ERTH656/CEE623/ERTH654

Analytical Groundwater Flow Solutions

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Steady One-Dimensional Flow

For ground water flow in the x-direction in a confined aquifer:

$$d^2h/dx^2 = 0$$

Integrate twice:

$$h = Cx + h_0$$
$$= -qx/K + h_0$$

$dh/dx = -q/K$, according to Darcy's law ($q = Q/A$)

This states that head varies linearly with flow in the x-direction.

Response of ideal aquifers to pumping

- Assumptions:
  - Governing equation:
    - compressibility is strictly vertical
    - water release is instantaneous as head drops
    - vertically integrated flow equation (vertical gradients are negligible)

Response of ideal aquifers to pumping

- Aquifer characteristics
  - homogeneous and isotropic aquifers
  - constant thickness
  - hydraulic head is uniform prior to pumping
  - aquifer is horizontal and infinitely large in the horizontal direction
- Well and pumping characteristics
  - single, fully penetrating well pumping at a fixed rate
  - well diameter is infinitesimally small

Steady Radial Flow to a Well-Confined

For horizontal flow, Q at any radius r, from Darcy's law,

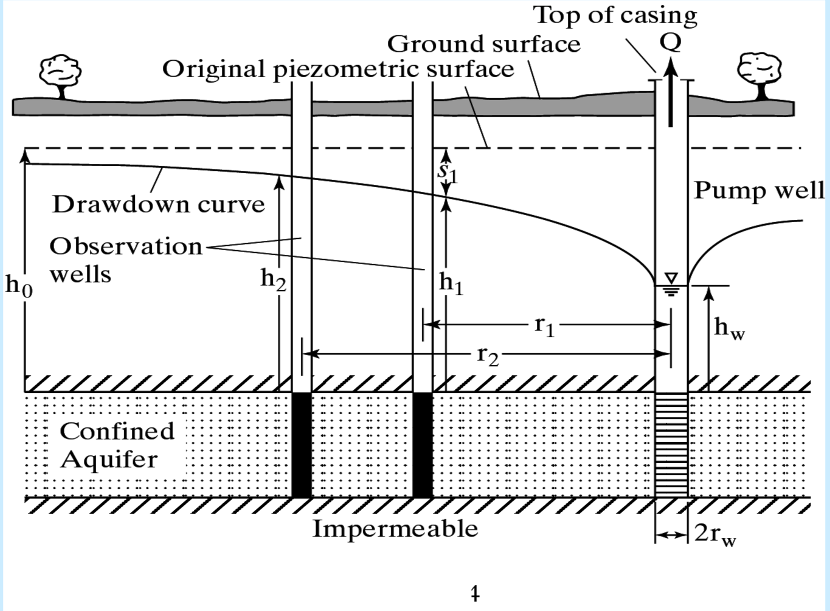
$$Q = -2\pi rbK\, dh/dr$$

for steady radial flow to a well where Q,b,K are const

Steady Radial Flow to a Well-Confined

Integrating after separation of variables, with
$h = h_w$ at $r = r_w$ at the well, yields Thiem Eqn

$Q = 2\pi Kb[(h-h_w)/(\ln(r/r_w))]$



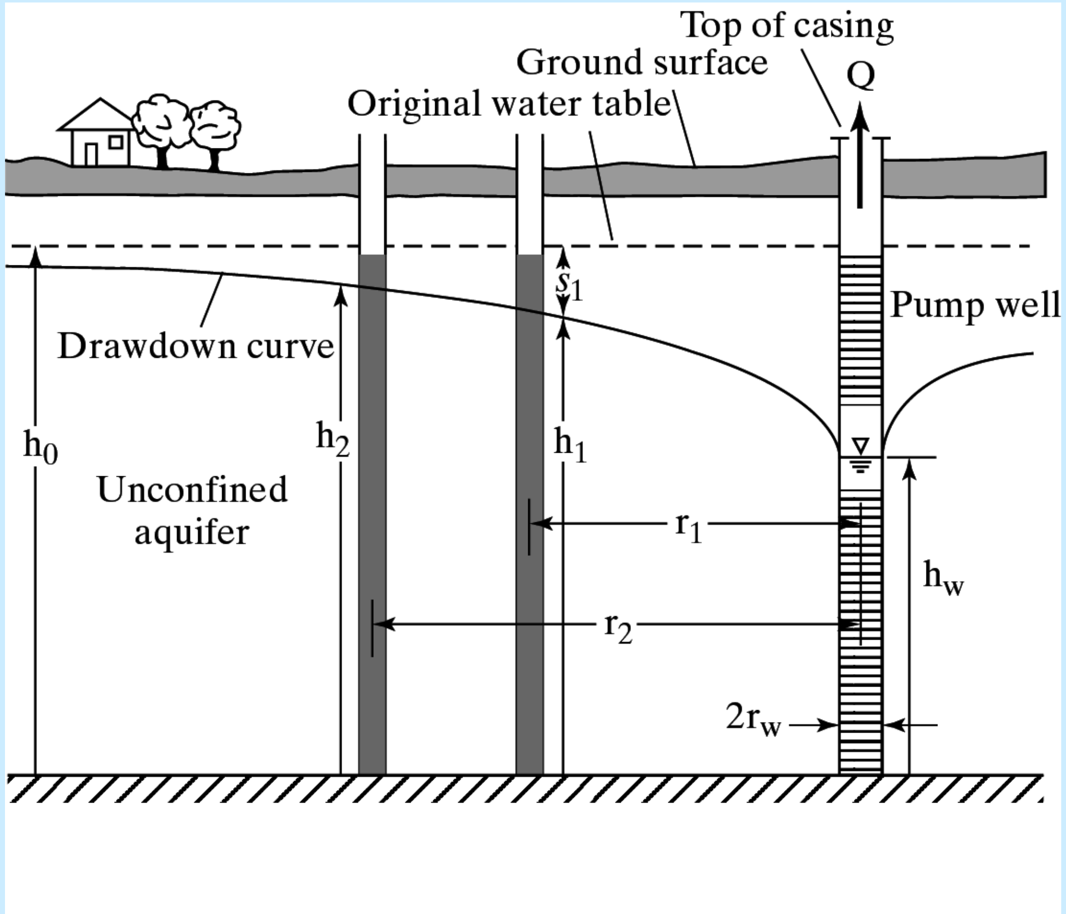
Radial flow to a well penetrating an extensive confined aquifer.

Steady Radial Flow to a Well-Confined

- Near the well, transmissivity, *T*, may be estimated by observing heads h_1 and h_2 at two adjacent observation wells located at r_1 and r_2, respectively, from the pumping well

$$T = Kb = \frac{Q \ln(r_2 / r_1)}{2\pi(h_2 - h_1)}$$

Steady Radial Flow to a Well-Unconfined



Radial flow to a well penetrating an unconfined aquifer.

Steady Radial Flow to a Well-Unconfined

- Using Dupuit's assumptions and applying Darcy's law for radial flow in an unconfined, homogeneous, isotropic, and horizontal aquifer yields:

  $$Q = -2\pi Kh\, dh/dr$$

  integrating,

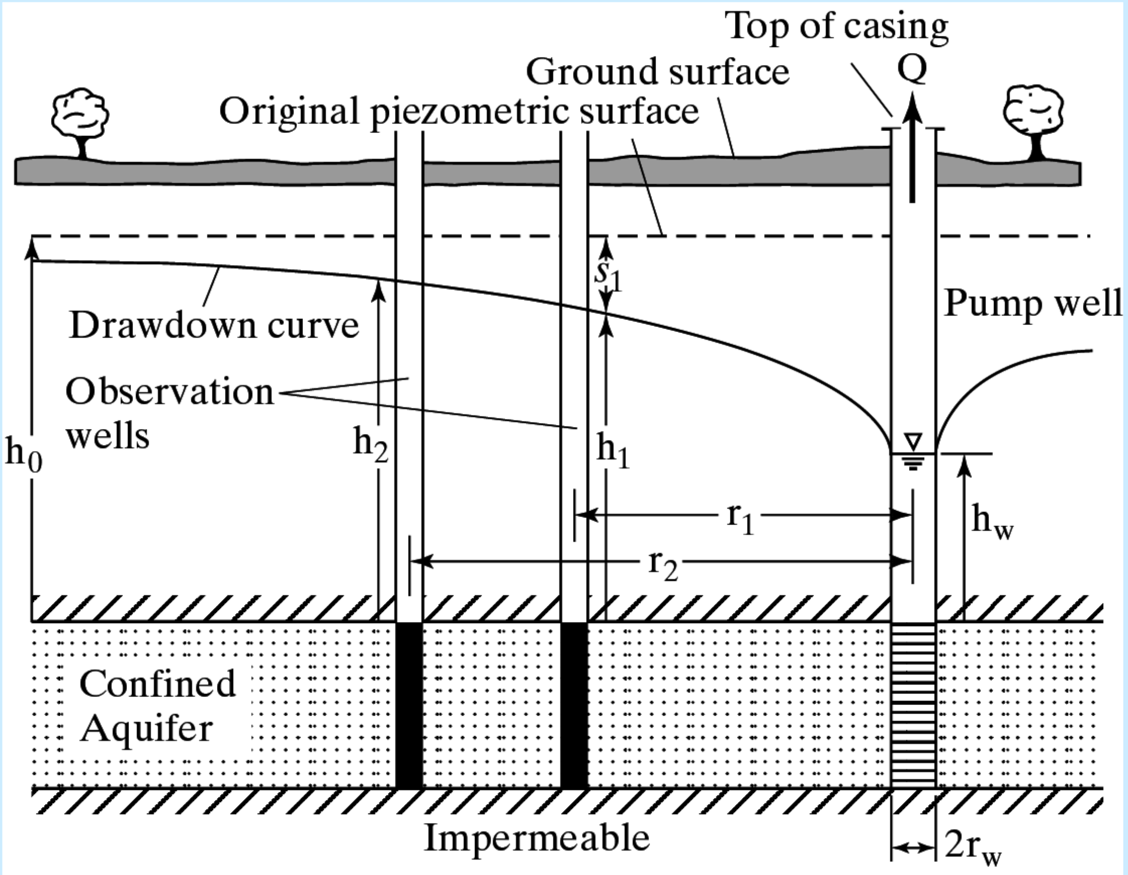
  $$Q = \pi K[(h_2^2 - h_1^2)/ln(r_2/ r_1)$$

  solving for K,

  $$K = [Q/\pi(h_2^2 - h_1^2)]ln\ (r_2/ r_1)$$

  h_1 and h_2 are observed at adjacent wells, distances r_1 and r_2 from the pumping well.

Transient condition: The Theis solution



Radial flow to a well penetrating an extensive confined aquifer.

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)=S_s\frac{\partial h}{\partial t}$$

Saturated 3-D equation

Transient condition: The Theis solution

Two-dimensional groundwater flow in a confined aquifer with transmissivity T and storativity S:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T}\frac{\partial h}{\partial t}$$

Can be written as:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t}$$

$$r = \sqrt{x^2 + y^2}$$

Initial condition:

$$h(r,0) = h_o \qquad \text{for all } r$$

Boundary condition at r= ∞:

$$h(\infty,t) = h_o \qquad \text{for all } t$$

At the well face (Darcy's law):

$$r\frac{\partial h}{\partial r} = \frac{Q}{2\pi T} \qquad \text{for } t > 0$$

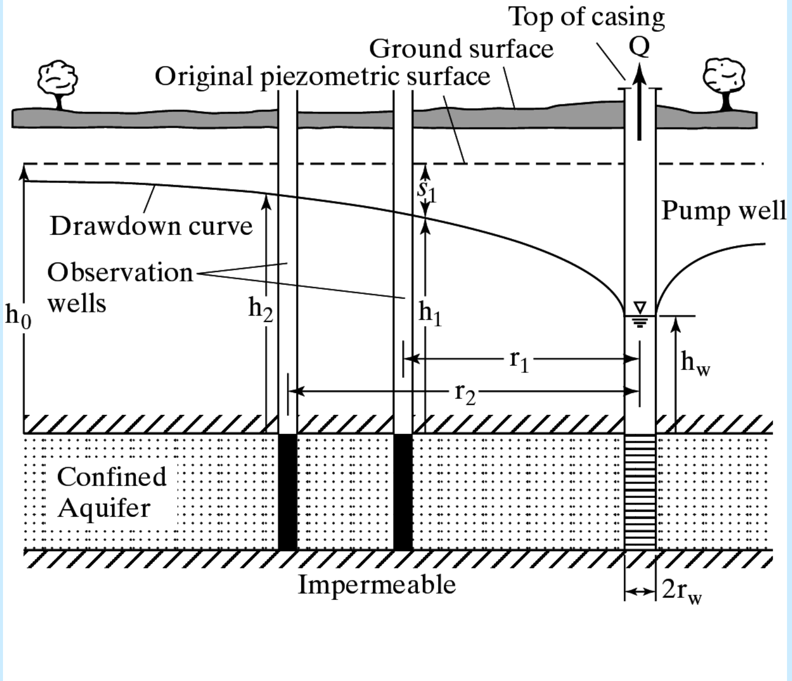
Transient condition: The Theis solution

$$s = \frac{Q}{4\pi T}\int_u^{\infty}\frac{e^{-x}}{x}dx = \frac{Q}{4\pi T}W(u)$$

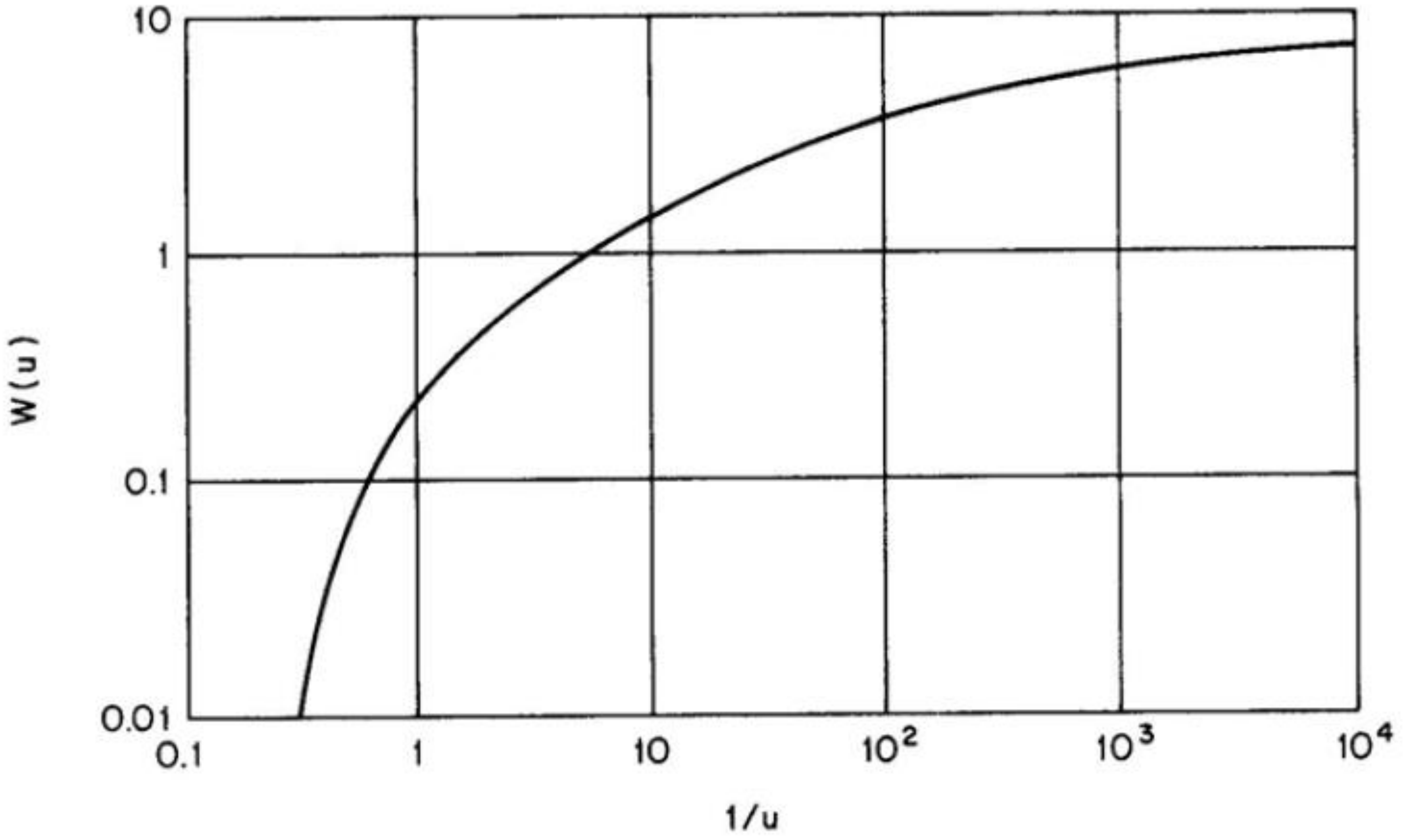
s = drawdown = h_0 – h; h_0 = initial head; h = head at time t; Q = pumping rate; T = transmissivity

$$u = \frac{Sr^2}{4Tt}$$

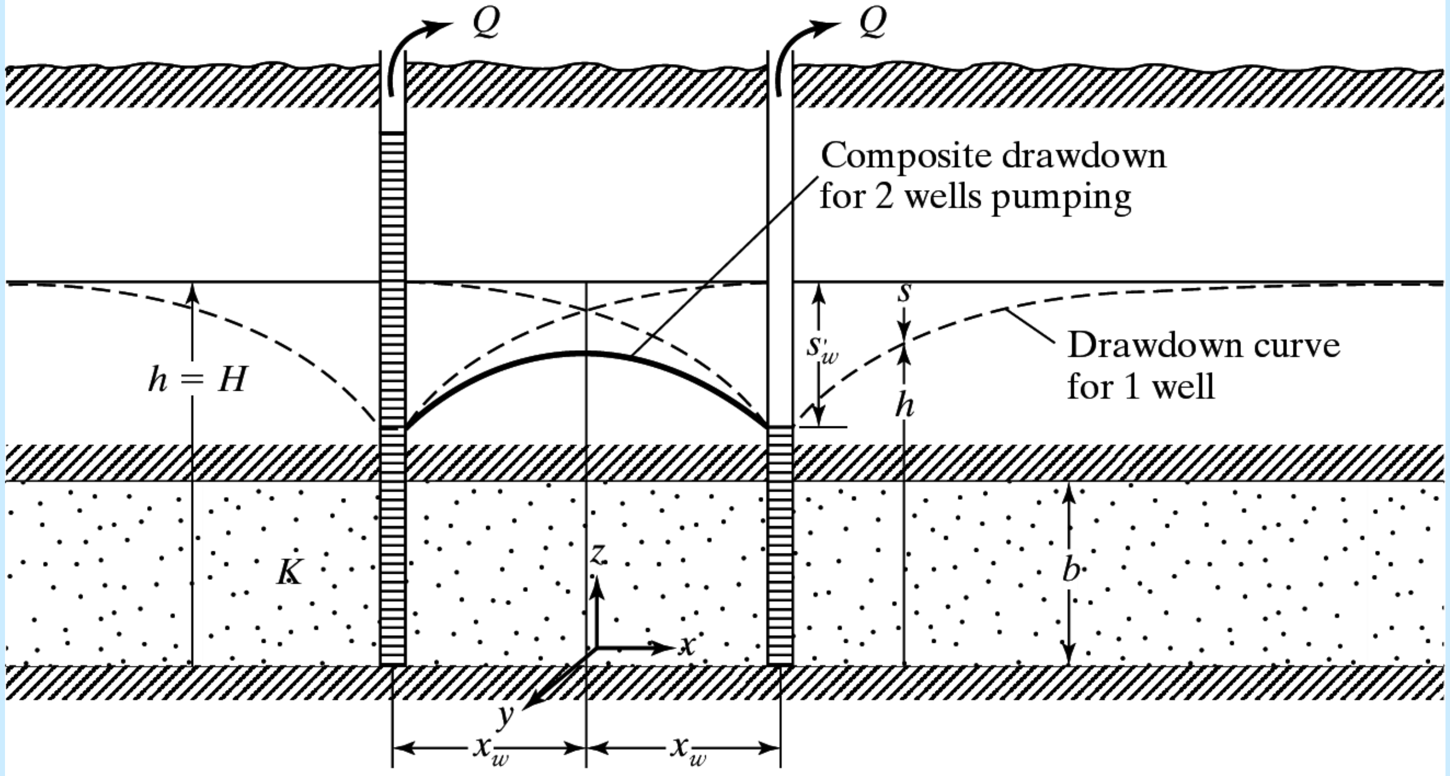
S = storativity (storage coefficient); r = distance from the well; t = time; W(u) = well function



Radial flow to a well penetrating an extensive confined aquifer.

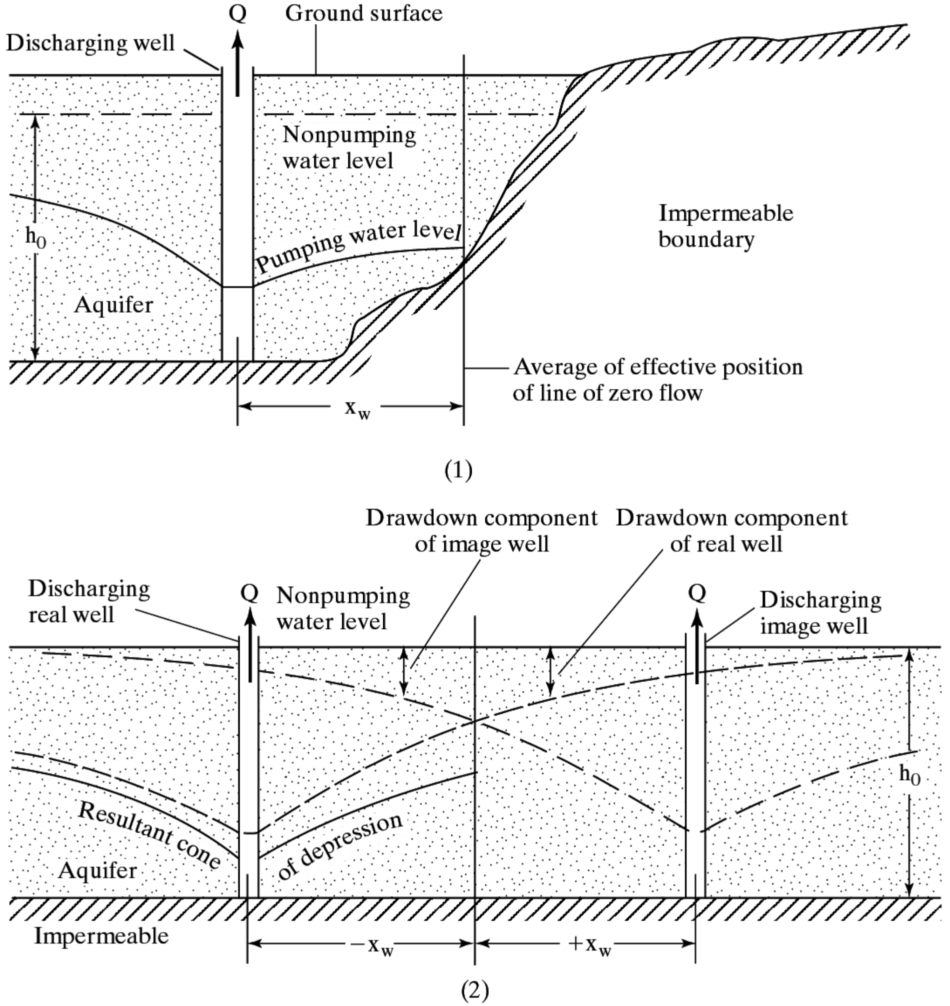

10
1
0.1
0.01
W(u)
0.1
1
10
10^2
10^3
10^4
1/u

Multiple-Well Systems



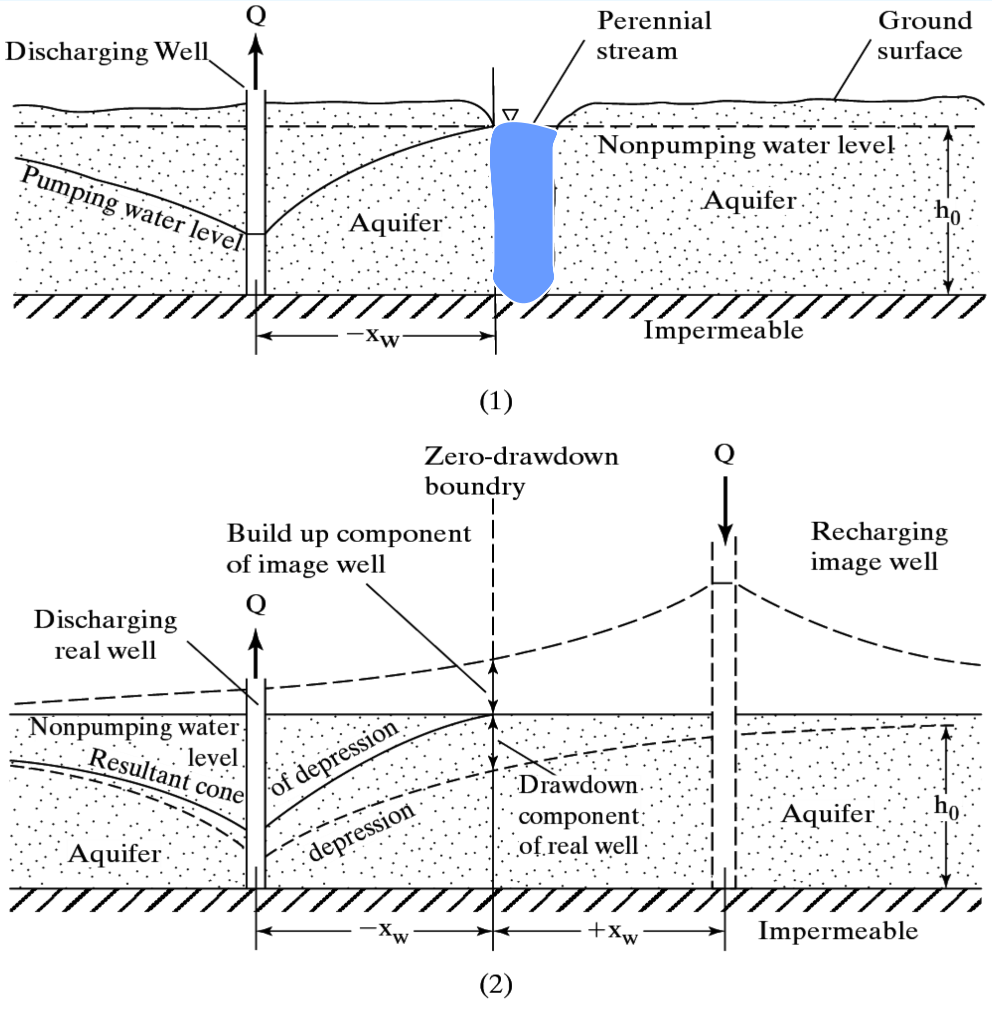
Individual and composite drawdown curves for two wells in a line.

Impermeable boundary



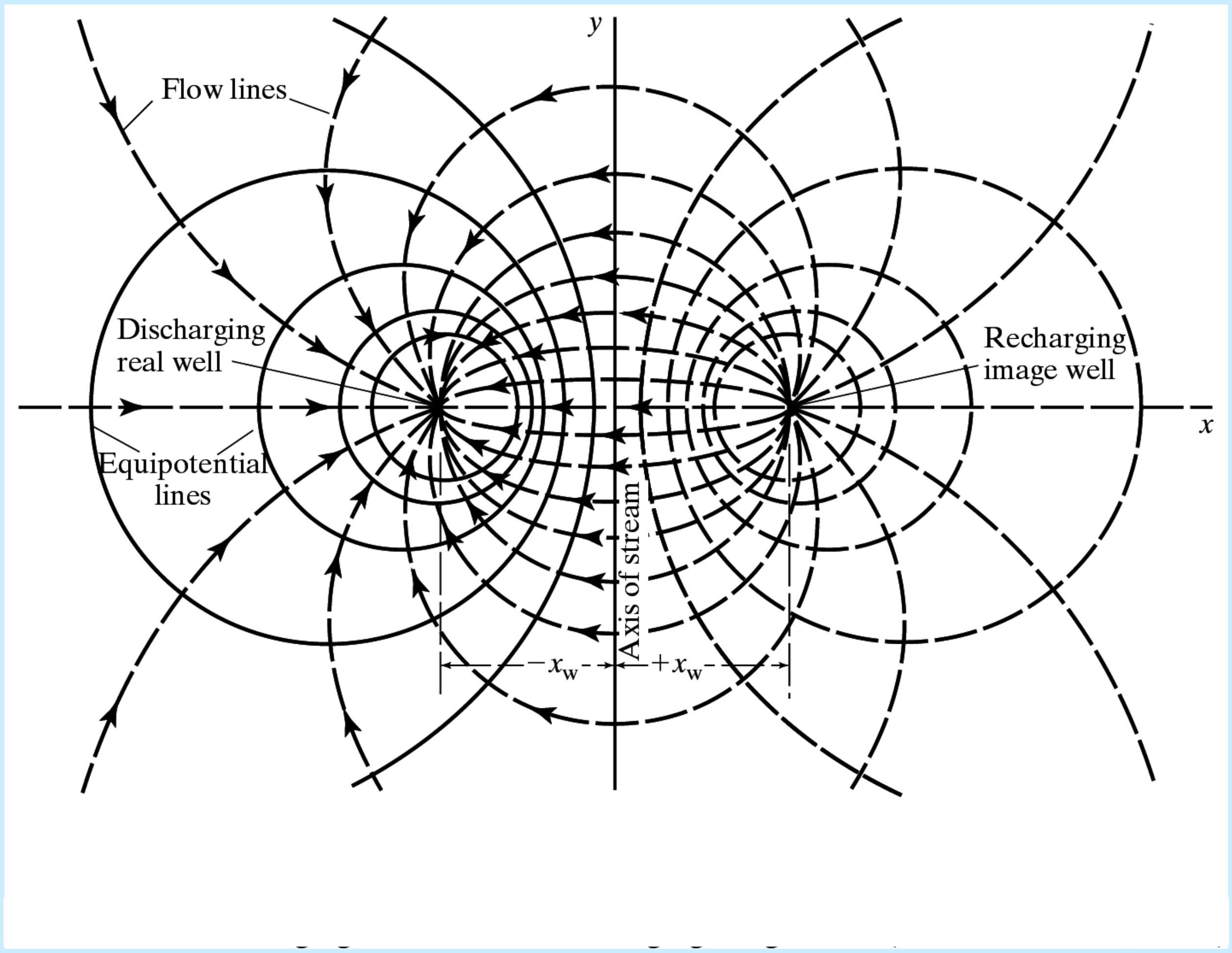
(1) Discharging well near an impermeable boundary. (2) Equivalent hydraulic system in an aquifer of infinite areal extent. (After Ferris et al., 1962.)

Perennial stream

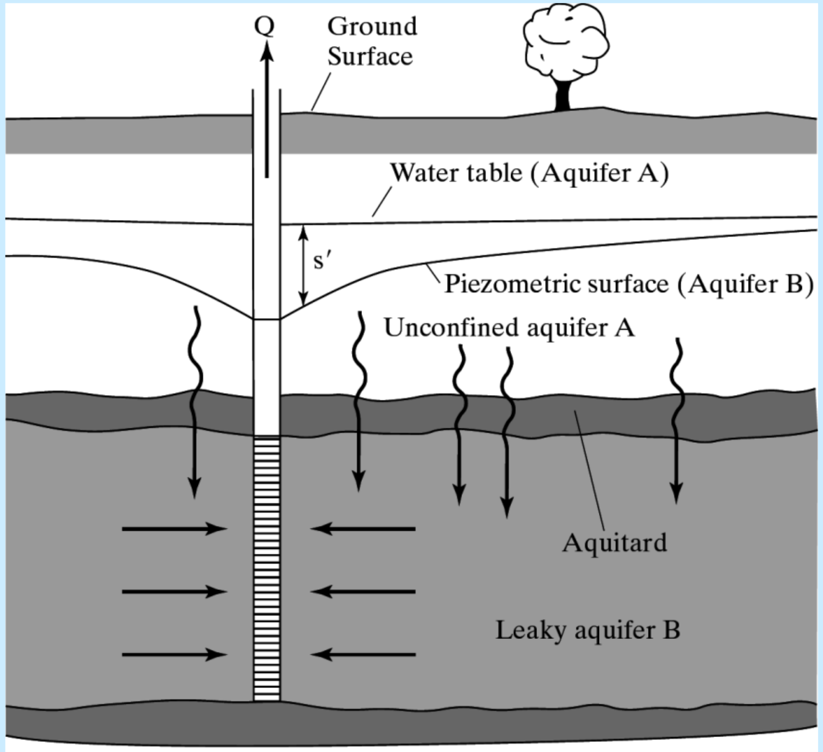


Sectional views. (1) Discharging well near a perennial stream.
(2) Equivalent hydraulic system in an aquifer of infinite areal extent.

Injection-Pumping Pair of Wells



Leaky aquifers



Well pumping from a leaky aquifer.

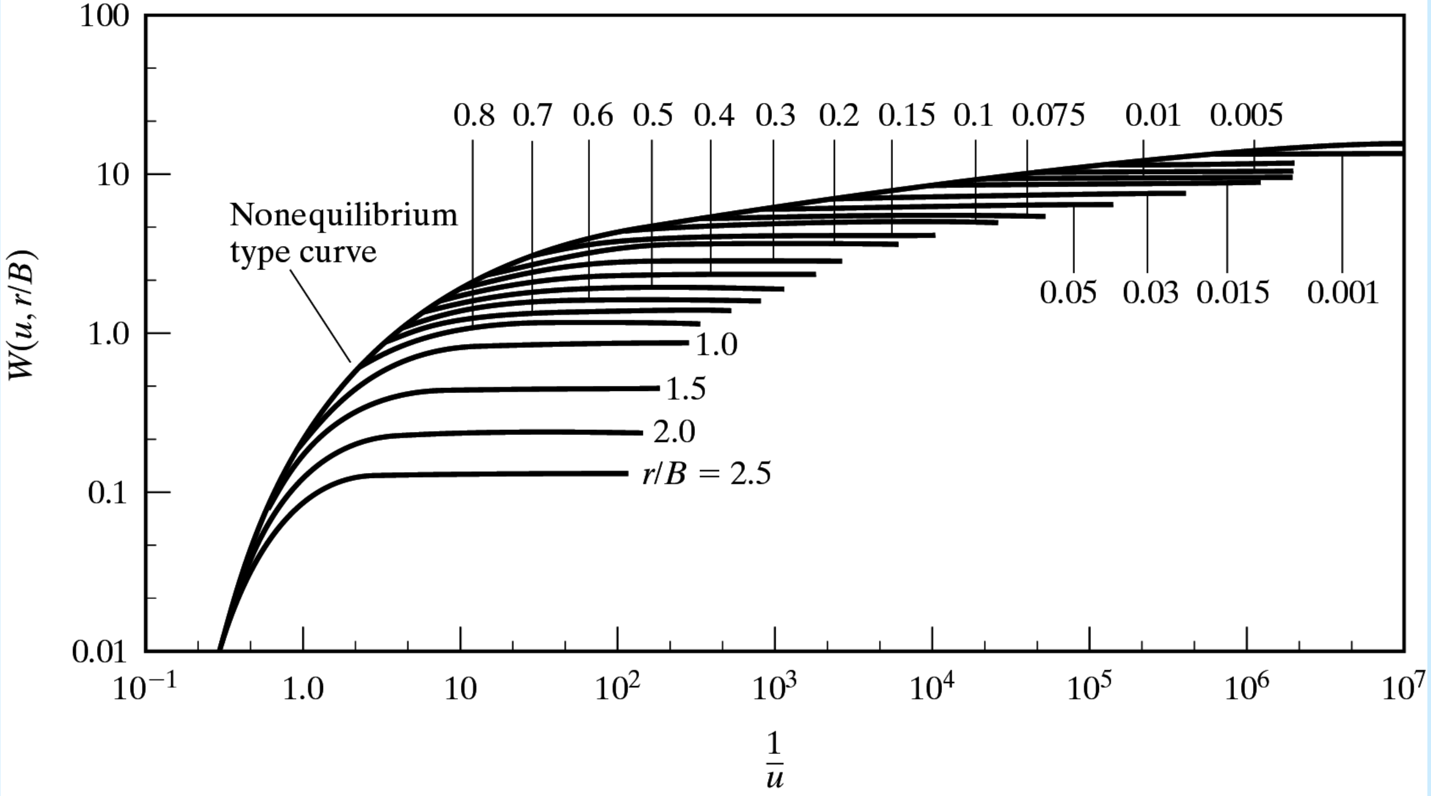
- When pumping starts from a well in a leaky aquifer, drawdown of the piezometric surface can be given by:

$$s = (Q/4\pi T)W(u,r/B)$$

$$r/B = r/\ T/(K' / b')$$

T = transmissivity of the aquifer

K' = vertical hydraulic conductivity
b' = thickness of the aquitard



Type curves for analysis of pumping test data to evaluate storage coefficient and transmissivity of leaky aquifers. (After Walton, 1960, Illinois State Water Survey.)

Other well analytical solutions

- Partially penetrating well
- Two or 3 layer system
- Large diameter well
- Unsaturated/saturated condition