A Sequential Route-Building Algorithm Employing a Generalised Savings Criterion
Author(s): R. H. Mole and S. R. Jameson
Source: *Operational Research Quarterly (1970-1977)*, Vol. 27, No. 2, Part 2 (1976), pp. 503-511
Published by: Palgrave Macmillan Journals on behalf of the Operational Research Society
Stable URL: https://www.jstor.org/stable/3008819
Accessed: 23-03-2020 10:08 UTC

Opl Res. Q., *Pergamon Press* 1976. *Vol.* 27, 2, ii, *pp.* 503 *to* 511. *Printed in Great Britain*

A Sequential Route-building Algorithm Employing a Generalised Savings Criterion

R. H. MOLE and S. R. JAMESON

Department of Management Sciences, University of Manchester, Institute of Science and Technology

Sequential and concurrent route building methods for planning multiple drop delivery journeys are contrasted, and it is concluded that the former would offer important practical advantages if the quality of the routes could be improved. A new method for vehicle scheduling, based upon sequential generation of vehicle routes, is described in this paper. The possibility of several journeys being made by each vehicle is actively considered, and vehicle utilisation is high in consequence. The Clarke and Wright savings criterion is generalised, and each journey is automatically free from intersections. The method is also computationally efficient as evidenced by a study of the sensitivity with respect to fleet size, of changes in vehicle capacity and a driven distance constraint for a single depot problem with 225 customers.

INTRODUCTION

THE DESIGN of multi-drop vehicle routes emanating from a central depot has received considerable attention from a number of authors. Heuristic procedures have been developed in the face of the inherently combinatorial aspects of the problem, but each method seems to display disadvantages of real practical importance for problems of reasonable size.

Most of the commercially available computer packages generate vehicle routes concurrently rather than sequentially, and employ the "savings" criterion of Clarke and Wright.[1] Such programs typically require a prior calculation of a "savings file" at considerable expense, which naturally deters many would-be users. Webb[2] and Yellow[3] have proposed alternative computational approaches to dispense with a savings file, but neither has gained widespread acceptance. Savings based sequential route building algorithms[3–5] do not require a savings file but tend to produce markedly inferior results unless additional strategems are employed. Yellow's segmentation of delivery areas was generally advantageous although the development of systematic, rather than intuitive, segmentation procedures was not reported.

The computing time is prohibitive for the multiple Travelling Salesman approach of Eilon and Christofides[6] which randomly generates a feasible set of routes prior to resequencing by 3-optimal procedures. (The r-optimal tour method for the Travelling Salesman Problem is due to Lin[7]; a tour is said to be r-optimal if improvements cannot be effected by replacing any r links by any other set of r links). Wren and Holliday[8] take a feasible

TABLE 1. RESULTS OF SEVERAL CONCURRENT AND SEQUENTIAL ROUTE-BUILDING METHODS FOR TEN TEST PROBLEMS[6]

Route distances (Vehicle numbers) Computing time in minutes.

	Concurrent route building methods			Sequential route building methods	
Problem Number [6]	Clarke and Wright [1] *	Eilon and Christofides [6] *	Wren and Holliday[8] **	Clarke and Wright[4] ***	Yellow [3] ****
1.	119 (2) 0.1	114 (2) 0.3			
2.	290 (4) 0.1	290 (4) 0.3			
3.	598 (4) 0.1	585 (4) 6.0	593 (4) 0.3	648 (4)	585 (4) 0.1
4.	955 (5) 0.1	949 (5) 1.5	954 (5) 1.4	949 (5)	964 (5) 0.1
5.	963 (5) 0.2	875 (4) 8.0	888 (4) 1.7	1017 (5)	
6.	1427 (8) 0.2	1414 (8) 2.4	1406 (8) 2.2	1427 (8)	
7.	839 (5) 0.2	810 (4) 2.4	812 (4) 1.7	850 (5)	843 (5) 0.1
8.	585 (6) 0.6	556 (5) 6.0	551 (5) 0.4		606 (5) 0.1
9.	900 (10) 1.3	876 (10) 12.0	863 (10) 0.9		907 (10) 0.1
10.	887 (8) 2.5	863 (8) 30.0	851 (8) 1.3		954 (8) 0.2

* IBM 7090
** KDF 9 for problems 3,4,5,6,7
1906A " " 8,9,10
*** IBM 7090
**** IBM 360/50 (time-shared)

set of routes, which are generated concurrently by the application of simple rules, and investigate the improvements which follow the reallocation of customers between routes and the resequencing of customers within routes.

Table 1 contrasts the total route distances and numbers of vehicles produced for each of ten test problems[6] by the various methods. The computing times are included where these were reported. Wren and Holliday succeed in producing routes with the lowest aggregate distance and numbers of vehicles. However problem 10 with 100 customers requires 0.9 min execution time (ICL 1906A) and computation times increase rapidly with the size of the problem (N.B. Sub-routine "pair" was not used in problem 10, saving significant computer time). It would therefore be of real practical importance to search for new methods which require less execution time.

The sequential savings based approaches are very much faster than any of the concurrent route building algorithms, although the total route distances are much longer on average. This fact suggests the development of sequential savings based methods incorporating refining heuristics in the spirit of the Wren and Holliday approach. It also concentrates attention upon modified forms of savings criterion due to Gaskell[4] and later tested by Webb[5] and McDonald.[9]

The other reason for developing the approach advocated above is the freedom with which vehicles may be scheduled as the routes are planned. Thus the possibilities of several out and return journeys by the same vehicle may be actively pursued, and additional factors such as planned maintainance for vehicles may be included with relative ease. Vehicle scheduling is a misnomer when applied to concurrent route building procedures, which must then be followed by procedures to allocate routes to vehicles (actually a one-dimensional "cutting-stock problem"). Of course in the absence of a mileage constraint on distance driven the vehicle scheduling activity becomes synonomous with route planning since, in principle, only one vehicle "shift" is required. The vehicle numbers given in Table 1 are thus the "journeys" made from the depot in the case of problems 1,2,6,8,9,10 where the driven distance is unconstrained.

A GENERALISED SAVINGS CRITERION

The Clarke and Wright savings function $S(P, Q)$ which results from linking two customers P and Q, a distance d_{PQ} apart and at distances of d_{OP} and d_{OQ} from the depot O, is given by

$$S(P, Q) = d_{OP} + d_{OQ} - d_{PQ} \tag{1}$$

But if a customer C is included on a route between customers A and B the generalised savings $SAV_C(A, B)$ which would result is given by

$$SAV_C(A, B) = 2\,d_{OC} + (d_{AB} - d_{AC} - d_{BC}) \tag{2}$$

The bracketed term is best interpreted as a negative "strain" associated with diverting the route to C between A and B where "strain" is defined as

$$ST_C(A, B) = d_{AC} + d_{BC} - d_{AB} \tag{3}$$

thus

$$SAV_C(A, B) = 2\,d_{OC} - ST_C(A, B) \tag{4}$$

Notice that if customer P is adjacent to the depot then $SAV_C(P, O) = S(C, P)$. But customer C is best included in the route between customers I and J where

$$ST_C(I, J) = \underset{\substack{\text{All adjacent} \\ \text{customers } A, B}}{\text{MINIMUM}} \{ST_C(A, B)\} \tag{5}$$

Thus $SAV_C(I,J) \geqslant S(C,P) \geqslant O$ by definition, and the savings rationale is preserved.

There is a choice of customer C to introduce into the route, and customer K is to be preferred when neither distance nor capacity constraints are violated

$$SAV_K(L, M) = \underset{\substack{\text{s.t. } C \text{ is load feasible, and} \\ ST_C(I, J) \text{ is distance feasible}}}{\text{MAXIMUM}} \{SAV_C(I, J)\} \tag{6}$$

Gaskell's π criterion is a modified savings function $MS(P, Q)$ given by

$$MS(P, Q) = S(P, Q) - \pi d_{PQ} \tag{1A}$$

Positive values of π lay additional stress upon the proximity of customers P and Q than is implied by criterion 1 where $\pi = 0$. This encourages the formation of routes which tend to be more "radial" than "circumferential". But (1A) is no longer necessarily non-negative, and indeed the savings rationale becomes blurred in favour of an intuitive approach. Webb[10] has tested criteria such as linking customers in order of proximity to neighbours, and other proximity heuristics employ the combined distances to the two nearest neighbours,[11] a minimum strain criterion,[8,12] and even the distances of customers from the depot.[10] These are mostly special cases of the following parameterised criterion of equation 2 above.

$$MSAV_C(A, B) = \lambda d_{OC} + \mu d_{AB} - d_{AC} - d_{BC} \tag{2A}$$

or

$$MSAV_C(A, B) = \lambda d_{OC} - MST_C(A, B) \tag{3A}$$

where

$$MST_C(A, B) = d_{AC} + d_{BC} - \mu d_{AB} \tag{4A}$$

The most advantageous position to introduce customer C is given by

$$MST_C(I, J) = \underset{\substack{\text{All adjacent customers } A, B \\ \text{where } ST_C(A, B) \text{ is distance} \\ \text{feasible.}}}{\text{MINIMUM}} \{MST_C(A, B)\} \tag{5A}$$

Customer K is to be preferred for inclusion in the route when

$$MSAV_K(L, M) = \underset{\text{Load feasible customers } C}{\text{MAXIMUM}} \{MSAV_C(I, J) \tag{6A}$$

Values of parameters λ and μ which are associated with the route building criteria named above are given in Table 2. In the case where customer C is introduced between depot O and customer P then equation (2A) becomes $MSAV_C(P, O) = (\lambda - 1)\, d_{OC} + \mu d_{OP} - d_{PC}$. So that if $\mu = \lambda - 1$ this criterion provides a ranking identical to that given by Gaskell's π criterion of (1A) for π numerically equal to $(\lambda - 1)^{-1} - 1$ for λ in the range $1 \leqslant \lambda \leqslant 2$.

Criterion (1A) is sufficient to identify which pair of customers will initiate a new route provided that $\mu = \lambda - 1$. Otherwise $MS(P, Q) \neq MS(Q, P)$ and some other criterion is to be preferred to initiate the route, e.g. the customer

TABLE 2. SPECIAL CASES OF THE GENERALISED SAVINGS CRITERION

$\lambda = 2$	$\mu = 1$	Generalised Clarke and Wright criterion
$\lambda = 0$	$\mu = 1$	Minimum 'strain' criterion
$1 \leq \lambda \leq 2$	$\mu = \lambda - 1$	Generalised Gaskell π criterion
$\lambda \to \infty$	μ finite	Ranking in order of distance from depot
$\lambda = 0$	$\mu = 0$	Proximity to two nearest neighbours ranking

furthest from the depot, or having the largest delivery. Once a route has been opened, further customer additions follow using criterion (6A).

A NEW SEQUENTIAL ROUTE BUILDING ALGORITHM

The sequential route building algorithm described here may be thought of in terms of a repeating sequence of three steps. In the first step one determines the most advantageous placing of each customer who has not yet received delivery on the emerging route, using criterion (5A). Criterion (6A) is employed in the second step to identify the next customer to be placed on the emerging route. In step three the possible resequencing of customers on the emerging route is explored using 2-optimal methods. Consequently this procedure is more general than those previously described in the literature. For example, all the savings based algorithms repeat step two alone, since it is implicitly assumed that additional customers are only included between the depot and the first or last customer of a route. Again, whereas other authors have applied resequencing techniques[6,8,9] they have not embedded them into their route building algorithms but reserve them until the routes are completed.

The key to the efficient implementation of our approach lies in the procedure for updating the modified strains of customers who have not yet received delivery. It is not necessary to apply criterion (5A) afresh each time the route is extended. If customer C has a minimum modified strain with respect to customers I, J where $ST_C(I, J)$ remains distance feasible, then it is only necessary to check the modified strains $MST_C(R, U)$ and $MST_C(R, V)$ where R has been included between U and V at the previous stage. If $MST_C(I, J)$ is no longer distance feasible, or if the last customer R was included in the route between I and J, it is only necessary to apply criterion (5A) afresh if customer C is load feasible with respect to the emerging route. The test for 2-optimality may be effectively accomplished since on the inclusion of a customer R, the set of links leaving R embedded need not be considered candidates for exchange in the first instance, as this possibility has previously been considered and rejected. If resequencing does occur then (5A) must be applied afresh for each load feasible customer C.

It should be clear that as λ grows then (6A) tends to favour the production of routes which become circumferential, and as μ grows the presence of long links between successive customers is discouraged by (5A). if μ is numerically equal to $\lambda - 1$ a scheme due to Webb[2] is employed for quickly identifying the maximum modified savings without complete elaboration of the possibilities. Otherwise the customer furthest from the depot is used to initiate the first journey made by each vehicle. The second and subsequent journeys are generated by a minimum strain criterion (5), since it is likely that the driving distance remaining within the legal restriction is the binding constraint.

When all the customers have been placed on journeys from the depot a "refine" procedure is activated. A customer will be transferred from one route to another if the strain is thereby reduced, and the vehicle and route constraints are not violated. Such transfers typically correct a basic pathology of sequential route building algorithms. The last (few) customer(s) on each journey may have been included despite the possibly large associated strain(s) simply because a vehicle load or distance constraint had not yet been met, and this tends to result, in turn, in high vehicle utilisation at the expense of lengthened route distances. When no further transfers are possible the existing strains are set to infinity so that an attempt can be made to redistribute the customers for the lightest laden vehicle among the remaining vehicles. If a reduction in the number of vehicles is thereby achieved the "refine" procedure is again activated afresh (the 2-optimality condition is maintained throughout). This approach may be likened to procedures "inspect", "single" and "delete" of Wren and Hollidays' algorithm.

Table 3 gives the aggregate distances, and numbers of vehicles required, for problems 1 through 10 collated by Eilon and Christofides for each of five groups of parameter combinations (which subsume nine parameter combinations altogether). It can be seen from the table that the aggregate distance (and vehicle numbers) for the most favourable parameter values for the individual problems at 7443 (54) compares most favourably with the Clarke and Wright figures of 7563 (57), and are only marginally worse than those of 7332 (54) obtained by Eilon and Christofides or the 7322 (54) obtained by Wren and Holliday where it has been assumed that results of 114 (2) and 290 (4) apply for problems 1 and 2. However the total computing time for the nine parameter combinations applied to the batch of ten problems was only *12.5 sec.* plus 2.1 sec. compile time. This is an order of magnitude faster than the computing times reported by other authors (see Table 1). Some of the improvement is doubtless due to the speed of the CDC 7600 computer employed, but exact comparisons are rendered difficult by the idiosyncratic nature of individual machines—execution times for the CDC 7600 depend appreciably upon the amount of output that is required. However, most of the improvement is directly attributable to the sequential route building approach. The aggregate route distance (and vehicle numbers) also show much improvement over earlier sequential route building methods; Figures of 4667 (25) may be contrasted with 4891 (27) for the Clarke and Wright criterion (cases 3,4,5,6,7 only), and figures of 4722 (36) may be contrasted with 4859 (37) obtained by Yellow also using the Clarke and Wright criterion, with segmentation of the delivery area (cases 3,4,7,8,9,10 only).

TABLE 3. RESULTS OF NEW SEQUENTIAL ROUTE-BUILDING ALGORITHM; EILON AND CHRISTOFIDES CASES 1–10

	Route distance (Vehicle numbers)					
Problem	Proximity ranking $\lambda = 0$ $\mu = 0$	Minimum strain criterion $\lambda = 0$ $\mu = 1$	Generalised C+W Savings criterion $\lambda = 2$ $\mu = 1$	Generalised Gaskell criterion *	Augmented minimum strain criterion **	Best result
1.	119 (2)	121 (2)	119 (2)	119 (2)	119 (2)	119 (2)
2.	290 (4)	294 (4)	290 (4)	290 (4)	290 (4)	290 (4)
3.	694 (4)	633 (4)	642 (4)	598 (4)	594 (4)	594 (4)
4.	994 (5)	994 (5)	979 (5)	949 (5)	949 (5)	949 (5)
5.	937 (5)	951 (5)	923 (4)	890 (4)	891 (4)	890 (4)
6.	1422 (8)	1465 (8)	1422 (9)	1422 (8)	1422 (8)	1422 (8)
7.	836 (5)	856 (5)	834 (4)	892 (4)	834 (4)	812 (4)
8.	672 (5)	603 (5)	597 (5)	590 (5)	575 (5)	575 (5)
9.	1065(10)	1017(10)	934(10)	910(10)	919(10)	910(10)
10.	948 (8)	882 (8)	955 (8)	926 (8)	882 (8)	882 (8)
Column totals	7977(56)	7816(56)	7695(54)	7506(54)	7475(54)	7443(54)

* Best of 4 runs $\lambda = 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2$; $\mu = \lambda - 1$

** Best of 5 runs $\lambda = 0, \frac{1}{2}, 1, 1\frac{1}{2}, 2$; $\mu = 1$

When judged upon the basis of the ten test cases, an augmented minimum strain criterion where $\lambda \geqslant 0$ and $\mu = 1$ appears to offer the best compromise between route quality and execution time. This conclusion is also supported by limited computational experience with much larger problems. The execution time is shorter because the distance feasibility checks become simpler due to $MST_C(A, B) = ST_C(A, B)$, and less frequent use has to be made of the 2-optimality routine.

The customer data for cases 8–10 were aggregated to yield a 225 customer problem, and the depot location was chosen to correspond to case 10. Table 4 shows the sensitivity of the total route distance and vehicle numbers to changes in the vehicle capacity and permissible driven distance. It can be seen that the total mileage is relatively insensitive to variations in the driven distance constraint within the range 100–150 units, whereas the fleet size is reduced quite appreciably. In contrast, both the total mileage and fleet size are significantly reduced with increases in vehicle capacity in the range 180–220 units. This behaviour may be attributed to the high proportion of vehicles making multiple journeys. The total computing time for this exercise was 160 sec. on the CDC 7600.

TABLE 4. RESULTS OF THE NEW SEQUENTIAL ROUTE-BUILDING METHOD. CUSTOMER DATA[6] FOR CASES 8–10 HAVE BEEN COMBINED; DEPOT LOCATION AS FOR CASE 10

Route distance (Vehicle numbers)

		Driven distance constraint		
		100	125	150
Vehicle Capacity	180	1775 (19)	1742 (16)	1742 (13)
	200	1656 (18)	1625 (15)	1608 (12)
	220	1572 (17)	1523 (14)	1553 (12)

The best of 5 runs of an augmented minimum strain criterion

where $\lambda = 0, \frac{1}{2}, 1, 1\frac{1}{2}, 2$

$\mu = 1$

DISCUSSION

The algorithm described in this paper may be readily modified to suit specific purposes. For example, an early version of the program was written in BASIC, and a run of case 10 with 100 customers required only 10 K bytes of storage (this program did not include the "refine" procedure). Again the program can be extended to schedule delivery vehicles from several depots. There is also scope for substituting a 3-optimal routine for the present 2-optimal routine, and upgrading the "refine" procedure.

The general approach is therefore most flexible and has advantages where true vehicle scheduling, as opposed to mere route planning, is essential. Thus (a) The possibility of multiple journeys by the same vehicle is actively pursued (b) Constraints upon vehicle capacity and the suitability of delivery to certain customers may be specified separately for each vehicle (and thus driver). (c) Constraints upon driven distance, number of journeys, and delivery territory may be easily specified for each vehicle (and thus driver). (d) Priority customers may be assigned to given vehicles. (e) It is not necessary to pre-assign a delivery to a particular day if several acceptable alternatives are available because, upon approaching the deadline it becomes a priority and is treated as in (d) above. (f) Planned vehicle maintainance may be incorporated readily. (g) The algorithm is sufficiently fast to enable sensitivity analyses with respect to fleet requirements to be completed prior to implementing changes in vehicle capacity, or driven distance.

ACKNOWLEDGEMENTS

The authors would like to record their appreciation of one of the referees for his careful reading of the original paper and his several helpful comments.

REFERENCES

[1]G. CLARKE and J. W. WRIGHT (1964) Scheduling of vehicles from a central depot to a number of delivery points. *Ops Res.* **12,** 568.

[2]M. H. J. Webb (1965) Vehicle routing and scheduling by computer. O.R.S. conference.
[3]P. C. Yellow (1970) A computational modification of the savings method of vehicle scheduling. *Opl Res. Q.* **21,** 281.
[4]T. J. Gaskell (1967) Bases for vehicle fleet scheduling. *Opl Res. Q.* **18,** 281.
[5]M. H. J. Webb (1972) Relative performance of some sequential methods of planning multiple delivery journeys. *Opl Res. Q.* **23,** 361.
[6]S. Eilon and N. Christofides (1969) An algorithm for the vehicle dispatching problem. *Opl Res. Q.* **20,** 309.
[7]S. Lin (1965) Computer solutions to the travelling salesman problem. *Bell System Tech. J.* **44,** 2245.
[8]A. Wren and A. Holliday (1972) Computer scheduling of vehicles from one or more depots to a number of delivery points. *Opl Res. Q.* **23,** 333.
[9]J. J. McDonald (1972) Vehicle scheduling—a case study. *Opl Res. Q.* **23,** 433.
[10]M. H. J. Webb (1972) *Transporting Goods by Road.* Weidenfeld & Nicolson, London.
[11]M. H. J. Webb (1971) Some methods of producing approximate solutions to travelling salesman problems with hundreds or thousands of cities. *Opl Res. Q.* **22,** 49.
[12]*Deliver: A New Suite of Programs for Vehicle Scheduling.* Sanaco Computer Services, Birmingham.