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Comparison of Local Grid Refinement Methods for MODFLOW

by Steffen Mehl[1], Mary C. Hill[2], and Stanley A. Leake[3]

Abstract

Many ground water modeling efforts use a finite-difference method to solve the ground water flow equation, and many of these models require a relatively fine-grid discretization to accurately represent the selected process in limited areas of interest. Use of a fine grid over the entire domain can be computationally prohibitive; using a variably spaced grid can lead to cells with a large aspect ratio and refinement in areas where detail is not needed. One solution is to use local-grid refinement (LGR) whereby the grid is only refined in the area of interest. This work reviews some LGR methods and identifies advantages and drawbacks in test cases using MODFLOW-2000. The first test case is two dimensional and heterogeneous; the second is three dimensional and includes interaction with a meandering river. Results include simulations using a uniform fine grid, a variably spaced grid, a traditional method of LGR without feedback, and a new shared node method with feedback. Discrepancies from the solution obtained with the uniform fine grid are investigated. For the models tested, the traditional one-way coupled approaches produced discrepancies in head up to 6.8% and discrepancies in cell-to-cell fluxes up to 7.1%, while the new method has head and cell-to-cell flux discrepancies of 0.089% and 0.14%, respectively. Additional results highlight the accuracy, flexibility, and CPU time trade-off of these methods and demonstrate how the new method can be successfully implemented to model surface water–ground water interactions.

Introduction

Many numerical models of ground water flow use finite-difference methods to discretize and solve the governing partial differential equation. These models often require a highly refined finite-difference grid such that the solution can be accurately simulated in areas of interest, for example, (1) regions of large variations in hydraulic gradient occurring over relatively small spatial scales caused by, for example, pumping/injection wells (water supply wells and artificial aquifer recharge), rivers, and drains, (2) areas of site-scale contaminant migration within a regional model, and (3) heterogeneity structures such as thin lenses, confining units, faults, pinch-outs, and fractures.

Using a fine grid over the entire domain (referred to here as global refinement) can be computationally intensive—both in terms of CPU time and memory requirements—and intractable in some cases; other alternatives are needed. This work investigates three methods to address this problem. First, the methods and their characteristics are discussed. Two cases are then used to compare the methods in terms of accuracy, CPU time, and memory requirements.

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Description of the Methods

Variable Spacing

A common approach to provide more refinement in an area of interest is to use a finite-difference grid with variable spacing such that the grid spacing is small where needed and gradually increases away from this area. While this approach reduces the computational time compared to globally refining the grid, it generally results in peripheral refinement because conventional finite-difference

grids require that the grid lines in each direction extend out to the model boundaries. This requirement has two important implications: (1) if refinement is needed in more than one area of the domain, using a variably spaced grid often results in a relatively fine grid over the entire domain, thus losing much of its computational advantage over global refinement and (2) in addition to introducing extra nodes and thus more computations, this approach can produce finite-difference cells with large aspect ratios, which can lead to numerical errors (de Marsily 1986, 351).

This approach demonstrates the shortcomings of a traditional finite-difference method vs. a finite-element method in that the grid is not flexible. The variably spaced finite-difference grid really is a brute-force solution to the problem of local-grid refinement (LGR). Despite these drawbacks, this method of refinement remains an accurate and viable alternative in some circumstances. Nevertheless, working with these grids (construction, data input, and postprocessing) is more arduous than with uniformly spaced grids.

Telescopic Mesh Refinement with No Feedback

The drawbacks of the variably spaced grid were significant enough that other methods of LGR were sought. One common alternative is often called telescopic mesh refinement (TMR). This technique combines two or more different-sized finite-difference grids—usually a coarse grid, which incorporates regional boundary conditions, and a locally refined grid, which focuses on the area of interest. In ground water modeling, the link between the coarse and local grids is most commonly accomplished by first simulating the coarse grid and using its results to interpolate heads and fluxes, or a combination of both, onto the boundaries of the local grid (for example, Ward et al. 1987; Leake et al. 1998; Davison and Lerner 2000; Hunt et al. 2001).

This approach is fairly straightforward, flexible, and relatively easy to implement but has some serious limitations. Because the coupling communication between the two grids occurs in one direction, from the coarse grid to the local grid, there is no feedback from the local grid to the coarse grid. Thus, after running the coarse- and local-grid models, significant discrepancy can occur in either the boundary heads or fluxes, whichever one was not used to couple the grids, leaving the modeler unsure of whether the coupling is adequate or not. Naturally, these discrepancies along the boundary are propagated into the interior of the model where high accuracy is desired, but the extent of this propagation is unknown and can vary. The heads and the fluxes across the interfacing boundary can be checked for both models, but if they do not match, there is no formal mechanism for achieving better agreement.

While this approach is attractive for its simplicity and computational efficiency, it can be inaccurate. The problem is that in some cases this method works well, while in other cases it does not, and thus the burden is placed on the modeler to check the consistency between the two grids to determine how well this approach works for their particular case (Leake and Claar 1999, 5–7). It is the authors' opinion that these checks are seldom done in practice, and therefore use of this approach without recognizing and quantifying the potential pitfalls can produce misleading results.

Methods with Feedback

The shortcomings of the traditional TMR approach were recognized, and alternative methods were developed that have a rigorous numerical coupling while incurring a minimum of computational cost. Much of the work involving the coupling of two different grids was pioneered by the petroleum industry. Edwards (1996) concisely reviews many of these methods, including their strengths and weaknesses. Recent alternatives within the ground water modeling field involve directly coupled (Haefner and Boy 2003; Schaars and Kamps 2001) and iteratively coupled methods (Székely 1998; Mehl and Hill 2002a, 2004; Mehl 2003). Both approaches incorporate feedbacks from the local grid to the coarse grid, ensuring that heads and fluxes are consistent between both grids.

Figure 1a shows a locally refined grid embedded within coarser grid. The directly coupled method modifies the finite-difference equations to account for the irregular geometry along the interface of the two grids. Unfortunately, this produces matrices that are different from the diagonally dominant banded matrices that result from a conventional finite-difference discretization. Therefore, solvers that can accommodate these more difficult matrices are required. In contrast, the iteratively coupled method uses both flux and head boundary conditions, which are updated iteratively, to couple both grids. In this case, the coarse-grid solution is used to interpolate heads onto the boundary nodes of the locally refined model. After simulating the local model, the fluxes across the adjoining interface are calculated and applied as a specified-flux boundary on the coarse model for the subsequent iteration (Figure 1b). This process is repeated until the head and

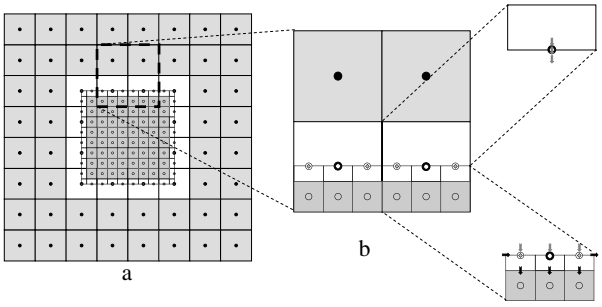

Figure 1. (a) Two-dimensional schematic of the locally refined grid. The interface area denoted by the dashed line is shown in greater detail in (b), and illustrates flux balance across the local refinement interface. Darker shading is material represented by the local grid, lighter shading is material represented by the coarse grid, and no shading is material at the interface. ● node of the coarse grid only; ○ node of the coarse grid that is shared with a boundary node of the local grid; ◍ node of the local grid only. The coarse grid is inactivated here after the initial coarse-grid simulation, so the coarse grid has a hole in it; ◈ specified-head boundary node of the local grid determined by interpolation from the coarse grid.

flux change along the interfacing boundary is deemed negligible. Because the coupling occurs through the boundary conditions, and these appear on the right-hand side of the matrix equations, the finite-difference stencil is not modified and the matrices maintain their regular structure. Therefore, standard solvers can be used without modification. This method is being developed for MODFLOW and is publicly available with MODFLOW-2005 (Mehl and Hill 2005).

In either case, the coupling is more rigorous than the traditional TMR approaches, but the price for this rigorous coupling comes in the form of increased computational costs. It will be shown later that the accuracy of this rigorous coupling is necessary to produce good results for applications involving river-aquifer interactions. Thus, it is the authors' opinion that these approaches provide a good trade-off between the computational efficiency and flexibility of TMR methods and the accuracy of variably spaced grids and therefore are a good solution to the problem of LGR.

Comparisons

Two synthetic test cases were designed to contain features that emulate those important in many ground water models. The first test case exhibits contrasts in transmissivity and sharp changes in gradient caused by a pumping well, while the second test case examines the representation of river-aquifer interactions in three dimensions. Using the uniform fine grid as the reference solution, comparisons of the locally refined solutions are based on the head and flux solution within the locally refined area, CPU time, and memory requirements, and are used to draw conclusions about advantages and drawbacks of the LGR methods considered.

Test Case 1–Two-Dimensional, Heterogeneous with Pumping

The first synthetic test case simulates steady-state flow with a pumping well in a heterogeneous transmissivity field, as shown in Figure 2. The block-like transmissivity structure can be represented explicitly on all the grids without changing values across the interface between grids. A model with a uniform fine grid over the entire domain provided a reference solution for comparison with results from three methods of LGR. The fine grid has 450 rows and 972 columns, with cell dimensions of 1.028 and 1.0 m in the east-west and north-south directions, respectively. For the LGR methods, the embedded grid has 100 rows and 154 columns with cell dimensions the same as the fine grid. The area of the locally refined model is shown enclosed in a dashed line in Figure 2, which includes the pumping well that extracts 5.5 m^3/s. The coarser outer grid has 50 rows and 108 columns, with cell dimensions of 9.25 and 9.0 m in the east-west and north-south directions, respectively. The ratio of refinement is 9:1 (nine local cells span the width of one coarse grid cell).

The variably spaced grid is implemented in MODFLOW-2000 and has grid spacing equivalent to the local grid in the area surrounding the well, with increasing grid spacing away from this area. Two TMR methods are considered in this example. They are implemented using MODTMR (Leake and Claar 1999) and represent the traditional TMR methods using one-way coupling with either heads or fluxes (labeled TMR-Head and TMR-Flux, respectively). For the TMR-Flux simulations, a single head along the boundary was fixed so that a unique solution could be obtained (Leake and Claar, 1999, 7). The iteratively coupled method used here is described by Mehl and Hill (2002a) and was implemented in MODFLOW-2000.

For all methods, the heads and fluxes within the interior 36% of the local model domain are compared to the uniform fine-grid results, and the discrepancies are used to judge the accuracy of the local grid solution methods. The model grids were designed such that the grid spacing and node locations within the interior of the refined region are identical so that interpolation errors would not affect the calculation of the discrepancies. At each location, the percent head discrepancies are calculated as the difference between the head from the fine-grid model and the head from one of the other models normalized by dividing by the head from the fine grid and multiplying by 100 to obtain a percent. The absolute values were averaged to provide an overall measure of discrepancies. This same procedure was used to investigate discrepancies in the cell-to-cell fluxes in the same interior 36% of the local model domain. These results are shown in Table 1. CPU times are also compared.

The results shown in Table 1 indicate that there is a clear trade-off between accuracy of the variably spaced grid and CPU time of TMR methods. The iteratively coupled method provides a compromise in this trade-off. Another interesting result is that coupling using fluxes (TMR-Flux) provides more accurate flux results along the boundary, but less accurate flux results in the interior compared to coupling with heads (TMR-Head), as pointed out by Mehl and Hill (2002b). This result is counterintuitive and demonstrates one of the pitfalls of the TMR methods—lack of consistency on the boundary can propagate both head and flux discrepancies into the interior of the refined region, diminishing the accuracy where it is needed most. The errors are propagated from the

Table 1
Comparison of Head and Flux Discrepancies from the Fine-Grid Solution and CPU Times for Several Grid Refinement Schemes for the Interior 36% of the Local Model Domain

Gridding	Average % Head Discrepancy	Average % Cell-to-Cell Flux Discrepancy	CPU Time (s)
Fine grid ("truth")	0.000	0.000	716
Variably spaced	0.023	0.034	57
TMR-Head	0.393	2.140	3
TMR-Flux	6.801	7.074	4
Iteratively coupled	0.089	0.140	28

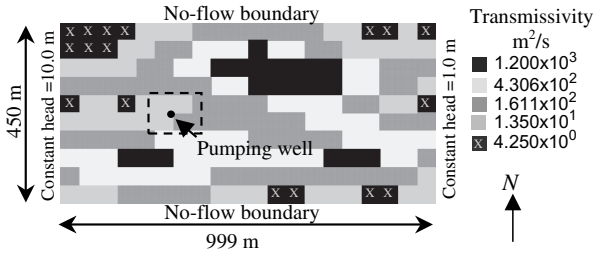

Figure 2. Synthetic test case that is two dimensional and heterogeneous. Area of local refinement is indicated by the dashed rectangle.

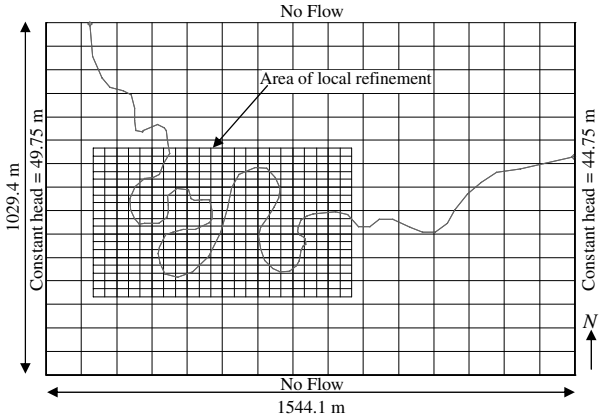

Figure 3. Meandering river and planar area of refinement. The ratio of refinement is 3:1 and extends in all three dimensions.

boundary into the interior via a diffusion process, as discussed by Mehl and Hill (2004). Even though the iteratively coupled method has a feedback, discrepancies are still introduced along the boundary interface from the abrupt change in grid spacing and resolution, and therefore it is not as accurate as the variably spaced grid.

Test Case 2—Three-Dimensional, Homogenous Model of River-Aquifer Interactions

A second synthetic test case was created to test the ability of the LGR methods to represent small-scale features of river-aquifer interactions. Figure 3 shows the plan view of a meandering river in contact with a homogenous, unconfined aquifer and the planar area of local refinement. The refinement extends vertically through half the thickness of the aquifer. The river has a linear drop in stage from inlet to outlet and is represented using MODFLOW's River Package. Constant-head boundaries are placed on the east and west sides to provide a background gradient; no-flow boundaries span the north and south edges of the domain. The hydraulic conductivity of the riverbed is equal to that of the aquifer to maximize the interaction between the two. The riverbed conductance is assigned according to the area of the river that intersects each cell. For the locally refined grids, all the properties (riverbed conductance, vertical conductance, etc.) are treated the same as they are in the globally refined grid.

For this case, no comparisons are made using a variably spaced grid, which is particularly awkward to work with because of the meandering river. The river leakages in the locally refined grids are compared to the river leakages obtained using a globally refined model with the equivalent local-grid spacing throughout the entire domain. The ratio of refinement is 3:1 in all three grid dimensions; thus, 27 local-grid cells occupy the volume of a single coarse-grid three-dimensional cell. Table 2 shows the average of the absolute value of the percent discrepancies of the river leakage. Essentially, the numbers indicate how much improvement in memory and CPU time is lost to accuracy by using local instead of global refinement.

The results in Table 2 show that the decrease in RAM and CPU time can be significant for the locally refined grids vs. the globally refined equivalents. The globally refined grid requires 1,476,225 nodes and 107 MB of RAM, while the locally refined model requires 154,850 nodes and 13.7 MB of RAM. The result also suggests that little accuracy is lost by refining the grid locally instead of globally, if a feedback is included (iteratively coupled). However, if a feedback is not included (TMR-Head), discrepancies in river leakage relative to the fine-grid solution are 11.6% for this problem.

Conclusions

The three methods of LGR considered here—(1) a variably spaced grid; (2) a traditional TMR method; and (3) a rigorously coupled LGR method—all have advantages and drawbacks. The variably spaced grid can be viewed as a brute-force solution that is accurate but can be computationally intensive and lacks flexibility and elegance. The TMR methods are conceptually simple, easy to implement, and computationally very efficient but can be inaccurate in ways that often are not obvious to the

Table 2
Comparison of Memory, CPU Time, and Mean River Leakage Discrepancies in the Locally Refined Grid vs. a Globally Refined Grid. Values in Parentheses Represent the Percent Reduction vs. the Globally Refined Grid

Discretization	Grid Size	RAM Usage (MB)	CPU Time (s)	% River Leakage Discrepancy
Globally refined	405 × 405 × 9	107	1913	0
TMR-Head	163 × 190 × 5	13.7	76	11.6
Iteratively coupled	163 × 190 × 5	13.7 (87%)	811 (58%)	1.72

user. Use of these methods without carefully considering the potential errors can produce misleading model results. Local grid refinement methods that include a feedback to rigorously couple the grids produce results that are consistent between both grids and thus have better accuracy than the traditional TMR methods. These methods are more flexible and computationally more efficient than variably spaced grids. These trade-offs make this a good approach to use for models requiring LGR.

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