Answer of Mr. ALLEN:

That may be correct. I am not sure. The collar count data indicated that there was some expansions in the upper zone but when you get into areas of very small rebound, of tenths of a foot, such as you are speaking of, you are not sure of your data. So, assumming that this happens, the amount was small and we did not take it into account in subsidence. We are speaking only of measured subsidence of the surface related to the oil zones. So it is possible that the compaction of oil zones is greater that what we have actually shown here. These would be minimum figures.

Intervention of Prof. Kenneth E. LEE (USA):

Question:

You mentioned that if the sand has any cementation in it, it will not compact.This does not seem to follow logically. We know that all materials compress under changing load. And I wondered if you had any data from other fields where the material is essentially a cemented material that show definitely that this does not happen?

Answer of Mr. ALLEN:

I think we are speaking of the difference between qualitative and quantitative here, and what will compact and what any compaction means. There is no question that any material with pores can be compacted when enough load is put on it. There are many studies which have been made on cemented sandstones. I have a bibliography of them, and I believe I used some of them as references in this paper, in which cemented sandstones had been stressed and the deformations measured. Now, when the sandstones are cemented, deformations are found, usually within the elastic range. You find that within the range of pressures, that might be suffered in producing oil field reservoirs, the strains which are imposed normally will not crush a cemented sandstone. They will deform slightly elastically. There is a very slight reduction of pore volume, the amount is perhaps a few percent of what we see by rearrangement of sand grains.

ANALYSIS OF BOREHOLE EXTENSOMETER DATA FROM CENTRAL CALIFORNIA

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ABSTRACT

In subsiding areas of central California, highly sensitive borehole extensometers provide data that define the compression characteristics of the compacting artesian aquifer systems. The extensometer records are combined with hydrographs for the confined and unconfined aquifers to form stress-strain diagrams on which the annual cycles of head decline and recovery generate a series of open loops. Gross compaction, elastic expansion, and net permanent compaction are clearly defined. The average level of preconsolidation stress, the elastic compressibility at lesser stresses, and the much larger plastic compressibility at higher stresses can often be determined from such diagrams. At a given location, these compressibilities may differ by as much as two orders of magnitude. Under favorable circumstances, estimates of the average vertical permeability and average compressibility of the fine-grained strata can be derived.

RÉSUMÉ

Dans les régions d'affaissement en Californie centrale, des extensomètres ultra-sensibles installés dans les trous de forage tubés, fournissent des renseignements qui indiquent les caractéristiques de compression des nappes aquifères artésiennes. Les enregistrements de l'extensomètre sont combinés avec les relevés du débit, en un temps donné, des nappes aquifères artésiennes ou libres, afin de former des diagrammes de pression-tension sur lesquels les cycles annuels de diminution ou de regain de pression artésienne produisent une série de boucles ouvertes. La compaction brute, l'expansion élastique et la compaction permanente nette sont clairement indiquées. Le niveau moyen de la pression de préconsolidation, la compressibilité élastique à des pressions moindres et la compressibilité plastique à des pressions plus grandes peuvent souvent être déterminées d'après ces diagrammes. A certains endroits, ces compressibilités peuvent varier au point d'avoir deux ordres de grandeur. Dans des circonstances favorables, les évaluations de la moyenne de perméabilité verticale et de la moyenne de compressibilité des couches à grain fin peuvent être obtenues.

INTRODUCTION

Recording borehole extensometers of the taut-cable type described by Lofgren (1961) have been used by the US Geological Survey since 1955 in investigations of subsidence due to ground-water withdrawal in central California. As experience was gained with the instruments, it became apparent that the relationship between the recorded compaction of the confined aquifer system and the decline of artesian head was both complex and fundamental to an understanding of the mechanics of subsidence. The purposes of the present paper are to summarize certain new techniques for analyzing this relationship and to present some preliminary results of their application.

STRESS AND COMPACTION AT PIXLEY, CALIF.

The multiple-extensometer installation near the town of Pixley, Calif., is described briefly in Lofgren's contribution to the present symposium. His figure 11 presents the basic head-change and compaction data used in the following analysis. Figure 1 of my paper shows, in the uppermost graph, the history of compaction between depths of 355 and 760 feet (108 to 232 meters), an interval that corresponds approximately to the zone of principal pumpage from the confined aquifer system. The second graph is a somewhat generalized history of the changes in the stress applied to all strata within this depth interval, as a result of changes in head in the confined and overlying unconfined aquifer systems. (See Lofgren, 1968, for the method of calculating stresses.) For convenience, stresses in this illustration and throughout this paper are expressed in equivalent units of water head (1 ft of water head equals 0.433 lb in^{-2} or 0.030 kg cm^{-2}; 1 m of head equals 0.1 kg cm^{-2}). The stress-change graph is plotted with stress increasing downward to emphasize its close correlation with declining artesian head.

From 1958 through 1968, 3.4 ft (1.04 m) of compaction has been recorded in the 355- to 760-ft interval. Despite the fact that the stress fluctuated through about the same range year after year, each major episode of stress increase was accompanied by additional permanent compaction. As stresses diminished during the fall and winter seasons of head recovery, compaction ceased, and in most years a slight expansion of the aquifer system was recorded. However, no simple and quantitatively consistent relationship between stress change and compaction is evident in the data.

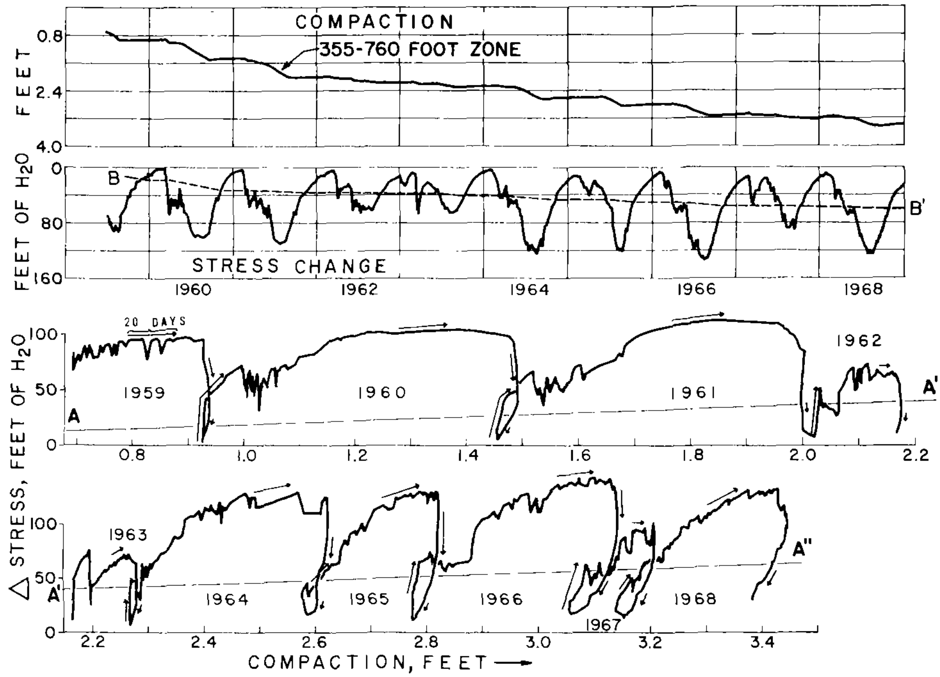

FIGURE 1. *History of compaction and stress change, and the relationship between stress change and compaction near Pixley, Tulare County, California*

THEORY OF AQUIFER-SYSTEM COMPACTION

The well known hydrodynamic (Terzaghi) theory of soil consolidation can provide a semi-quantitative explanation for the phenomenon of repeated permanent compaction during successive cycles of loading and unloading through about the same stress range. In the context of this problem a central tenet of consolidation theory states that an increase in stress applied to a "clay" stratum (aquitard) becomes effective as a compressive grain-to-grain load only as rapidly as the heads (pore pressures) in the aquitard can decay toward equilibrium with the head in the adjacent aquifer(s). Because of the low permeability and relatively high compressibility of the interbedded aquitards, the consolidation (compaction) of a multi-layered aquifer system in response to increased applied stress is a strongly time-dependent process, and complete or "ultimate" consolidation is not attained until a steady-state vertical distribution of head exists throughout the aquifer system. Transient heads in the aquitards higher than those in the adjacent aquifers (termed residual excess pore pressures) are a direct measure of the remaining primary consolidation that will ultimately occur under the existing stress. When pore-pressure equilibrium is attained throughout the aquitard, it is said to be 100 percent consolidated for the prevailing stress and no further permanent compaction will occur if the same stress is repeatedly removed and reapplied. (The possible role of secondary, or nonhydrodynamic, consolidation in aquifer-system compaction is not well known, but is assumed in this discussion to be minor.)

For a single homogeneous aquitard, bounded above and below by aquifers in which the head is instantaneously and equally lowered, the time, t, required to attain any specified dissipation of average excess pore pressure is a direct function of: (1) the volume of water that must be squeezed out of the aquitard in order to establish the denser structure required to withstand the increased stress, and (2) the impedance to the escape of this water. The product of these two parameters constitutes the aquitard time constant. For a specified stress increase, the volume of water is determined by the volume compressibility m_v, of the aquitard, the compressibility, β_w, of the water, and the thickness, b', of the aquitard. The impedance is determined by the vertical permeability, K', and thickness of the aquitard. Thus, the required time, t, is a function of the time constant, τ, where

$$\tau = \frac{S_s'(b'/2)^2}{K'} \tag{1}$$

and where S_s' is the specific storage of the aquitard, defined as

$$S_s' = S_{sk}' + S_{sw} \tag{2}$$

in which

$$S_{sk}' = m_v \gamma_w = \frac{\Delta b'}{b' \Delta h_a} \tag{2a}$$

and

$$S_{sw} = n \beta_w \gamma_w \tag{2b}$$

S_{sk}' is the component of specific storage due to compressibility of the aquitard, S_{sw}, is the component due to the compressibility of water, h_a is the average head in the aquitard, n is the porosity, and γ_w is the unit weight of water. For consolidating aquitards $S_{sk}' >>> S_{sw}$.

For convenience, it is customary to define a dimensionless time factor, T, such that

$$T = \frac{t}{\tau} \tag{3}$$

when T equals unity, t equals the time constant. The degree of consolidation, $U\%$, at any time, t, is then expressed as a function of T, the form of the functional relation being determined by the initial conditions of the problem. For the commonly used time-consolidation functions, $U\%$ is somewhat more than 90 percent when T is unity. Detailed development of the time-consolidation theory summarized above may be found in Scott (1963, pp. 162-197).

STRESS-STRAIN ANALYSIS

From the foregoing, it seems evident that reliable techniques for determining, preferably *in situ*, the specific storage (or compressibility), vertical permeability, and the thickness of every stratum in the aquifer system would provide the data required for accurately analyzing and predicting the time-dependent compaction of the aquifer system as a whole. Such an undertaking would be technically formidable and in most cases prohibitively expensive.

Under favorable circumstances, however, the bulk compressibility and average time-response characteristics of the confined-aquifer system as a whole can be approximated from the stress-strain relationship defined by the recorded head changes in the aquifers and the recorded compaction of the total system. To illustrate this relationship, the basic data presented in the two upper graphs in figure 1 have been combined in the lower part of the illustration to form a stress-strain diagram. Following the usual convention, stress is plotted increasing upward and strain (in this case cumulative compaction, not unit strain) increasing to the right. In subsidence studies, it is convenient to deal with the modulus of compressibility, which is here designated α when it applies to the response of the entire aquifer system in the elastic range. Accordingly, "slopes" of the stress-strain curve will be characterized numerically by strain per unit stress, the reciprocal of the conventional ratio, and the values will increase as the slopes become flatter.

The yearly cycles of stress increase and decrease, resulting from seasonal demand for irrigation water, produce a series of annual stress-strain loops, each of which is identified in figure 1 by irrigation year. Although time is not displayed as a variable in this type of illustration, it is obviously a strong influence on the shape of the stress-strain loops, because of the time-dependent character of consolidation. In order to convey some idea of the different rates of change of stress and strain at various times during the annual cycle, I have added arrows, each of which indicates by its length and direction the change in stress and strain during a 20-day period represented by the portion of the stress-strain curve adjacent to the arrow.

The descending segments of the annual loop are of particular interest since they represent the resultant of two opposing tendencies – one toward continuing compaction and one toward elastic expansion in response to decreasing applied stress. Expansion of the more permeable strata of the aquifer system must be essentially concurrent with the observed rise in head in wells. However, the first reduction of stress may produce only a slight reduction in compaction rate. Evidently, initial expansion of the aquifers is concealed by continuing compaction of the interbedded aquitards as water continues to be expelled under the influence of higher pore pressures remaining within the medial regions of the beds.

Consolidation theory requires that the maximum excess pore pressure, which is in the middle of a doubly-draining aquitard, be related to the same parameters (embodied in the time factor, T) that control the time-consolidation function. It is, therefore, inevitable that there be, at the end of a relatively short pumping season, a large range of maximum excess pore pressures in a sequence of aquitards of widely varying thicknesses and physical properties. Thus, as head in the aquifers rises and stress declines, the thinnest and (or) most permeable aquitards, containing the least excess pore pressure, will quickly assume an elastic response; but the thickest and (or) least permeable beds may continue to compact at diminishing rates through most or perhaps all of the period of head recovery and stress relief.

Evidence for this type of behavior is contained in the continuously curving stress-strain line characteristic of much of the descending portions of the annual loops (fig. 1). If the lower part of the descending curve approximates a straight line with a positive slope, as it clearly does, for instance, in 1967 and 1968, we probably are justified in assuming that essentially all excess pore pressures have been exceeded by the rising heads and that the entire aquifer system is expanding in accordance with its elastic modulus.

Because stress is expressed in units of water head, h, the compaction-stress-ratio under steady-state conditions equals S_k, the component of the aquifer-system storage coefficient, S, attributable to deformation of its skeleton (Riley, 1968). ($S = S_s \cdot m$, where m is the thickness of the aquifer system.) Therefore, for the confined aquifer system at Pixley we calculate, from the reciprocal slope of the straight-line approximation:

$$S_{ke} = \frac{\Delta m}{\Delta h} = \frac{-0.10}{-87} = 1.1 \times 10^{-3}$$

and

$$S_{ske} = \frac{S_{ke}}{m} = \frac{1.1 \times 10^{-3}}{405} = 2.8 \times 10^{-6} \text{ ft}^{-1} = 9.3 \times 10^{-6} \text{ m}^{-1}$$

and finally the compressibility of the aquifer system in the elastic range is computed as

$$\alpha = \frac{S_{ske}}{\gamma_w} = (2.8\times10^{-6})/0.433 = 6.6\times10^{-6}\,\mathrm{in}^2\mathrm{lb}^{-1} = 9.3\times10^{-5}\,\mathrm{cm}^2\mathrm{kg}^{-1}$$

where the subscript, $_e$, identifies the parameter as being in the elastic range. This value may be compared with β_w, the compressibility of water, which is $3.6\times10^{-6}\mathrm{in}^2\mathrm{lb}^{-1}$ ($5.1\times10^{-5}\mathrm{cm}^2\mathrm{kg}^{-1}$). The elastic compressibility slope is best defined in the 1967 and 1968 loops, but is approximated by the lowest segments of the loops in most years, except 1961 and 1964, when the expansion data were degraded by instrumental problems.

By extending construction lines with a slope equal to S_{ke} from the annual point of minimum stress to an intercept with the zero stress-change line, the net annual permanent compaction can be measured directly as the difference between the intercepts. For example in 1967, a maximum compaction of 0.15 ft (4.6 cm) was recorded, but the net permanent compaction was only 0.07 ft (2.1 cm).

The elastic hysteresis loops resulting from the steep ascent and subsequent flattening of the stress-strain curves in the first weeks of each pumping season are attributable in part to the frictional "dead-band" of the extensometer (0.01 to 0.03 ft) and in part to the hydrodynamic lag associated (even in the elastic range) with rapid stress increase. The effects of hydrodynamic lag, though relatively minor during the interval of slow elastic expansion, are sufficient, during the brief period of most rapid stress increase, to delay the appearance of a substantial percentage of the potential elastic compaction. Accordingly, the descending segments of the loops are used for calculating the elastic compressibility parameters.

By judiciously appraising the stress level at which the descending expansion curve approaches tangency with the S_{ke} line and the level at which the following elastic compaction curve crosses over the expansion curve, it is possible to select each year a stress level below which no appreciable excess pore pressures remain within the aquifer system. Although a distinctly subjective element enters into the selection of the stress level representing zero excess pore pressure, it was found that the straight line A-A′-A″ will closely fit the selected values. The intercepts of line A-A′-A″ on the annual expansion curves have been used to control the dashed line B-B′ on the time-stress graph. Line B-B′ defines the seasonal increments and long-term trend of decline in maximum excess pore pressure. Thus, it separates an overlying stress range, within which the sediments are fully consolidated, from an underlying range of higher stresses in which at least some of the aquitards are much less than 100 percent consolidated; therefore, the line B-B′ may be called a "preconsolidation line." Stress changes above the preconsolidation line cause only minor elastic deformations of the aquifer system, but each year, as stresses increase below the line, permanent "plastic" compaction resumes.

The slope of straight line A-A′-A″ approximately represents the ratio of the annual decrement of maximum (midplane) excess pore pressure to the annual increment of total compaction. By correcting for the annually increasing amounts of elastic expansion we can determine a corrected slope equal to the ratio of midplane pore-pressure decrement to permanent compaction increment. Since permanent compaction is a function of the decrement of average pore pressure, the corrected slope may be interpreted as characterizing the ratio of midplane excess pore-pressure decline to average excess pore-pressure decline. The apparent linear relation between these variables is reasonable if an initially sinusoidal distribution of excess pore pressure within the aquitards is assumed. During the first cycle of stress increase into a new range, a linear distribution (with depth) of initial excess pore pressure is more likely, but in all subsequent cycles in the same range a roughly sinusoidal initial distribution should prevail.

It is evident that if the stress repeatedly cycles through the same range – each year extracting an approximately equal percentage of the remaining excess pore pressure – the annual compaction increment will become progressively smaller and 100 percent consolidation will ultimately be approached; further stress cycles will then produce essentially congruent hysteresis loops of purely elastic compression and expansion. Under these conditions, the line A-A′-A″ will terminate at the top of the loops – a point representing full dissipation of midplane pore pressures and maximum compaction (plastic-plus-elastic) for the existing maximum stress. At this point, it is evident that the reciprocal slope of the line A-A′-A″ represents the compaction-stress ratio, which is S_k, the component of the "long-term" (steady-state) system storage coefficient attributable to plastic (-plus-elastic) skeletal compression. Furthermore,

$$\frac{S_k}{m} = S_{sk} = M_v \gamma_w ; \quad M_v \gg \alpha$$

where M_v is the overall compressibility of the entire aquifer system for the stress range involved.

Determination of S_k from the slope of line A-A′-A″ on figure 1 is somewhat complicated by the fact that the stresses did not cycle through exactly the same range each year. If the applied stress is abruptly increased beyond the maximum previous level, there will be superimposed on the existing time-consolidation pattern a new response equivalent to the time-consolidation curve for the new stress increment under initially uniform pore-pressure distribution (Scott, 1963, pp. 214-218). Episodes of this type took place near the end of the pumping seasons in 1960, 1961, 1964, and 1966.

The principal immediate effect is a moderate increase in compaction of perhaps 2 to 5×10^{-3} ft per ft of increase over previous maximum stress. A probable secondary result is a reduction in the increment of compaction during the next cycle of equal or lesser stress. In any case, the reasonably good fit of the straight line A-A′-A″, while admittedly based on field data too imprecise for rigorous analysis, suggests that, at Pixley, occasional moderate and short-lived excursions of stress to new maxima have little net effect on either the linearity or average slope of the zero-excess-pore-pressure line (A-A′-A″). Accordingly, the reciprocal slope of this line is taken to be representative of the gross compressibility of the aquifer system, and the system compressibility parameters are estimated as follows:

$$S_k = 5.7 \times 10^{-2}$$

$$S_{sk} = \frac{S_k}{m} = 1.4 \times 10^{-4}\,\mathrm{ft}^{-1} = 4.6 \times 10^{-4}\,\mathrm{m}^{-1}$$

$$M_v = \frac{S_{sk}}{\gamma_w} = 3.2 \times 10^{-4}\,\mathrm{in}^2\,\mathrm{lb}^{-1} = 4.6 \times 10^{-3}\,\mathrm{cm}^2\,\mathrm{kg}^{-1}$$

By fitting the steepest and flattest reasonable lines to the data, we can further estimate tha the minimum probable value of S_{sk} is $1.1 \times 10^{-4}\mathrm{ft}^{-1}$ and the maximum is $1.8 \times 10^{-4}\mathrm{ft}^{-1}$ Since M_v is about 2 orders of magnitude larger than β_w, the compressibility of water, for practical purposes S_{sk} is equal to S_s, the steady-state specific storage for the entire aquifer system during compaction. The derived estimate of $S_{sk} = S_s$, although useful for comparing one aquifer system with another, is not directly applicable to determining the average compressibility for the aquitards, since it is based on the total thickness, m, of the aquifer system. To calculate the average aquitard compressibility, the plastic storage coefficient, S_k, should be divided by the aggregate thickness of compacting aquitards to obtain the average specific storage for the aquitards, designated $\bar{S}_s$, from which the average plastic compressibility, $\bar{m}_v$, can be obtained.

Thus

$$\bar{S}_s = S_k/\Sigma b' = \frac{5.7\times 10^{-2}}{2.46\times 10^{2}} = 2.3\times 10^{-4}\,\text{ft}^{-1} = 7.5\times 10^{-4}\,\text{m}^{-1}$$

$$\overline{m}_v = \frac{\bar{S}_s}{\gamma_w} = 5.3\times 10^{-4}\,\text{in}^2\,\text{lb}^{-1} = 7.5\times 10^{-3}\,\text{cm}^2\,\text{kg}^{-1}$$

After an estimate of the gross aquifer-system storage coefficient has been obtained, the percent of ultimate consolidation, $U\%$, can be calculated for each annual episode of compaction. Thus

$$\frac{U\%}{100} = \frac{\Delta m}{\Delta h \cdot S_k} \tag{4}$$

where Δm is the annual increment of compaction beyond the elastic range, as measured on the stress-strain graph, and Δh is the increase in stress beyond the preconsolidation level. The computed percent consolidation ranges from a minimum of 4.6 percent in 1967 to a maximum of 8.3 percent in 1961.

The determination of $U\%$ raises the obvious possibility of calculating the system time constant by equation 3, provided the appropriate functional relation, $T = F(U\%)$, can be defined. The character of this function depends upon the nature of the time-loading function (instantaneous or time-dependent) and the distribution of initial excess pore pressures. None of the available analytical functions closely approximates the highly irregular and arbitrary loading functions defined by the increasing-stress segments of the time-stress graph (fig. 1). However, the long time constant of the system, indicated by the small annual percent consolidation, suggests that short-term variations in the rates of stress increase may not be of great importance. Accordingly, an attempt has been made to estimate the time constant, τ, using an approximate procedure developed by Terzaghi and described by Leonards (1962, pp. 169, 170). The procedure is based on the nearly correct assumption that at the end of a period in which stress increases uniformly with time, the consolidation is the same as if the entire load had been applied instantaneously half way through the loading period.

As applied to the Pixley data, the procedure requires generalizing the steeply declining part of the seasonal stress-increase curve as a straight line from time, t_0, when the time-stress graph crosses the preconsolidation line, B-B′, to t_1 the time of maximum stress. If there are more than one stress maxima of about the same magnitude, the earliest is used. For those years in which there was an appreciable reduction of stress in late spring. following an initial stress increase beyond the preconsolidation level, the last peak (minimum stress) before the major stress increase was used as t_0. (If this last peak rises above line B-B′, the next intercept of the descending time-stress curve on B-B′ is used as t_0.) The stress increase, Δh, between t_0 and t_1, and the total compaction, Δm, during the same period are entered in equation 4 calculate percent consolidation. The time factor, T, is then read from the $T = F(U\%)$ curve for instantaneous loading and sinusoidal distribution of initial excess pore pressure. (The function is tabulated in Leonards, 1962, p. 164, 165.) In accordance with the Terzaghi approximation, the time constant for aquifer-system compaction may then be estimated by

$$\tau = \frac{0.5(t_1 - t_0)}{T}$$

The calculated values for τ for each year are tabulated below.

Year	Time constant (years)	Year	Time constant (years)	Year	Time constant (years)
1960	4.1	1963	4.7	1966	5.1
1961	4.9	1964	4.6	1967	3.8
1962	4.9	1965	4.7	1968	5.0
				Average	4.6

Considering the crudeness of the analytical procedure, the values of the time constant are surprisingly consistent, particularly in view of the heterogeneity of the aquifer system, the large differences from year to year in the time-stress graphs, and the even greater variation (by a factor of 6) in the increments of compaction.

The average vertical permeability of the aquitards may be estimated by equation 1 as 3.0×10^{-3}ft yr^{-1} (2.9×10^{-9} cm sec^{-1}).

The compressibilities and time constants developed in this analysis certainly cannot be regarded as representative for all or most of the aquitards in the very heterogeneous confined aquifer system at Pixley. However, they may provide, to a first approximation, the numerical values required to construct a highly idealized model whose time-consolidation behavior is reasonably similar to that of the real aquifer system. Testing of the predictive capabilities of this model by numerical methods and electrical analogy is planned for the near future.

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MODEL EXPERIMENTS ON LAND SUBSIDENCE

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Abstract

In order to investigate the characteristic relations between land subsidence and withdrawal of ground water in the aquifer, some model experiments on a large scale were performed.

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