Chapter 561

Conover Test of Variances (Simulation)

Introduction

This procedure analyzes the power and significance level of the Conover homogeneity test. This test is used to test whether two or more population variances are equal. For each scenario that is set up, two simulations are run. One simulation estimates the significance level and the other estimates the power.

Technical Details

Computer simulation allows us to estimate the power and significance level that is actually achieved by a test procedure in situations that are not mathematically tractable. Computer simulation was once limited to mainframe computers. But, in recent years, as computer speeds have increased, simulation studies can be completed on desktop and laptop computers in a reasonable period of time.

The steps to a simulation study are

1. Specify how the test is carried out. This includes indicating how the test statistic is calculated and how the significance level is specified.

2. Generate random samples from the distributions specified by the <u>alternative</u> hypothesis. Calculate the test statistics from the simulated data and determine if the null hypothesis is accepted or rejected. Tabulate the number of rejections and use this to calculate the test's power.

3. Generate random samples from the distributions specified by the <u>null</u> hypothesis. Calculate each test statistic from the simulated data and determine if the null hypothesis is accepted or rejected. Tabulate the number of rejections and use this to calculate the test's significance level.

4. Repeat steps 2 and 3 several thousand times, tabulating the number of times the simulated data leads to a rejection of the null hypothesis. The power is the proportion of simulated samples in step 2 that lead to rejection. The significance level is the proportion of simulated samples in step 3 that lead to rejection.

Generating Random Distributions

Two methods are available in **PASS** to simulate random samples. The first method generates the random variates directly, one value at a time. The second method generates a large pool (over 10,000) of random values and then draws the random numbers from this pool. This second method can cut the running time of the simulation by 70%.

The second method begins by generating a large pool of random numbers from the specified distributions. Each of these pools is evaluated to determine if its mean is within a small relative tolerance (0.0001) of the target mean. If the actual mean is not within the tolerance of the target mean, individual members of the population are replaced with new random numbers if the new random number moves the mean towards its target. Only a few hundred such swaps are required to bring the actual mean to within tolerance of the target mean. This population is then sampled with replacement using the uniform distribution. We have found that this method works well as long as the size of the pool is at least the maximum of twice the number of simulated samples desired and 10,000.

Conover Test

Conover (1999) present a nonparametric test of homogeneity (equal variance) based on ranks. The test does not assume that all populations are normally distributed and is recommended when the normality assumption is not viable.

Suppose g groups each have a normal distribution with possibly different means and standard deviations $\sigma_1, \sigma_2, ..., \sigma_g$. Let $n_1, n_2, ..., n_g$ denote the number of subjects in each group, Y_{ki} denote response values, and N denote the total sample size of all groups. The test assumes that the data are obtained by taking a simple random sample from each of the g populations.

The formula for the calculation of Conover test is

$$T = \frac{1}{D^2}\left[\sum_{k=1}^{g} \frac{S_k^2}{n_k} - N\bar{S}^2\right]$$

where

$$Z_{ki} = |Y_{ki} - \bar{Y}_k|$$

$$R_{ki} = \text{Rank}(Z_{ki})$$

$$S_k = \sum_{i=1}^{n_k} R_{ki}^2$$

$$\bar{S} = \frac{1}{N}\sum_{k=1}^{g} S_k$$

$$D^2 = \frac{1}{N-1}\left[\sum_{k=1}^{g}\sum_{i=1}^{n_k} R_{ki}^4 - N\bar{S}^2\right]$$

If the assumptions are met, the distribution of this test statistic follows approximately the *Chi-squared* distribution with degrees of freedom g - 1.

Example 1 – Power for a Range of Standard Deviations

A one-way design is being employed to compare the means of four groups using an *F* test. Past experimentation has shown that the data within each group can reasonably be assumed to be normally distributed. The researcher would like to use the Conover test to evaluate the assumption that the variances of all four groups are equal (homogeneity). Previous studies have shown that the standard deviation within a group is about 5. The researcher wants to investigate the power when the standard deviation of the second group is 6, 8, or 10. Treatment means of 10, 20, 10, and 10 are anticipated. The researcher wants to compute the power for group sample sizes of 10, 20, 30, 40, and 50. The group sample sizes are equal in a particular scenario. The value of alpha is 0.05.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Power**
Alpha..**0.05**
Simulations **5000**
Random Seed..**5905034** (for Reproducibility)
Number of Groups **4**
Group Allocation ....**Equal (n = n1 = n2 = ···)**
n (Group Size) **10 20 30 40 50**
Group 1 Power Distribution............................**Normal(10 S1)**
Group 2 Power Distribution............................**Normal(20 S2)**
Group 3 Power Distribution............................**Normal(10 S1)**
Group 4 Power Distribution............................**Normal(10 S1)**
Parameter 2 Name **S1**
Parameter 2 Value(s)....................................**5**
Parameter 3 Name **S2**
Parameter 3 Value(s)....................................**7 8 9**
Specify Alpha Distributions ....**All equal to the group 1 distribution of the power simulation**

Output

Click the Calculate button to perform the calculations and generate the following output.

Simulation Summary and Numeric Results

Simulation Summary

Group	Power (H1) Simulation Distribution	Alpha (H0) Simulation Distribution
1	Normal(10 S1)	Normal(10 S1)
2	Normal(20 S2)	Normal(10 S1)
3	Normal(10 S1)	Normal(10 S1)
4	Normal(10 S1)	Normal(10 S1)

Number of Groups	4
Random Number Pool Size	10000
Number of Simulations	5000
Random Seed	5905034 (User-Entered)
Run Time	16.68 seconds

Numeric Results

Solve For: Power

Row	Power	Mean Group Sample Size n	Total Sample Size N	Target Alpha	Actual Alpha	Std Dev of H1 σ's	Mean of H1 σ's	S1	S2
1	0.140	10	40	0.05	0.058	0.9	5.5	5	7
2	0.241	20	80	0.05	0.058	0.9	5.5	5	7
3	0.324	30	120	0.05	0.056	0.9	5.5	5	7
4	0.443	40	160	0.05	0.056	0.9	5.5	5	7
5	0.513	50	200	0.05	0.051	0.9	5.5	5	7
6	0.220	10	40	0.05	0.062	1.3	5.7	5	8
7	0.410	20	80	0.05	0.057	1.3	5.7	5	8
8	0.586	30	120	0.05	0.052	1.3	5.8	5	8
9	0.693	40	160	0.05	0.055	1.3	5.7	5	8
10	0.813	50	200	0.05	0.056	1.3	5.7	5	8
11	0.317	10	40	0.05	0.064	1.8	6.0	5	9
12	0.569	20	80	0.05	0.055	1.7	6.0	5	9
13	0.802	30	120	0.05	0.058	1.8	6.0	5	9
14	0.894	40	160	0.05	0.050	1.7	6.0	5	9
15	0.954	50	200	0.05	0.054	1.7	6.0	5	9

Row	Identifies this line of the report. It will be used cross-reference this line in other reports.
σ	The standard deviation within a group. There is a (possibly different) σ for each group.
H0	The null hypothesis that all σ's are equal.
H1	The alternative hypothesis that at least one σ is different from the others.
Power	The probability of rejecting H0 when it is false. This is the actual value calculated by the power simulation.
n	The Mean Group Sample Size. The average of the individual group sample sizes.
N	The total sample size found by summing all group sample sizes.
Target Alpha	The planned probability of rejecting a true H0. This is the planned Type-1 error rate.
Actual Alpha	The alpha achieved by the test as calculated by the alpha simulation. Note that the alpha simulation is separate from the power simulation.
Std Dev of H1 σ's	The standard deviation of the group σ's used in the power simulation. This measures the magnitude of the difference among the σ's.
Mean of H1 σ's	The mean of the group σ's used in the power simulation. This measures the magnitude of the σ's.

Summary Statements
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A one-way design with 4 groups has sample sizes of 10, 10, 10, and 10. The null hypothesis is that the within-group standard deviations are equal (homogeneity) and the alternative hypothesis is that at least one within-group standard deviation is different from the others (heterogeneity). The total sample of 40 subjects achieves a power of 0.14 using the Conover Variance Test with a target significance level of 0.05 and an actual significance level of 0.058. The standard deviation of the within-group standard deviations is 0.9. The average within-group standard deviation assuming the alternative (power) distributions is 5.5. These results are based on 5000 simulations of the null distributions: Normal(10 S1); Normal(10 S1); Normal(10 S1); and Normal(10 S1) and of the alternative distributions: Normal(10 S1); Normal(20 S2); Normal(10 S1); and Normal(10 S1). Other parameters used in the simulation were: S1 = 5, and S2 = 7.
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References
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Conover, W.J. 1999. Practical Nonparametric Statistics. 3rd Edition. John Wiley & Sons. New York.
Devroye, Luc. 1986. Non-Uniform Random Variate Generation. Springer-Verlag. New York.
Gibbons, J.D. and Chakraborti, S. 2011. Nonparametric Statistical Inference, 5th Edition. CRC Press.
Milliken, G.A and Johnson, D.E. 1984. Analysis of Messy Data: Volume 1. Van Nostrand Reinhold. New York.
Winer, B.J., Brown, D.R, and Michels, K.M. 1991. Statistical Principles in Experimental Design. 3rd Edition.
   McGraw-Hill. New York.
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These reports show the output for this run. We will annotate the Numeric Results report.

Power

This is the probability of rejecting a false null hypothesis. This value is estimated by the power simulation. The Power and Alpha Confidence Interval report displayed next will provide estimates of the precision of these power values.

Mean Group Size n

This is the average of the group sample sizes.

Total Sample Size N

This is the total sample size of the study.

Target Alpha

The target value of alpha: the probability of rejecting a true null hypothesis. This is often called the significance level.

Actual Alpha

This is the value of alpha estimated by the alpha simulation. It should be compared with the Target Alpha. The Power and Alpha Confidence Interval report displayed next will provide estimates of the precision of these Actual Alpha values.

Std Dev of H1 σ's

This is the standard deviation of the hypothesized within-group standard deviations of the power (H1) simulation distributions. Under the H0, this value is zero. So this value represents the magnitude of the difference among the standard deviations that is being tested.

Mean of H1 σ's

This is the mean of the group standard deviations calculated from the power simulation distributions.

S1

These are the values entered for S1, the group standard deviation in the power simulation.

S2

These are the values entered for S2, the group two standard deviation of the power simulation.

Power and Alpha Confidence Intervals Report

Power and Alpha Confidence Intervals from Simulations

Row	Total Sample Size N	Power	Lower Limit of 95% C.I. of Power	Upper Limit of 95% C.I. of Power	Target Alpha	Actual Alpha	Lower Limit of 95% C.I. of Alpha	Upper Limit of 95% C.I. of Alpha	S1	S2
1	40	0.140	0.131	0.150	0.05	0.058	0.051	0.064	5	7
2	80	0.241	0.229	0.253	0.05	0.058	0.051	0.064	5	7
3	120	0.324	0.311	0.337	0.05	0.056	0.050	0.062	5	7
4	160	0.443	0.429	0.457	0.05	0.056	0.049	0.062	5	7
5	200	0.513	0.499	0.526	0.05	0.051	0.045	0.057	5	7
6	40	0.220	0.209	0.231	0.05	0.062	0.056	0.069	5	8
7	80	0.410	0.396	0.424	0.05	0.057	0.050	0.063	5	8
8	120	0.586	0.572	0.600	0.05	0.052	0.046	0.058	5	8
9	160	0.693	0.680	0.706	0.05	0.055	0.049	0.062	5	8
10	200	0.813	0.802	0.824	0.05	0.056	0.050	0.063	5	8
11	40	0.317	0.304	0.329	0.05	0.064	0.057	0.071	5	9
12	80	0.569	0.555	0.582	0.05	0.055	0.049	0.061	5	9
13	120	0.802	0.791	0.813	0.05	0.058	0.052	0.064	5	9
14	160	0.894	0.885	0.902	0.05	0.050	0.044	0.056	5	9
15	200	0.954	0.949	0.960	0.05	0.054	0.048	0.060	5	9

N	The total sample size found by summing all group sample sizes.
Power	The probability of rejecting H0 when it is false. This is the actual value calculated by the power simulation.
Lower and Upper Limits of a 95% C.I. for Power	The limits of an exact, 95% confidence interval for power based on the binomial distribution. They are calculated from the power simulation.
Target Alpha	The desired probability of rejecting a true null hypothesis at which the tests were run.
Actual Alpha	The alpha achieved by the test as calculated by the alpha simulation.
Lower and Upper Limits of a 95% C.I. for Alpha	The limits of an exact, 95% confidence interval for alpha based on the binomial distribution. They are calculated from the alpha simulation.

Total Sample Size N

This is the total sample size of the study.

Lower/Upper Limit of 95% C.I. for Power

These are the limits of an exact, 95% confidence interval for power using the power simulation. The confidence interval is based on the binomial distribution. The width of this confidence interval is directly related to the number of simulations that were used.

Lower/Upper Limit of 95% C.I. for Alpha

These are the limits of an exact, 95% confidence interval for alpha using the alpha simulation. The confidence interval is based on the binomial distribution. The width of this confidence interval is directly related to the number of simulations that were used. Since the target alpha is 0.05, 0.05 should be within these limits.

Detailed Results Reports

Details for Row 1 of the Numeric Results

Group	ni	Pct of N	H0 σ	H1 σ	H0 μ	H1 μ		
1	10	25	5	5.0	10	10.0		
2	10	25	5	7.0	10	20.0		
3	10	25	5	5.0	10	10.0		
4	10	25	5	5.0	10	10.0		

Group	N		Std Dev of H0 σ's	Std Dev of H1 σ's	Std Dev of H0 μ's	Std Dev of H1 μ's	Mean of H0 σ's	Mean of H1 σ's
All	40	100	0	0.9	0	4.3	5	5.5

(More Reports Follow)

These reports show the details of each scenario.

Row (in Title)

This is the row number of the Numeric Results report about which this report gives the details.

Group

This is the number of the group shown on this line.

ni

This is the sample size of each group. This column is especially useful when the sample sizes are unequal.

Percent of N

This is the percentage of the total sample that is allocated to each group.

H0 σ and H1 σ

These are the standard deviations that were obtained by the alpha and power simulations, respectively. Note that they often are not exactly equal to what was specified because of the error introduced by simulation.

H0 μ and H1 μ

These are the means that were used in the alpha and power simulations, respectively.

Std Dev of H0 (and H1) σ's

These are the standard deviations of the within-group standard deviations that were obtained by the alpha and power simulations, respectively. Under H0, this value should be near zero. The H0 value lets you determine if your alpha simulation was correctly specified. The H1 value represents the magnitude of the effect size (when divided by an appropriate measure of the standard deviation).

Std Dev of H0 (and H1) μ's

These are the standard deviations of the within-group means that were obtained by the alpha and power simulations, respectively. These values are essentially ignored by the test, but they are provided for completeness.

Mean of H0 (and H1) σ's

These are the average of the individual group standard deviations that were obtained by the alpha and power simulations, respectively. They give an overall value for the variation in the design.

Dropout-Inflated Sample Size Report

Dropout-Inflated Sample Size

| Average Group Sample Size n	Group	Dropout Rate	Sample Size ni	Dropout-Inflated Enrollment Sample Size ni'	Expected Number of Dropouts Di
10	1 - 4	20%	10	13	3
	Total		40	52	12
20	1 - 4	20%	20	25	5
	Total		80	100	20
30	1 - 4	20%	30	38	8
	Total		120	152	32
40	1 - 4	20%	40	50	10
	Total		160	200	40
50	1 - 4	20%	50	63	13
	Total		200	252	52

n	The average group sample size.
Group	Lists the group numbers.
Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
ni	The evaluable sample size for each group at which power is computed (as entered by the user). If ni subjects are evaluated out of the ni' subjects that are enrolled in the study, the design will achieve the stated power.
ni'	The number of subjects that should be enrolled in each group in order to obtain ni evaluable subjects, based on the assumed dropout rate. ni' is calculated by inflating ni using the formula ni' = ni / (1 - DR), with ni' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
Di	The expected number of dropouts in each group. Di = ni' - ni.

Dropout Summary Statements
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Anticipating a 20% dropout rate, group sizes of 13, 13, 13, and 13 subjects should be enrolled to obtain final group sample sizes of 10, 10, 10, and 10 subjects.
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This report shows the sample sizes adjusted for dropout. In this example, dropout is assumed to be 20%. You can change the dropout rate on the Reports tab.

Plots Section

Plots
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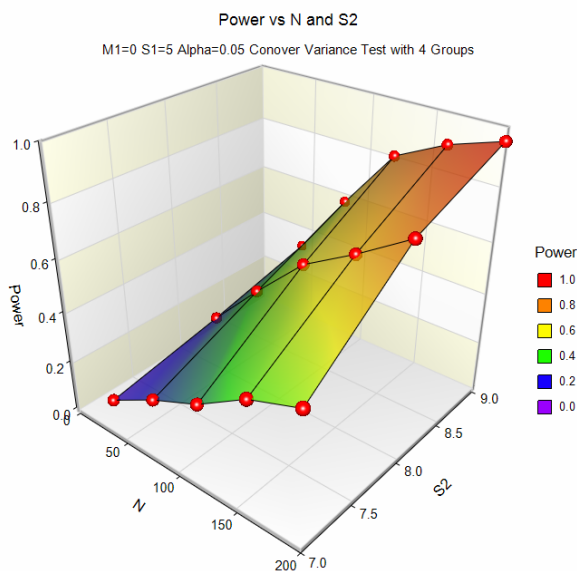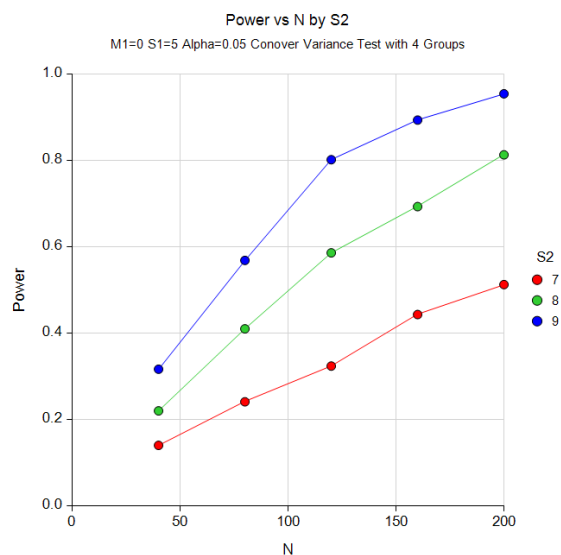
These plots give a visual presentation to the results in the Numeric Report. We can quickly see the impact on the power of increasing the standard deviation and the sample size.

Example 2 – Validation

To validate this procedure, we will compare its results to an example run in the *Tests for Two Variances* procedure. For this example, alpha was set to 0.05, the Scale was Standard Deviation, V1 was 1, and V2 was 2. The desired power was 0.8. The resulting sample size is N1 = N2 = 19.

For reproducibility, we'll use a random seed of 5730248.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Sample Size**
Power...**0.80**
Alpha..**0.05**
Simulations **5000**
Random Seed...**5730248** (for Reproducibility)
Number of Groups **2**
Group Allocation ....**Equal (n1 = n2 = ⋯)**
Group 1 Power Distribution............................**Normal(1 S1)**
Group 2 Power Distribution............................**Normal(1 S2)**
Parameter 2 Name **S1**
Parameter 2 Value(s).....................................**1**
Parameter 3 Name **S2**
Parameter 3 Value(s).....................................**2**
Specify Alpha Distributions ....**All equal to the group 1 distribution of the power simulation**

Output

Click the Calculate button to perform the calculations and generate the following output.

Simulation Summary

Group	Power (H1) Simulation Distribution	Alpha (H0) Simulation Distribution
1	Normal(1 S1)	Normal(1 S1)
2	Normal(1 S2)	Normal(1 S1)

Number of Groups	2
Random Number Pool Size	10000
Number of Simulations	5000
Random Seed	5730248 (User-Entered)
Run Time	3.42 seconds

Numeric Results
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Solve For: Sample Size
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Row	Target Power	Actual Power	Mean Group Sample Size n	Total Sample Size N	Target Alpha	Actual Alpha	Std Dev of H1 σ's	Mean of H1 σ's	S1	S2
1	0.8	0.818	25	50	0.05	0.049	0.5	1.5	1	2

Note that **PASS** calculates the sample size as 50 (25 per group). This is close to the analytic answer of 38 (19 per group) for the regular variance ratio test.