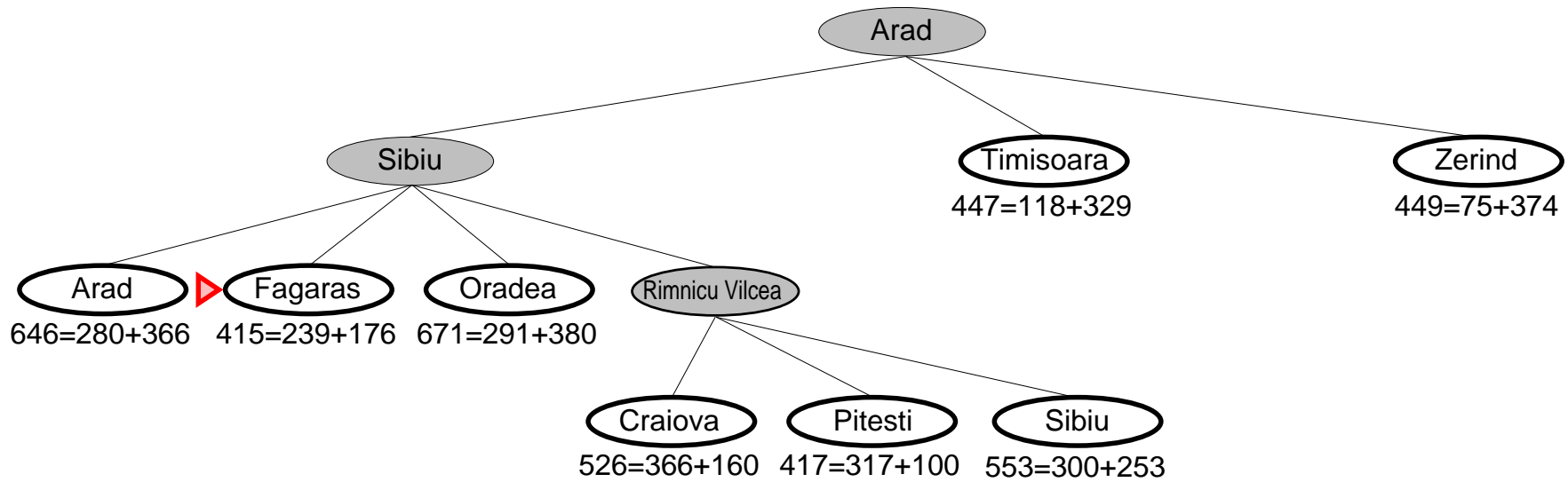
## A\* search example



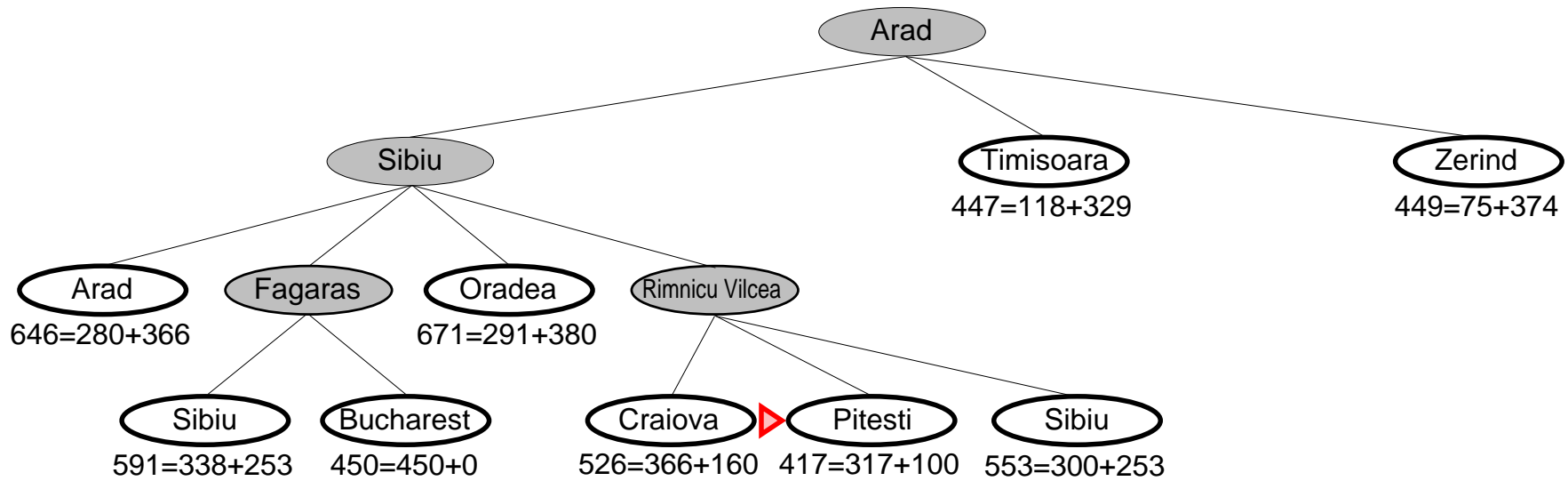
## A\* search example



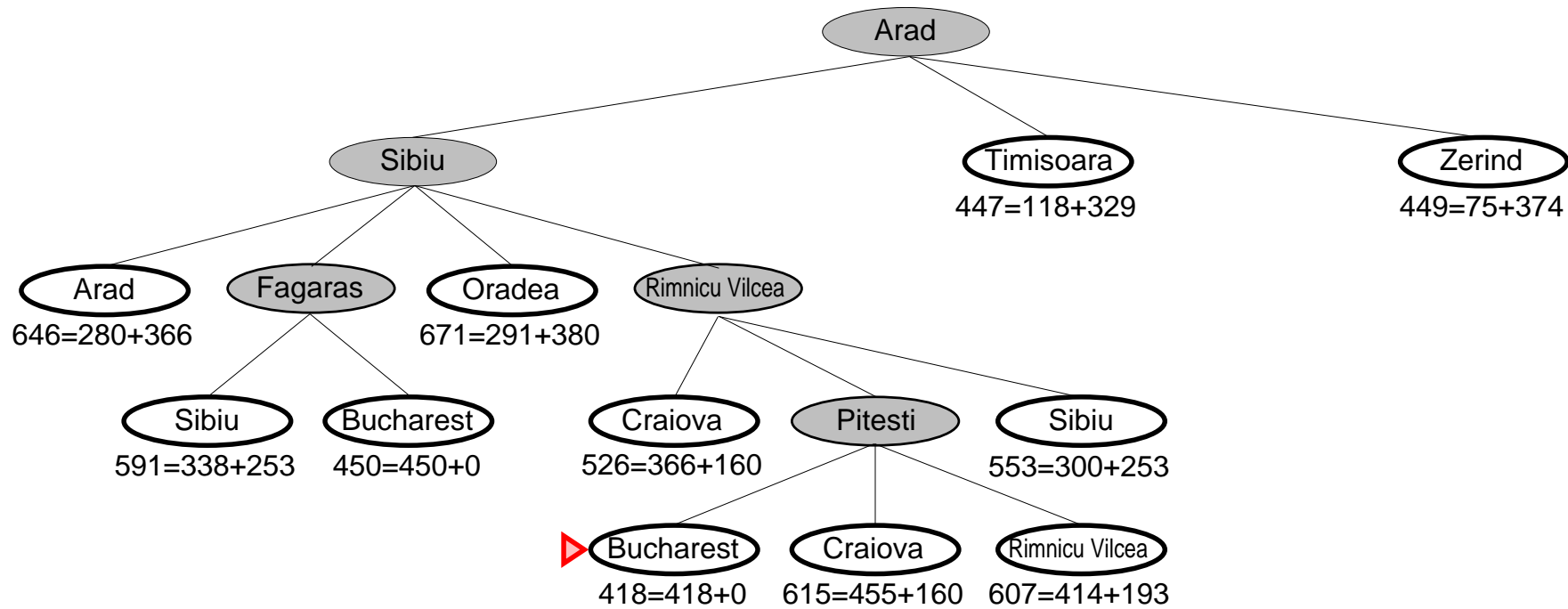
## A\* search example



## A\* search example

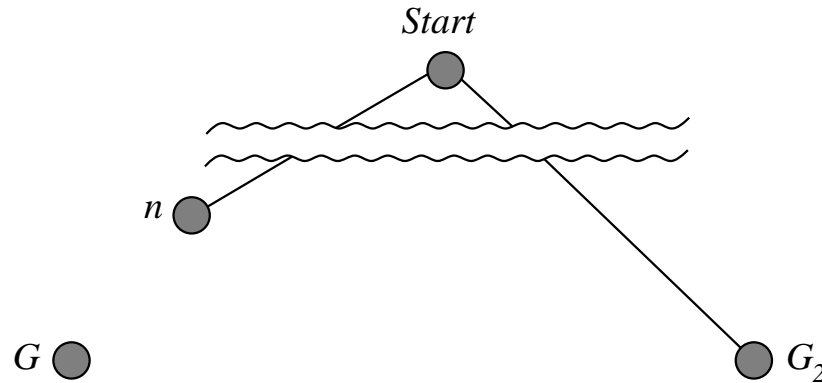


## A\* search example



## Optimality of $A^*$ (standard proof)

Suppose some suboptimal goal  $G_2$  has been generated and is in the queue. Let  $n$  be an unexpanded node on a shortest path to an optimal goal  $G_1$ .



$$\begin{aligned} f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\ &> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\ &\geq f(n) && \text{since } h \text{ is admissible} \end{aligned}$$

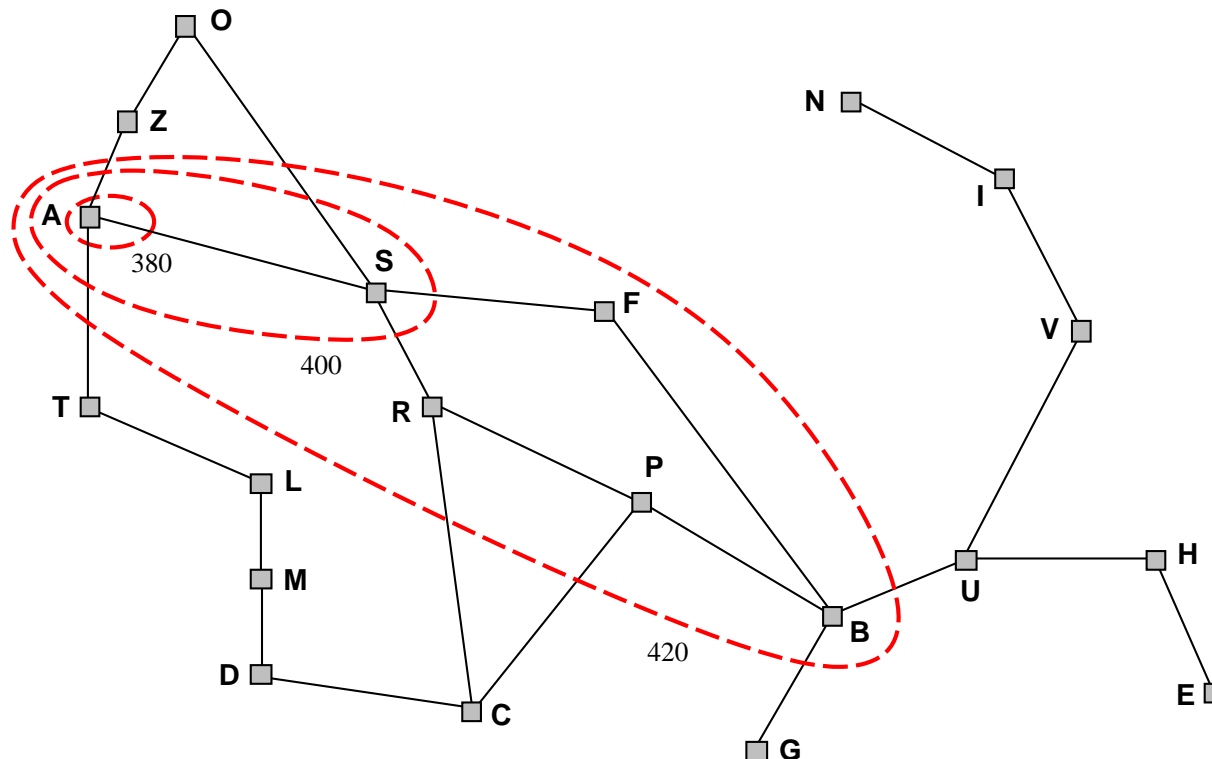
Since  $f(G_2) > f(n)$ ,  $A^*$  will never select  $G_2$  for expansion

## Optimality of $A^*$ (more useful)

**Lemma:**  $A^*$  expands nodes in order of increasing  $f$  value\*

Gradually adds “ $f$ -contours” of nodes (cf. breadth-first adds layers)

Contour  $i$  has all nodes with  $f = f_i$ , where  $f_i < f_{i+1}$





## Properties of $A^*$

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Optimal??

## Properties of $A^*$

Complete?? Yes, unless there are infinitely many nodes with  $f \leq f(G)$

Time?? Exponential in [relative error in  $h \times$  length of soln.]

Space?? Keeps all nodes in memory

Optimal?? Yes—cannot expand  $f_{i+1}$  until  $f_i$  is finished

$A^*$  expands all nodes with  $f(n) < C^*$

$A^*$  expands some nodes with  $f(n) = C^*$

$A^*$  expands no nodes with  $f(n) > C^*$

## Proof of lemma: Consistency

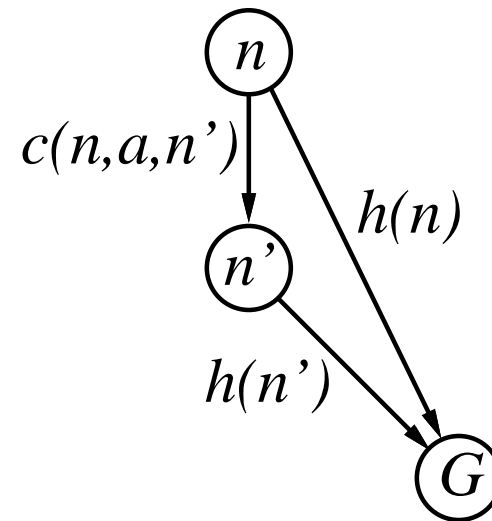
A heuristic is **consistent** if

$$h(n) \leq c(n, a, n') + h(n')$$

If  $h$  is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

I.e.,  $f(n)$  is nondecreasing along any path.



## Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$  = number of misplaced tiles

$h_2(n)$  = total **Manhattan** distance

(i.e., no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

$h_1(S) = ??$

$h_2(S) = ??$

## Admissible heuristics

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7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

$h_1(S) = ??$  6

$h_2(S) = ??$   $4+0+3+3+1+0+2+1 = 14$



## Dominance

If  $h_2(n) \geq h_1(n)$  for all  $n$  (both admissible)  
then  $h_2$  dominates  $h_1$  and is better for search

Typical search costs:

$d = 14$  IDS = 3,473,941 nodes

$A^*(h_1) = 539$  nodes

$A^*(h_2) = 113$  nodes

$d = 24$  IDS  $\approx$  54,000,000,000 nodes

$A^*(h_1) = 39,135$  nodes

$A^*(h_2) = 1,641$  nodes

Given any admissible heuristics  $h_a, h_b$ ,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates  $h_a, h_b$

## Relaxed problems

Admissible heuristics can be derived from the **exact** solution cost of a **relaxed** version of the problem

If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**, then  $h_1(n)$  gives the shortest solution

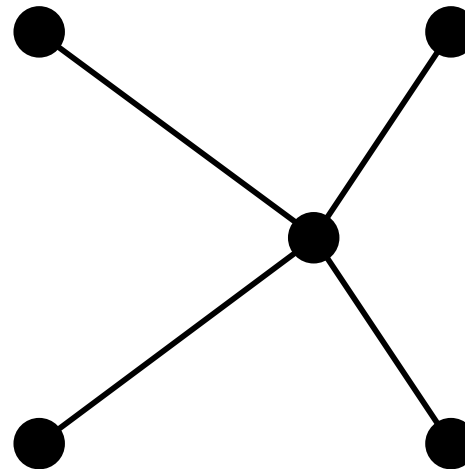
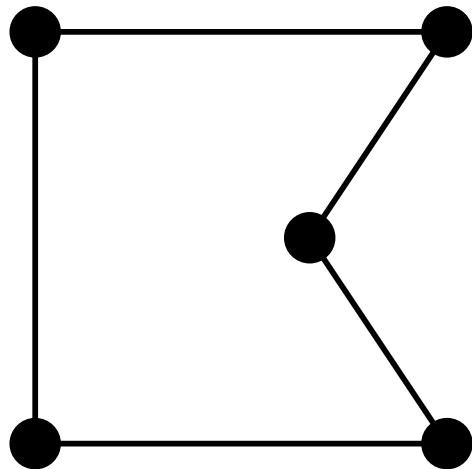
If the rules are relaxed so that a tile can move to **any adjacent square**, then  $h_2(n)$  gives the shortest solution

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

## Relaxed problems contd.

Well-known example: travelling salesperson problem (TSP)

Find the shortest tour visiting all cities exactly once



Minimum spanning tree can be computed in  $O(n^2)$   
and is a lower bound on the shortest (open) tour

## Summary

Heuristic functions estimate costs of shortest paths

Good heuristics can dramatically reduce search cost

Greedy best-first search expands lowest  $h$

- incomplete and not always optimal

A\* search expands lowest  $g + h$

- complete and optimal
- also optimally efficient (up to tie-breaks, for forward search)

Admissible heuristics can be derived from exact solution of relaxed problems