Introduzione alla computazione quantistica con QISKIT

Code link

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Quantum Research - CERN

CERN IT INNOVATION

QUANTUM
TECHNOLOGY
INITIATIVE

Programma

- Parte 1: Qiskit introduction
- Parte 2: QC for Quantum Machine Learning
- Parte 3: QC for Physics

Quantum Computing

[…] "Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical…" [1]

[1] Richard P. Feynman, Department of Physics, California Institute of Technology, International Journal of Theoretical Physics, Vol 21, Nos. 6/7, 1982

Classical Computation

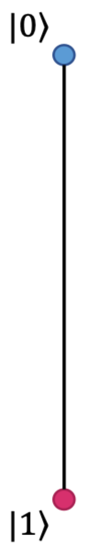
- **Based on classical binary logic**
- Reached incredibly peaks since late 40s
  - Many problems still can not be addressed adequately

Quantum Computation

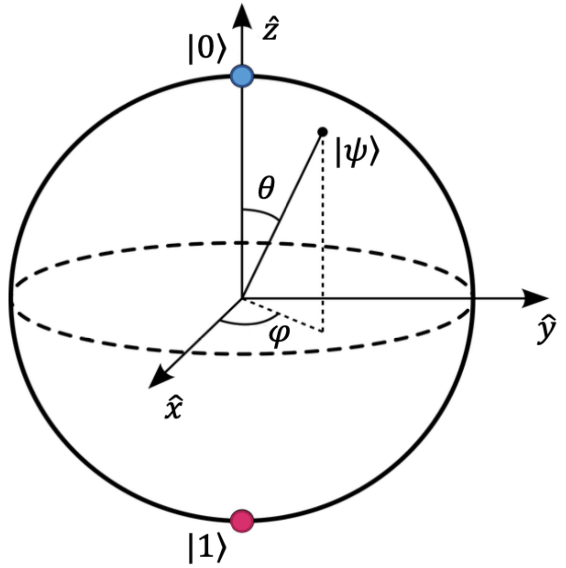
- **New frontier of computation**
- Started in early 80s
- First prototypal QC available since 2010s
  - Still in NISQ (Noisy Intermediate Scale Quantum) era

Quantum Information Theory

Unit of information



bit



qubit

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \left(\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle\right)e^{i\gamma}$$

where $\alpha, \beta \in \mathbb{C}$ and $\theta, \phi, \gamma \in \mathbb{R}$

Quantum logic gates

- Single qubit operations
  - **Hadamard gate:** creation of superposition $H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 \\ 1\end{pmatrix}$
  - **Pauli gates:** π rotations along main axes

$$\boldsymbol{\sigma_x} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \ \boldsymbol{\sigma_y} = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \ \boldsymbol{\sigma_z} = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

- Two-qubit operations

$$\boldsymbol{CNOT} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$$

$$\boldsymbol{C - \varphi} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & e^{-i\varphi}\end{pmatrix}$$

creation of entangleme

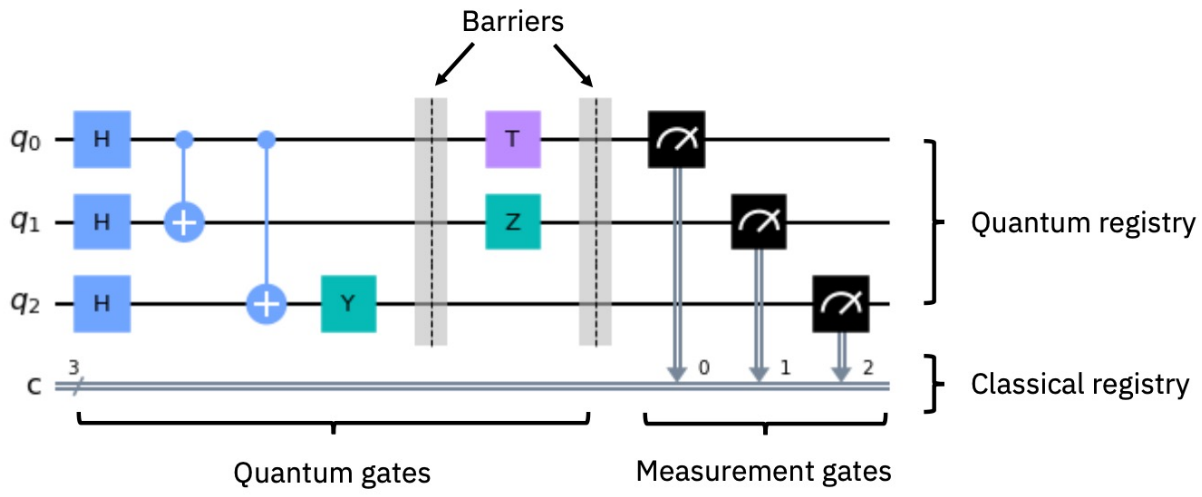
- Generic multi-qubit operations: decomposed in single-qubit and two-qubit gates

- **Universal gate sets**

Quantum Information Theory

Composing quantum gates: quantum circuits

- Set of actions to be performed to the selected qubits
  - qubits initialization
  - single-qubit gates, multi-qubit gates
  - measurements



Principles of quantum computation

- Quantum algorithm: set of quantum circuits performing certain task
  - Purely quantum, e.g. *Shor*
  - Hybrid classical-quantum, e.g. *VQE*

- Quantum Simulation: simulation of time evolution of quantum system
  - Analog Simulator
  - Digital Simulator: quantum logic gates, more flexible

Quantum Errors

- **Coherent errors**
  - Incorrect application of quantum gates
- **Incoherent errors**
  - Interaction with surrounding environment
  - Unavoidable

We are currently able to implement error mitigation techniques

Gates characterization

- **Search for systematic errors**
  - Parametrization of rotations using a set of small $(\varepsilon_x, \varepsilon_y, \varepsilon_z)$ tilt angles
  - State tomography of both single-qubit and two-qubit gates

Kraus decomposition

- **General and discrete approach**
- $f(p)$ is the effect of noise on the quantum state

$$f(p) = \sum_k \underbrace{E_k \rho E_k^\dagger}_{\text{Kraus operators}}$$

Lindblad master equation solving

- **Compute the continuum dynamics**
- Solving Lindblad master equation for ρ

$$\frac{d\hat{\rho}_s(t)}{dt} = \overbrace{-i\left[\hat{H}(t), \hat{\rho}_s(t)\right]}^{\text{coherent part}} + \underbrace{\mathcal{D}_{\rho_s}}_{\text{interaction with the environment: dissipator operator}}$$

Problems for a quantum computer

Modeling nature at the quantum level

Mathematics

Quantum chemistry,
Material science and
High energy physics

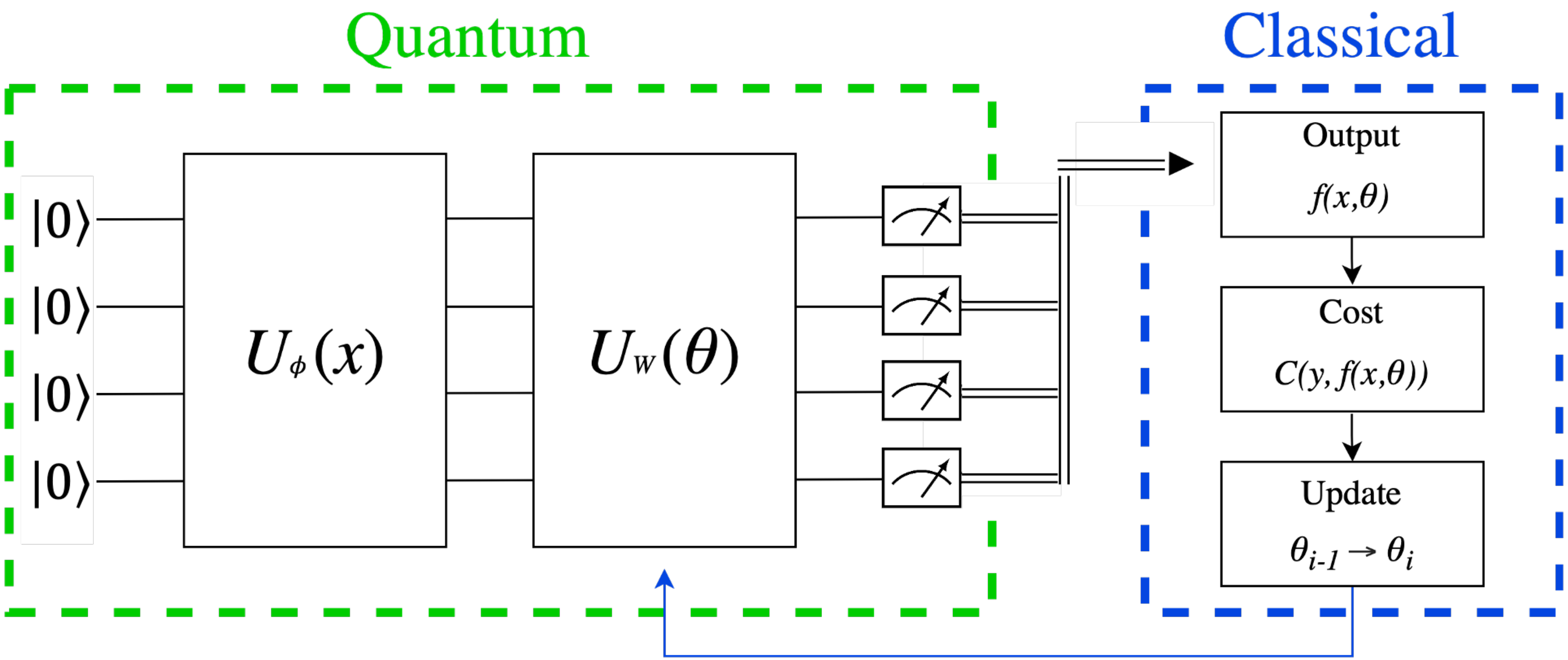
Algebraic algorithms
with provable speed-ups

- Shor’s Algorithm
- Grover’s Algorithm

Empirical heuristic
algorithms

- Optimization
- Machine learning

VARIATIONAL QUANTUM CIRCUIT (VQC)

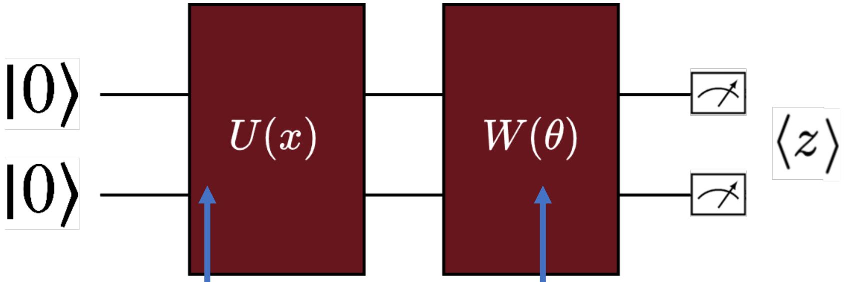


Based on a hybrid quantum-classical procedure

The iterative optimization of the parameters allows to design low-depth circuits, suitable to be run on NISQ devices

HYBRID QUANTUM-CLASSICAL APPROACH

Variational Quantum Circuit



Feature map

Parametrized circuit

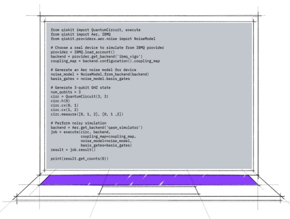
Measurement

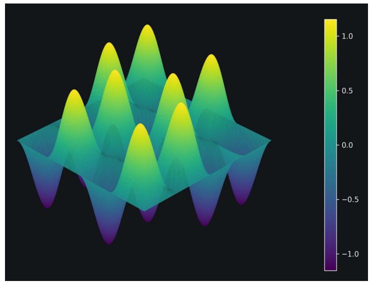
$$f(x,\theta) = \langle 0| U^\dagger(x) W^\dagger(\theta) Z W(\theta) U(x) |0\rangle$$

Parameter update

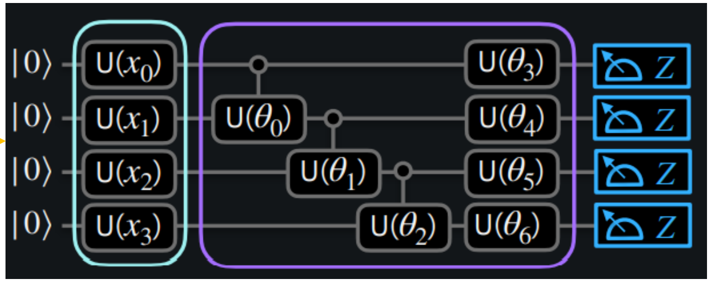
$\theta_i \rightarrow \theta_{i+1}$

Classical Optimizer



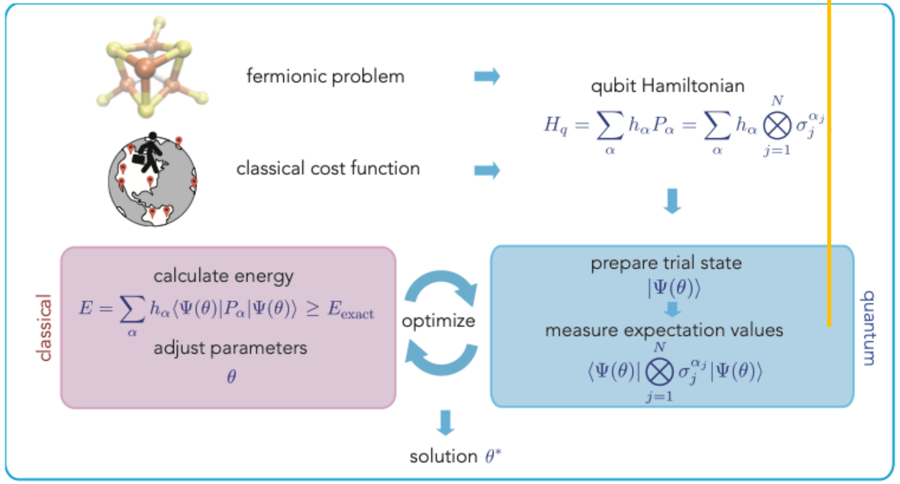


The VQE Algorithm



Find Ground state of a quantum system

Variational quantum eigensolver (**VQE**)[4] is aimed at finding the ground state energy E_g of a Hamiltonian H. So the cost function will contain the hamiltonian as operator in the cost function. According to the *Rayleigh-Ritz variatonal principle*, the cost is lower bounded to the E_g. In practice the Hamiltonian is represented as a linear combination of Pauli Operators $H = \sum_k c_k \sigma_k$ $(c_k \in \mathbb{R})$ so the cost function is obtained from linear combination of expectation values of σ_k. Since practical physical systems are generally described by sparse hamiltonian, the cost can be efficiently estimated on a quantum computer, with a computational cost grows poynomially with the system size[1].



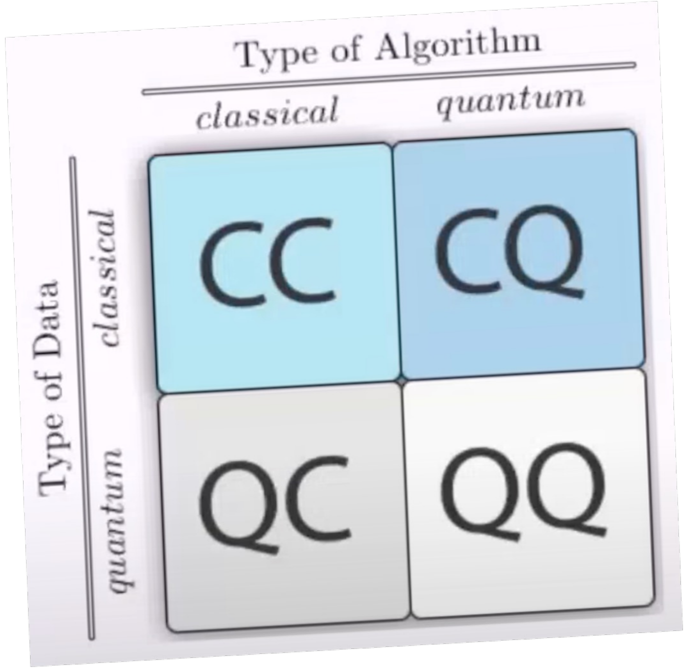
Variational quantum eigensolver method

$$E(\boldsymbol{\theta}) = \langle\phi(\theta)|H|\phi(\theta)\rangle = \sum_{ij}\langle\phi(\theta)|i\rangle\langle i|H|j\rangle\langle j|\phi(\theta)\rangle =$$

$$= \sum_i |\langle\phi(\theta)|i\rangle|^2 E_i = \sum_i |\langle\phi(\theta)|i\rangle|^2 (E_i - E_0) + E_0 \geq \boldsymbol{E_0}$$

$$\{|i\rangle\},\{|j\rangle\} = eigenvectors\ of\ H$$

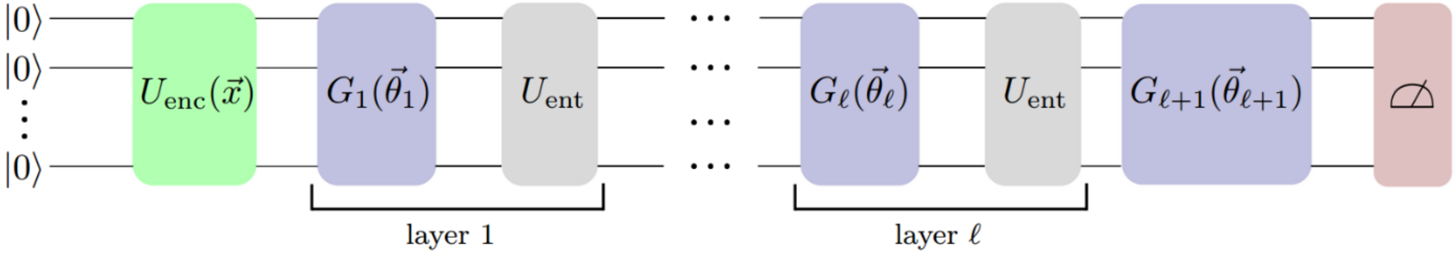
Hybrid Q-C Algorithms - QML



Noisy intermediate scale quantum devices

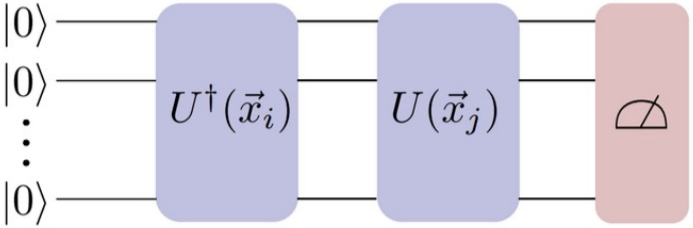
- Circuit width: limited number of qubits.
- Circuit depth: limited number of operations per qubit (small decoherence times).
- Hardware noise.

Variational algorithms - EXPLICIT



$\Rightarrow \langle \mathcal{O}_{\vec{x},\vec{\theta}} \rangle$

Kernel methods - IMPLICIT



$\Rightarrow K_{ij} = |\langle 0|U^\dagger(\vec{x}_i)U(\vec{x}_j)|0\rangle|^2$

Current hardware limitations: feature reduction needed for realistic datasets.

Thanks!

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