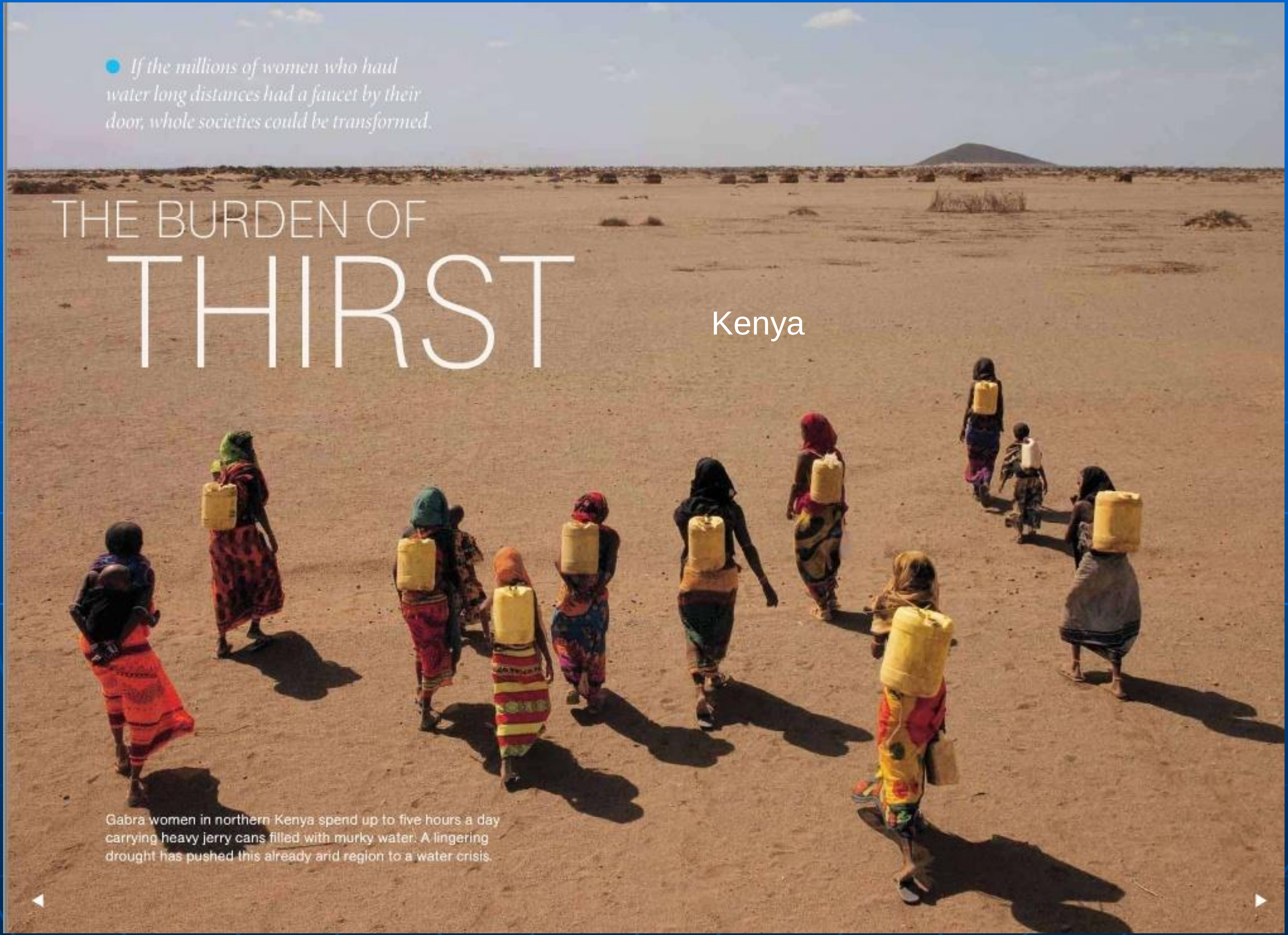
ERTH656/CEE623/ERTH654

Hydrogeology Refresher

Aly I. El-Kadi

Saline water in oceans = 97.2%
Ice caps and glaciers = 2.14%
Groundwater = 0.61%
Surface water = 0.009%
Soil moisture = 0.005%
Atmosphere = 0.001%



If the millions of women who haul water long distances had a faucet by their door, whole societies could be transformed.

THE BURDEN OF THIRST

Kenya

Gabra women in northern Kenya spend up to five hours a day carrying heavy jerry cans filled with murky water. A lingering drought has pushed this already arid region to a water crisis.


India

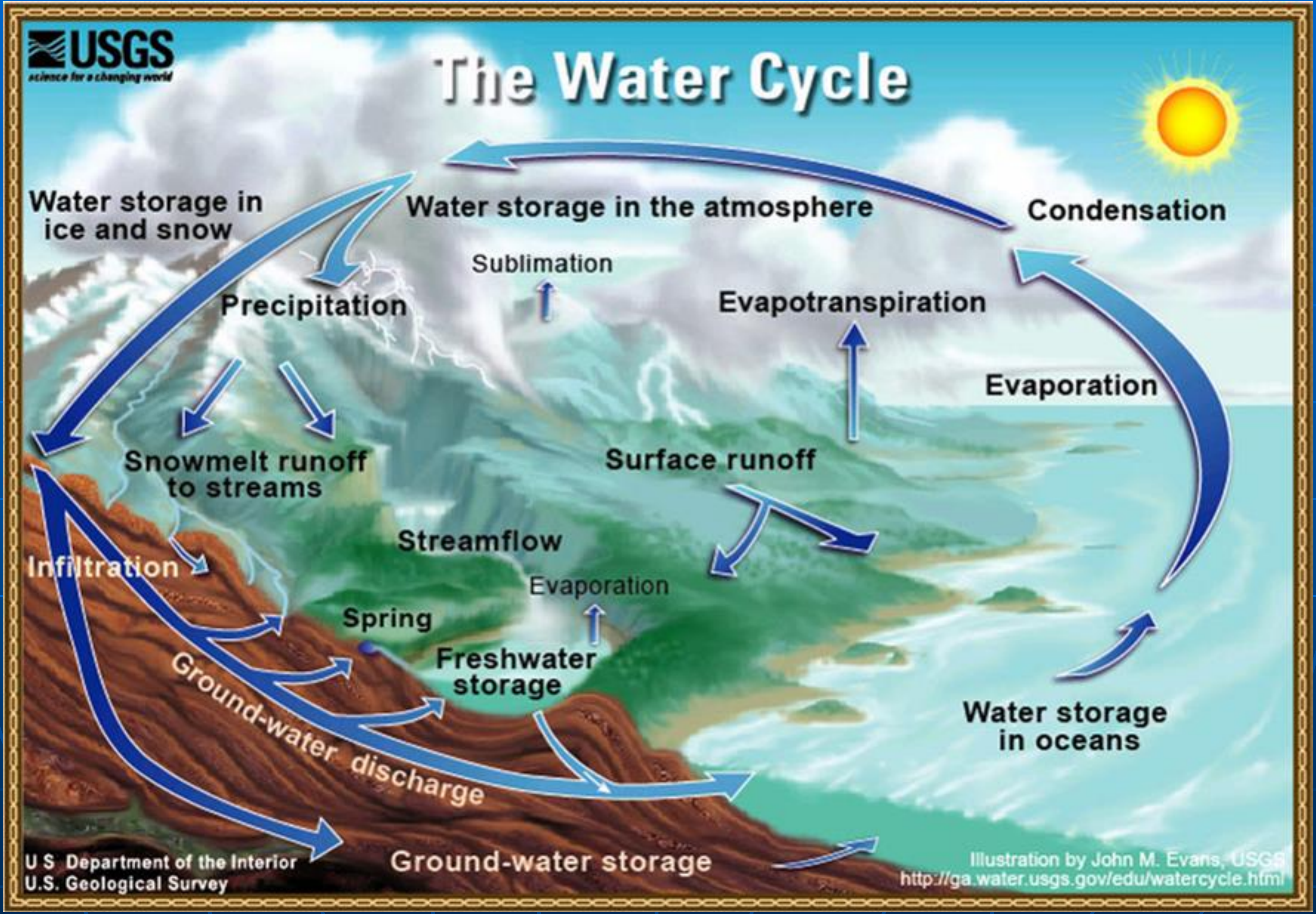
AMERICANS USE ABOUT 100 GALLONS OF WATER AT HOME EACH DAY • MILLIONS OF THE WORLD'S POOREST SUBSIST ON FEWER THAN FIVE GALLONS • 46 PERCENT OF PEOPLE ON EARTH DO NOT HAVE WATER PIPED TO THEIR HOMES • WOMEN IN DEVELOPING COUNTRIES WALK AN AVERAGE OF 3.7 MILES TO GET WATER • IN 15 YEARS, 1.8 BILLION PEOPLE WILL LIVE IN REGIONS OF SEVERE WATER SCARCITY

A seller of clean well water —ten cents a bag—has no problem finding a buyer in a slum in Luanda, Angola. In 2006 the prevalence of contaminated water in the city led to one of Africa's worst cholera epidemics, with 80,000 Angolans sickened.

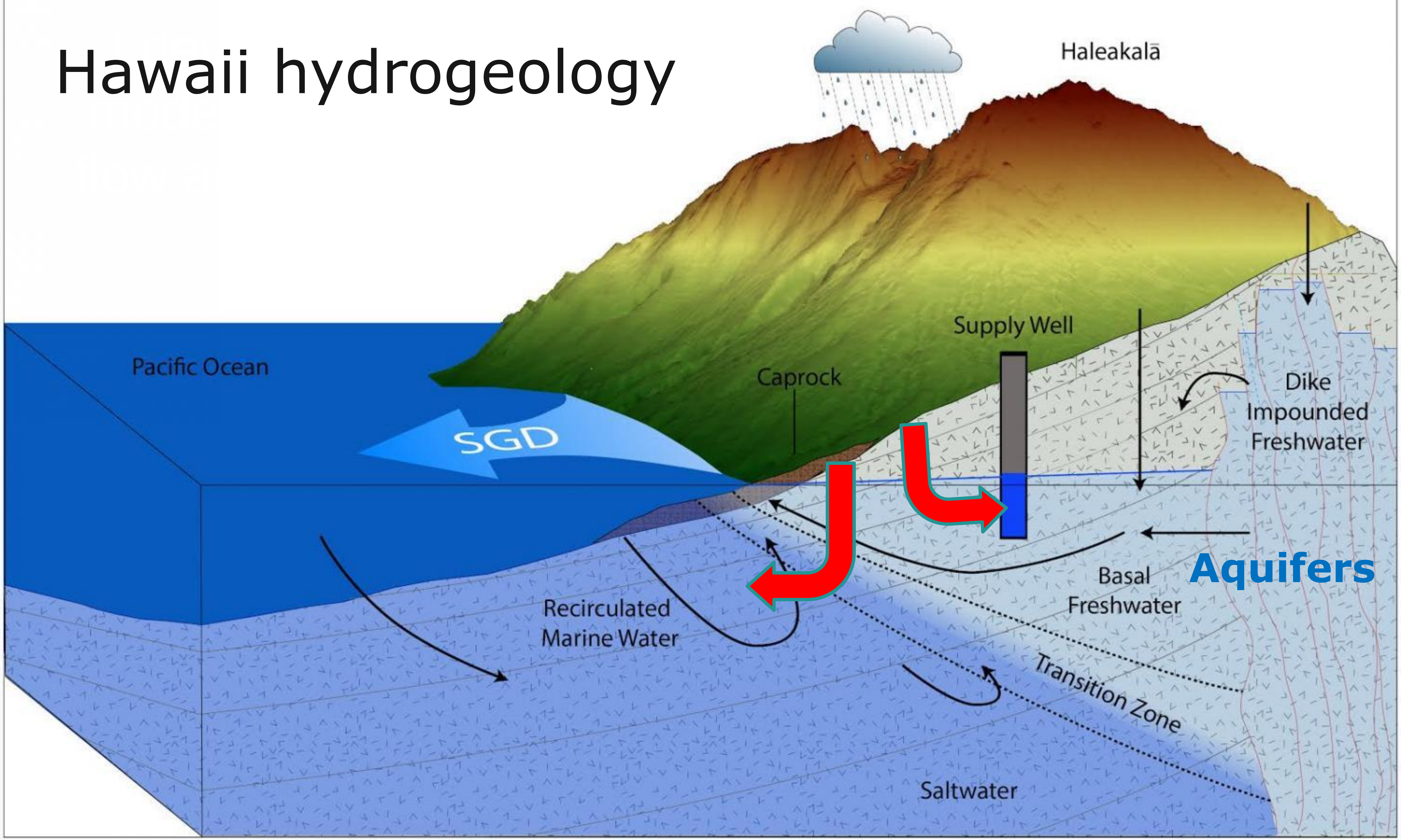
Angola

PHOTO: THOMAS HAVISHAM, PANOS

Bottled water costs 2000 times as much as tap water


USGS
science for a changing world
The Water Cycle
Water storage in ice and snow
Water storage in the atmosphere
Condensation
Sublimation
Precipitation
Evapotranspiration
Evaporation
Snowmelt runoff to streams
Surface runoff
Streamflow
Infiltration
Evaporation
Spring
Freshwater storage
Ground-water discharge
Water storage in oceans
Ground-water storage
U S Department of the Interior
U.S. Geological Survey
Illustration by John M. Evans, USGS
http://ga.water.usgs.gov/edu/watercycle.html

Hawaii hydrogeology



Dowsing

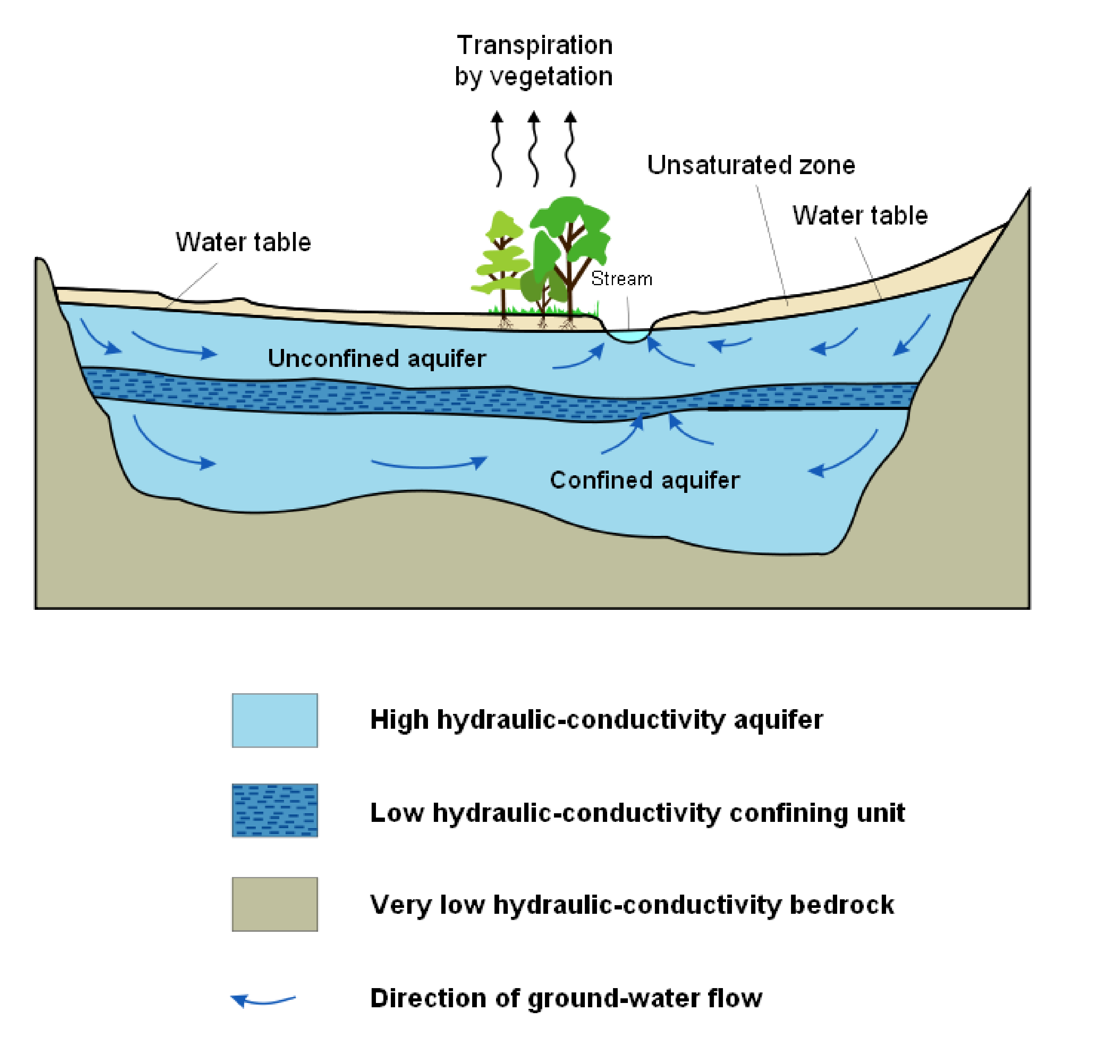
https://www.youtube.com/watch?v=J0PlekO4iWg

Aquifers

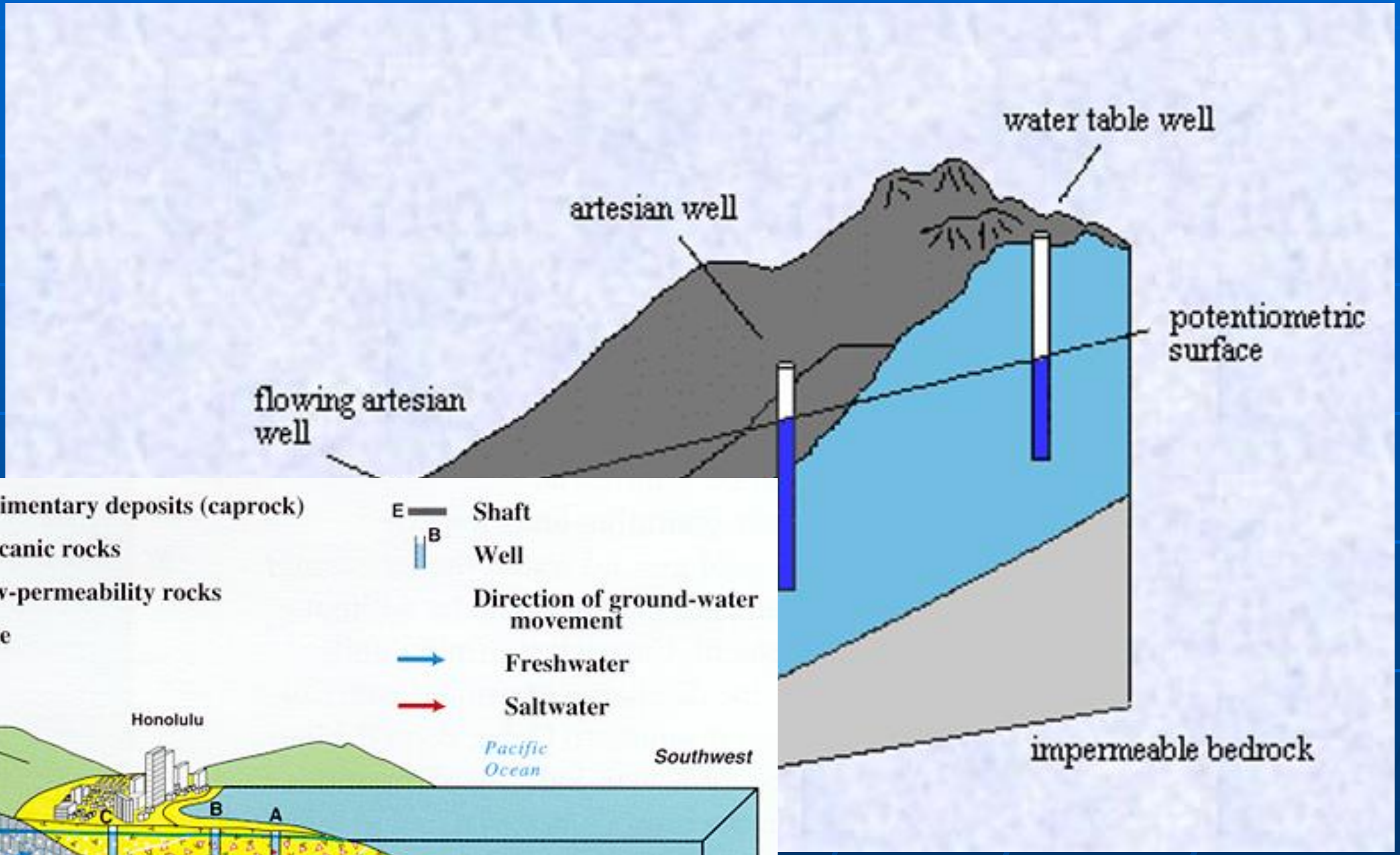
- Subsurface geological formations
- Store “reasonable” amount and transmit water “fast enough”
- Permeability $k > 10^{-12}$ cm^2
- Aquitard : k less than that

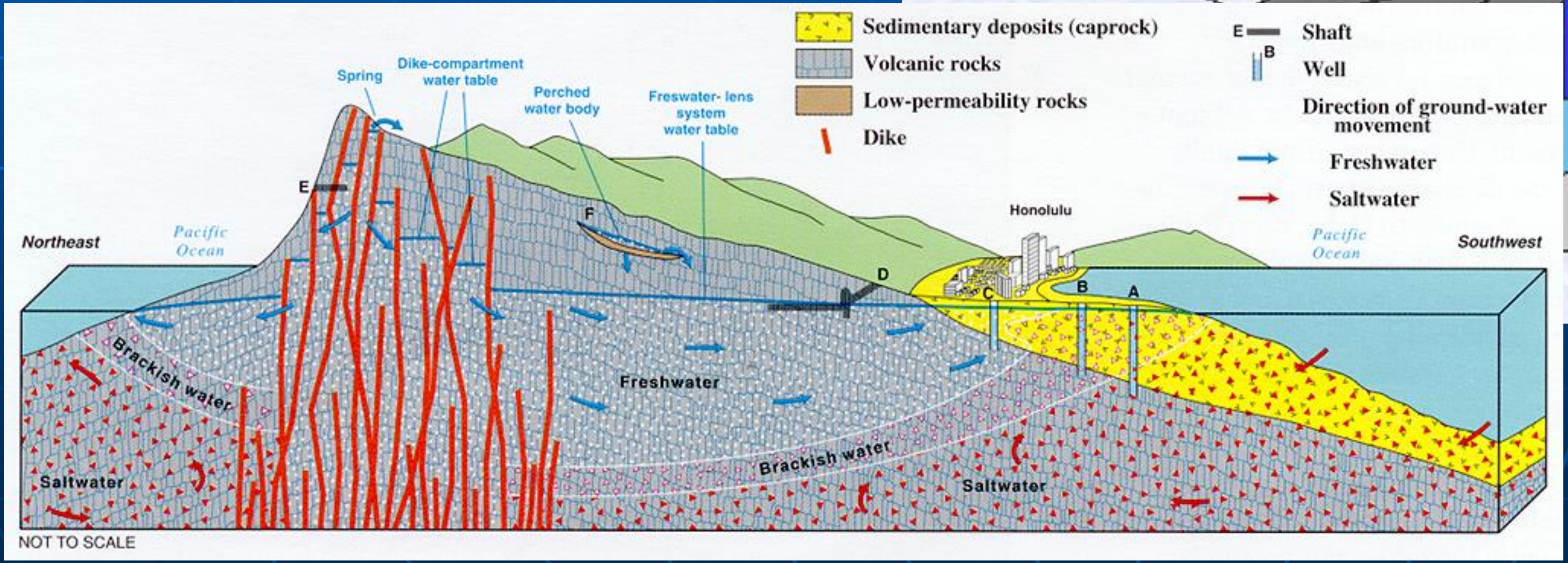
Aquifer types

1. Confined
2. Unconfined

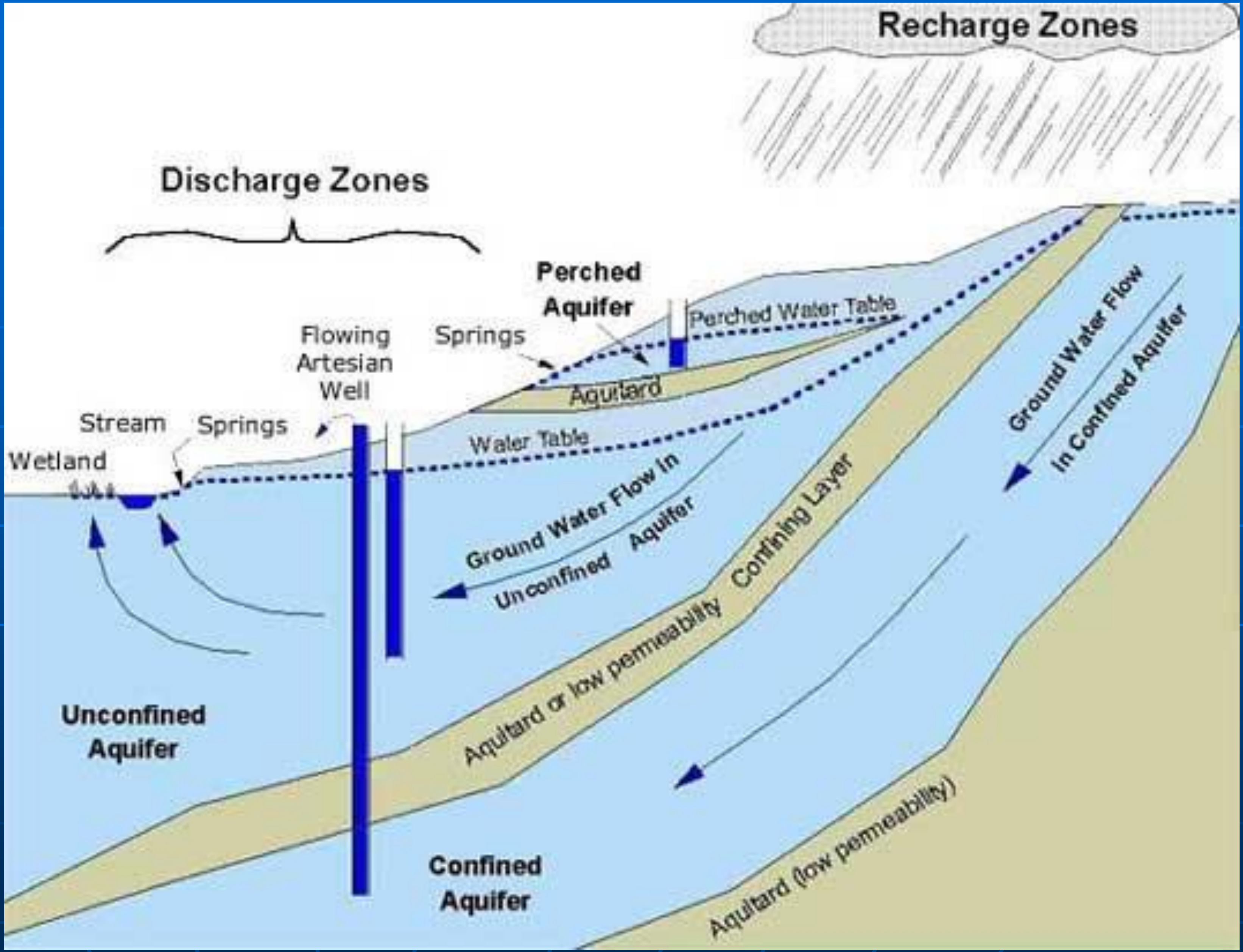


Changing from unconfined to confined conditions



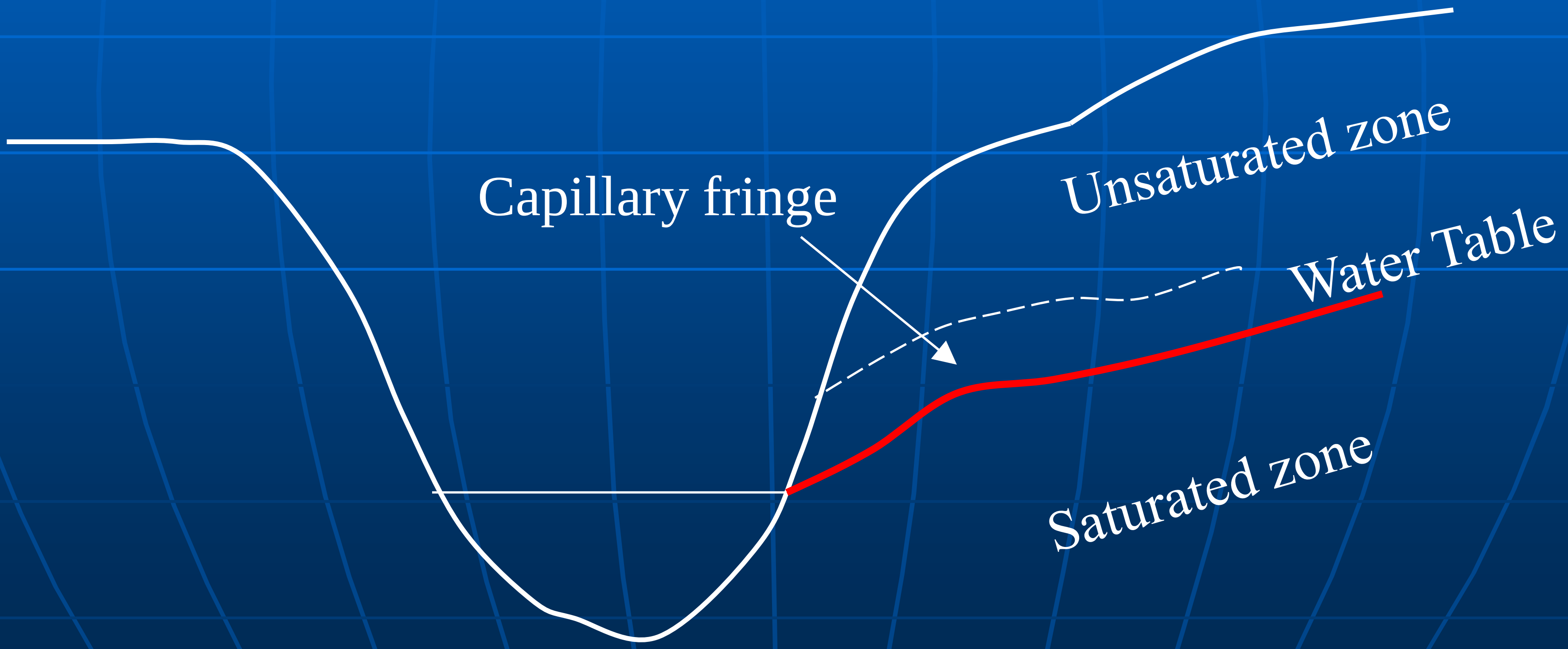


3. Perched aquifer



Water table

- For unconfined aquifers
- At water table: Pore water pressure = atmospheric pressure

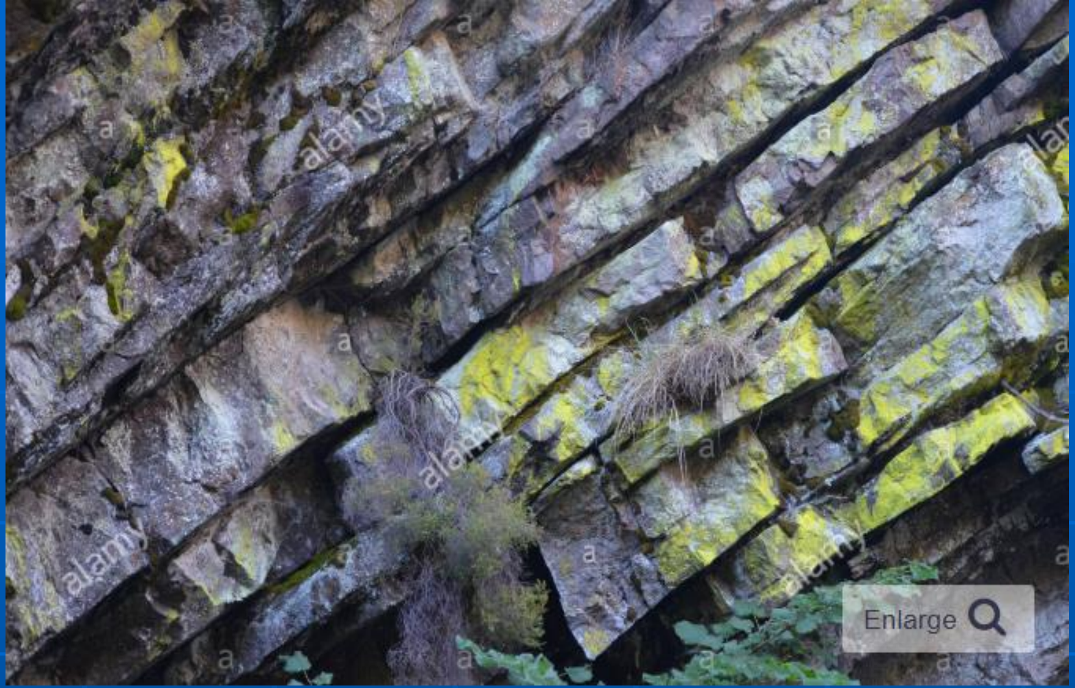


Aquifer parameters

Porosity

- Due to cracks or voids created by physical and chemical weathering processes

Fraction, e.g., 0.3 or 30%


Enlarge


Enlarge

Measuring porosity

- $$n = \frac{V_v}{V} = (1 - \frac{\rho_b}{\rho_d})$$ Fraction, e.g., 0.3 or 30%

  V_v = volume of void space

  V = total volume

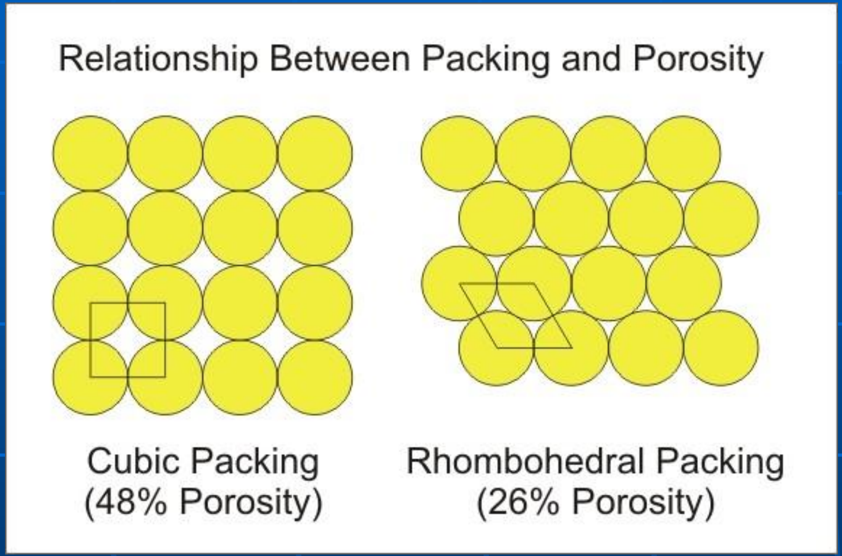
  ρ_b = bulk density

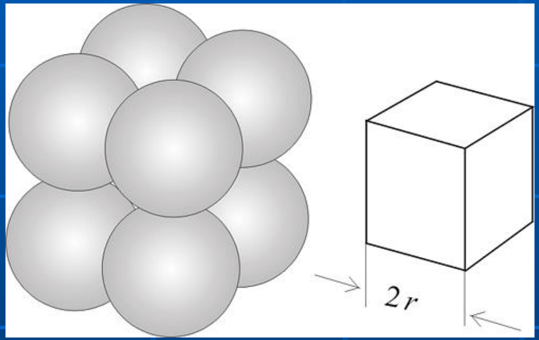
  ρ_d = particle density (~ 2.65 gm/cm3)

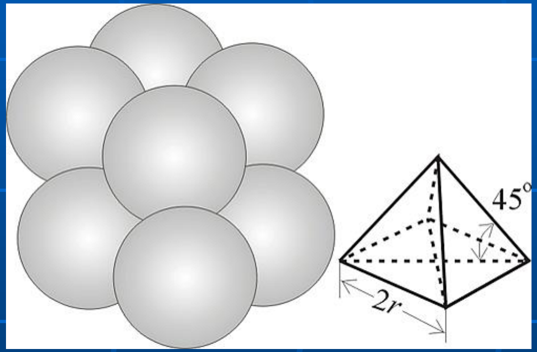
- Effective porosity: portion available for water flow

Sediments

Cubic packing n=48%
Rhombohedral Packing n=26%

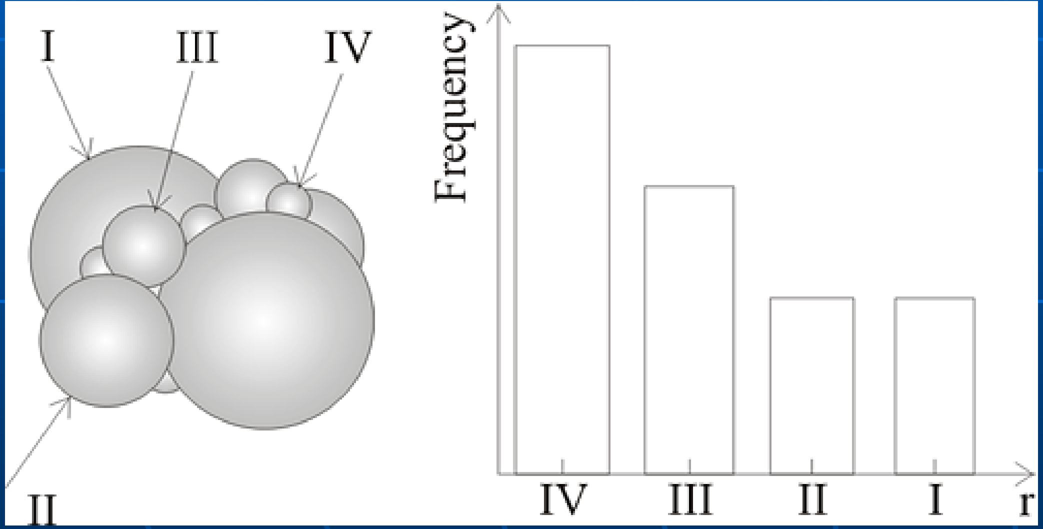




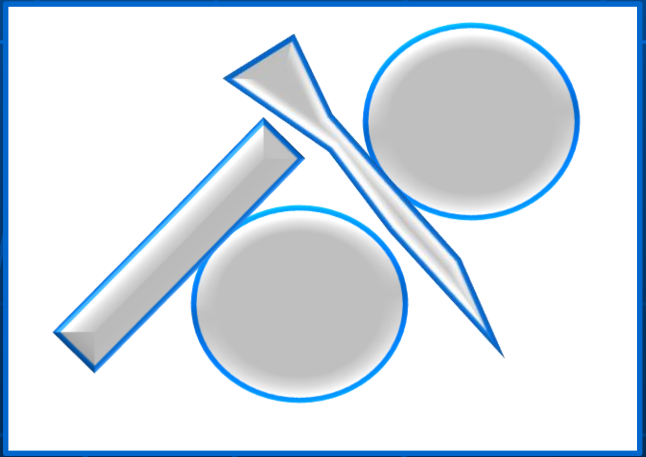


Porosity of well rounded material = 26-48% depending on packing

- Sorting: smaller Porosity
- Can use grain size distribution to classify sediments, which can be used to estimate porosity



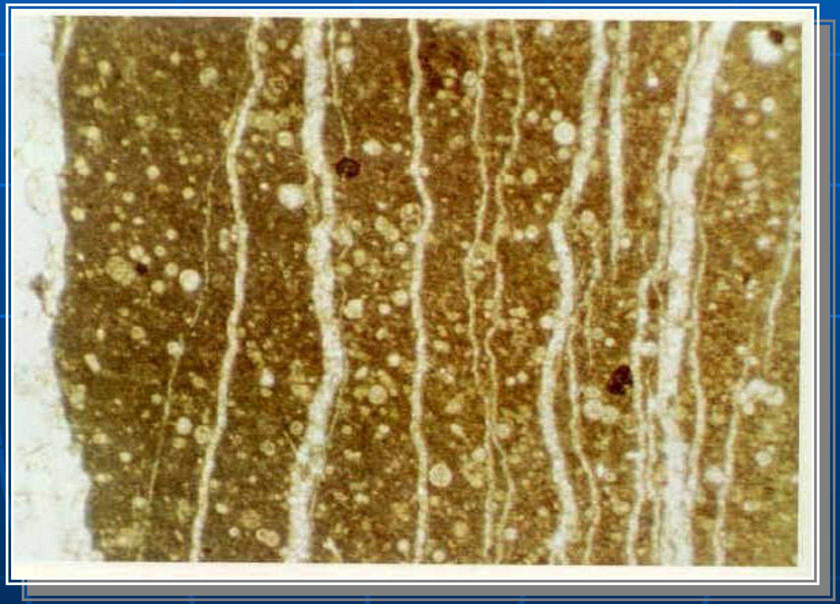
- Shape of grains & orientation



Grain Size of Sediments

http://www.brookes.ac.uk/geology/8320/grainsize.html

Soils with macro pores

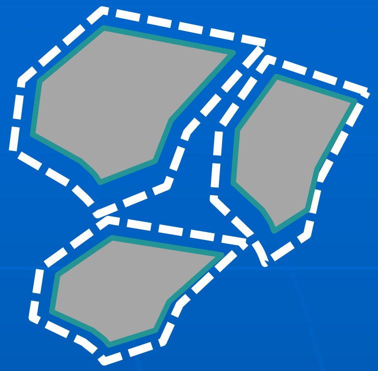


Range of Values of Porosity

	Porosity (%)
Unconsolidated deposits	
Gravel	25-40
Sand	25-50
Silt	35-50
Clay	40-70
Rocks	
Fractured basalt	5-50
Karst limestone	5-50
Sandstone	5-30
Limestone, dolomite	0-20
Shale	0-10
Fractured crystalline rock	0-10
Dense crystalline rock	0-5

Freeze and Cherry, *Groundwater,* Ch. 2, Table 2.4

Specific Yield



- $S_y = \dfrac{\textit{Volume of water drained by gravity}}{\textit{total volume of rock}}$
- Cannot release water when gravity force = surface tension (called pendular water)
- Porosity can be the same, S_y is smaller for finer material (larger surface area)

$n = S_y + S_r$

↑ ↖

Porosity specific retention

0 – 5%	clay
10-28%	fine sand
12-26%	coarse gravel

Storage parameters

- Specific storage (S_s)
  Volume of water absorbed or expelled per unit aquifer volume per unit change in head [1/L]

$$S_s = \rho_w g(\alpha + n\beta)$$

ρ_w = water density [M/L^3]
g = acceleration of gravity [L/T^2]
α = matrix compressibility [LT^2/M]
n = porosity [dimensionless]
β = water compressibility [LT^2/M]

- Storage coefficient (Storativity) (S)
  Volume of water absorbed or expelled per unit aquifer area per unit change in head [dimensionless]

Confined aquifer: Water is released due to compressibility:

$$S = S_s b$$

Unconfined aquifers: Water is released by draining the pores (voids) and compressibility; the latter can be ignored:

$$S = S_y + S_s h$$

$$S \sim S_y$$

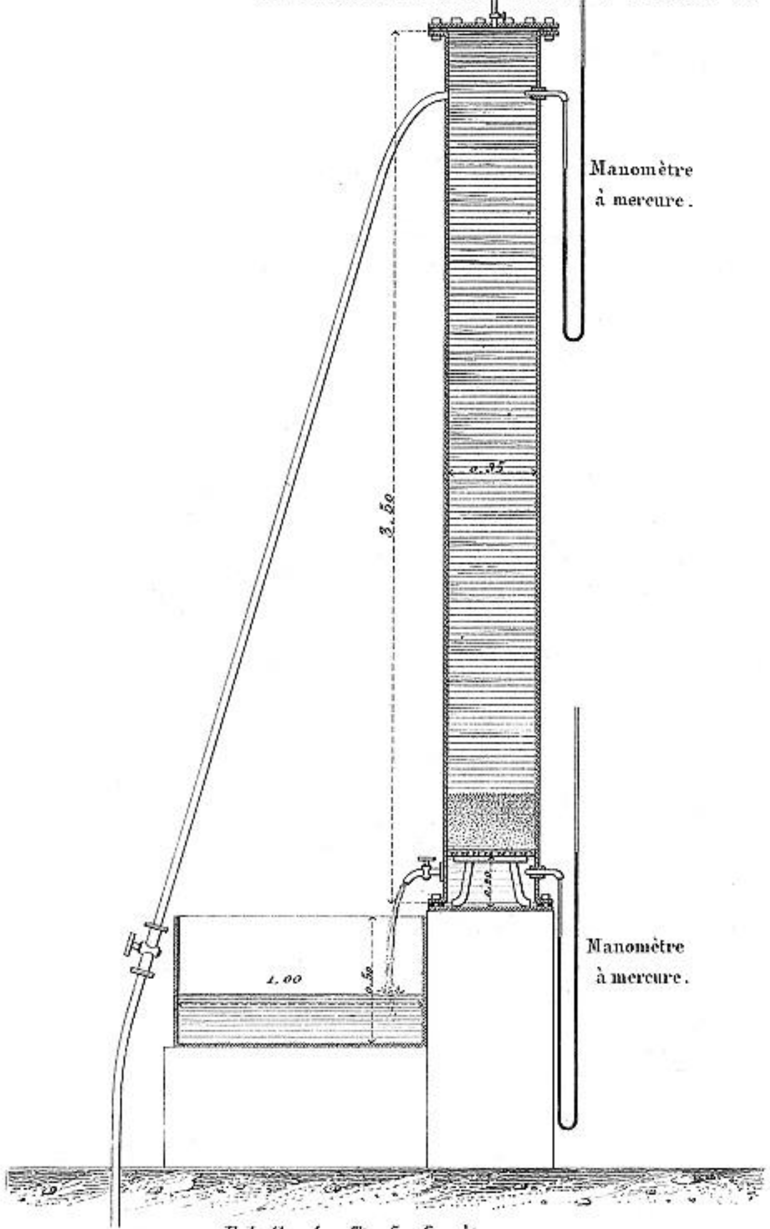
Hydraulic Conductivity

Darcy’s law

Henry Darcy (1856)

Fig. 3.

Appareil destiné a déterminer la loi de l'écoulement de l'eau à travers le sable.



Echelle de 0.m025 p.r mètre.

Discharge Q:

$Q \sim h_L$ ← hydraulic head difference

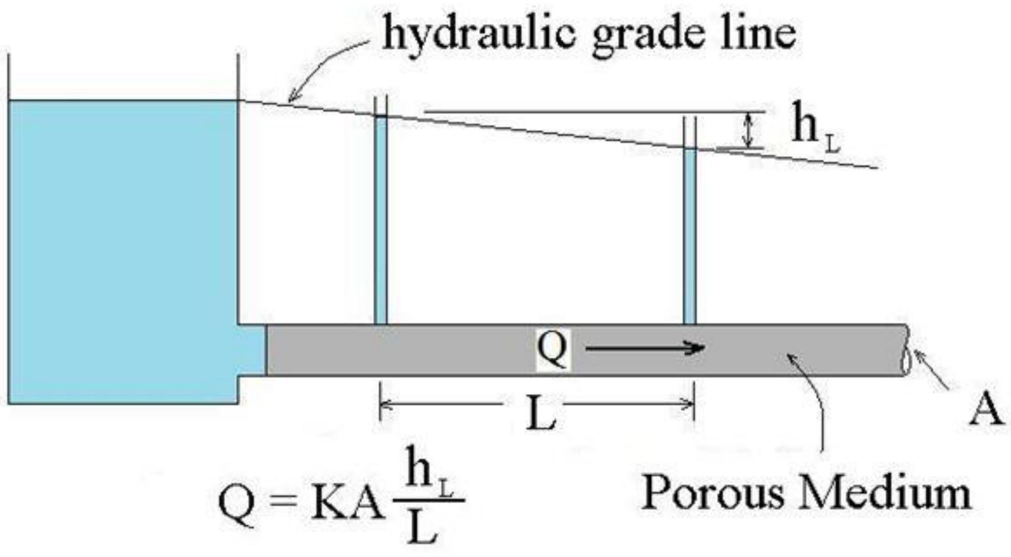
$Q \sim \frac{1}{L}$ ← length

$Q \sim A$ ← cross-sectional area

$$\boxed{Q = KA\frac{h_L}{L} \text{ or } Q = -KA\frac{dh}{dl}}$$

$\frac{dh}{dl}$ = hydraulic gradient (always negative)

Negative sign because water moves in direction of decreasing head



Darcy's Law Apparatus

K is related to permeability by fluid properties:

$$K = \frac{k\gamma}{\mu} = \frac{k\rho g}{\mu}$$

k = permeability [L^2]
γ, ρ, and μ = specific weight, density, and viscosity of fluid, respectively
g = acceleration of gravity

Laboratory measurements of K: Permeameters

Disturbed or undisturbed samples

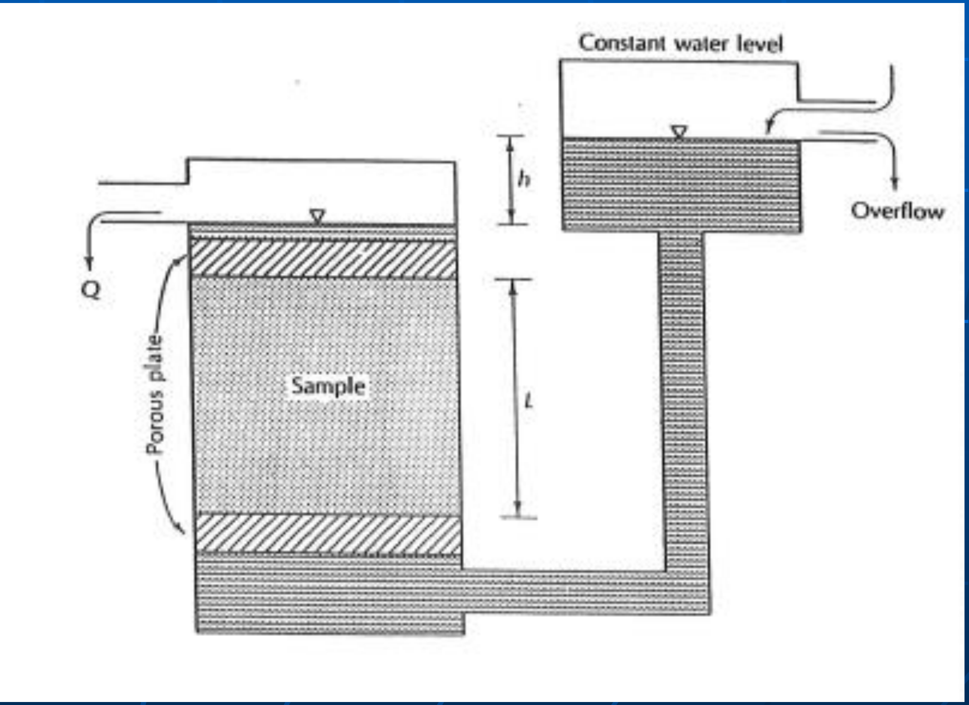
Constant-head permeameter

$$Q = -\frac{KAh}{L}$$

$$V = -\frac{KAh}{L}t$$

$$\boxed{K = \frac{VL}{Ath}}$$

V = volume collected in time t



Darcy's law is valid for Reynolds number R_N less than 10 (some studies suggest even less than one)

$$R_N = \frac{\rho v d}{\mu} \qquad \text{(dimensionless)}$$

d ~ diameter of particle

Example

ρ = 0.999x10^3 kg/m^3

μ = 1.14x10^{-5} kg/s/m

d = 0.0005 m

Calculate v such that Darcy's law is valid: $v = \frac{R_N \mu}{\rho d}$

For R_N = 1:

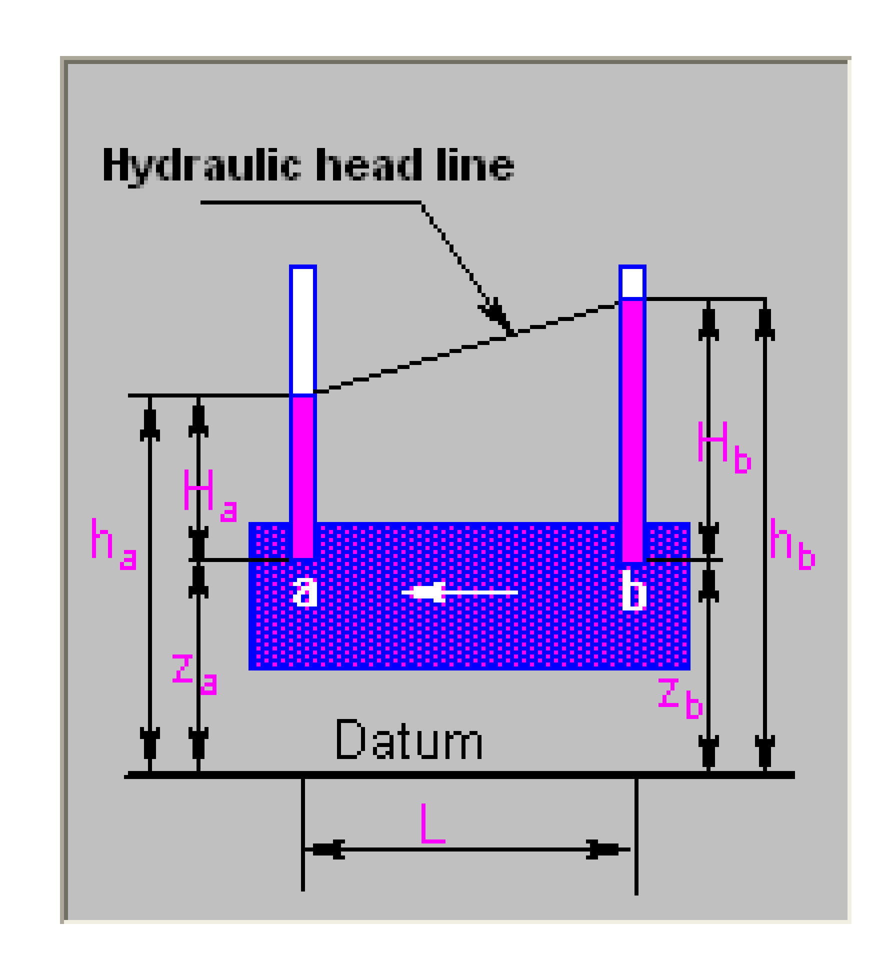
$$v = \frac{1\text{x}1.14\text{x}10^{-5}}{0.999\text{x}10^3\text{x}0.0005} = 0.0023 \text{ m/s}$$

Hydraulic (total) head

$h = z + H$ [L]
z = elevation [L]
H = pressure head [L]

$H = P/\rho_w g$

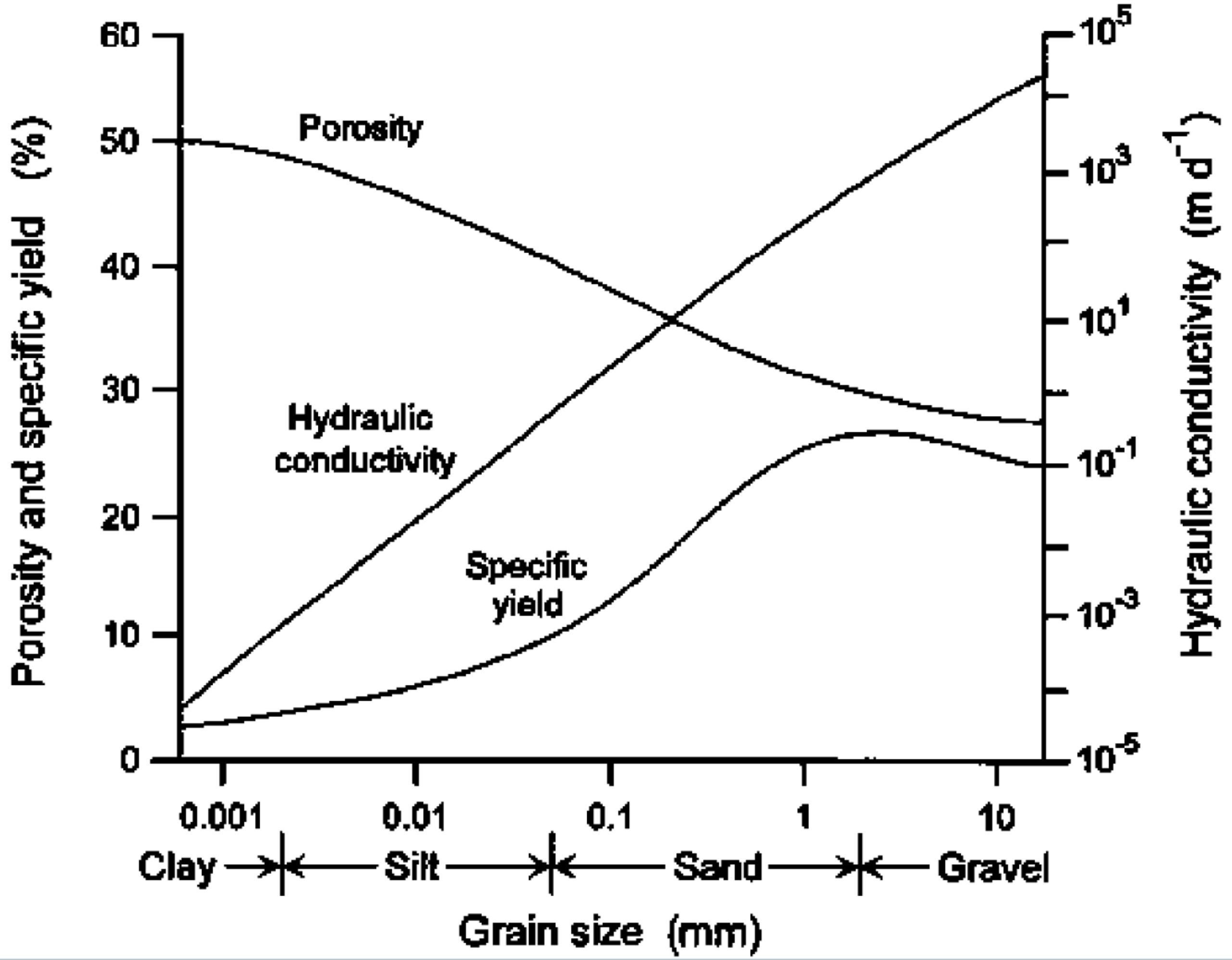
P = Pressure [M/L/T^2]
ρ_w = Density [M/L^3]
g = Gravity acceleration [L^2/T]



Darcy's experiment

Porosity, specific yield and hydraulic conductivity of granular materials

(Modified from Davis and De Wiest, 1966)



http://www.who.int/docstore/water_sanitation_health/wqassess/ch14.htm#b2-9.2.%20Characteristics%20of%20groundwater%20bodies

Transmissivity

T = Kb [L^2/T]

K = conductivity [L/T]

b = aquifer saturated thickness

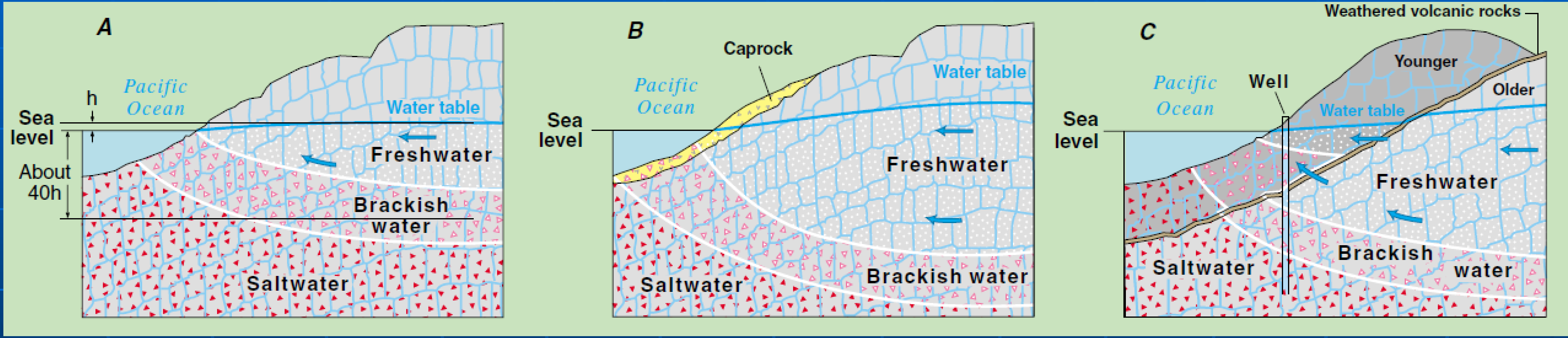
Assumes horizontal flow with uniform vertical head conditions

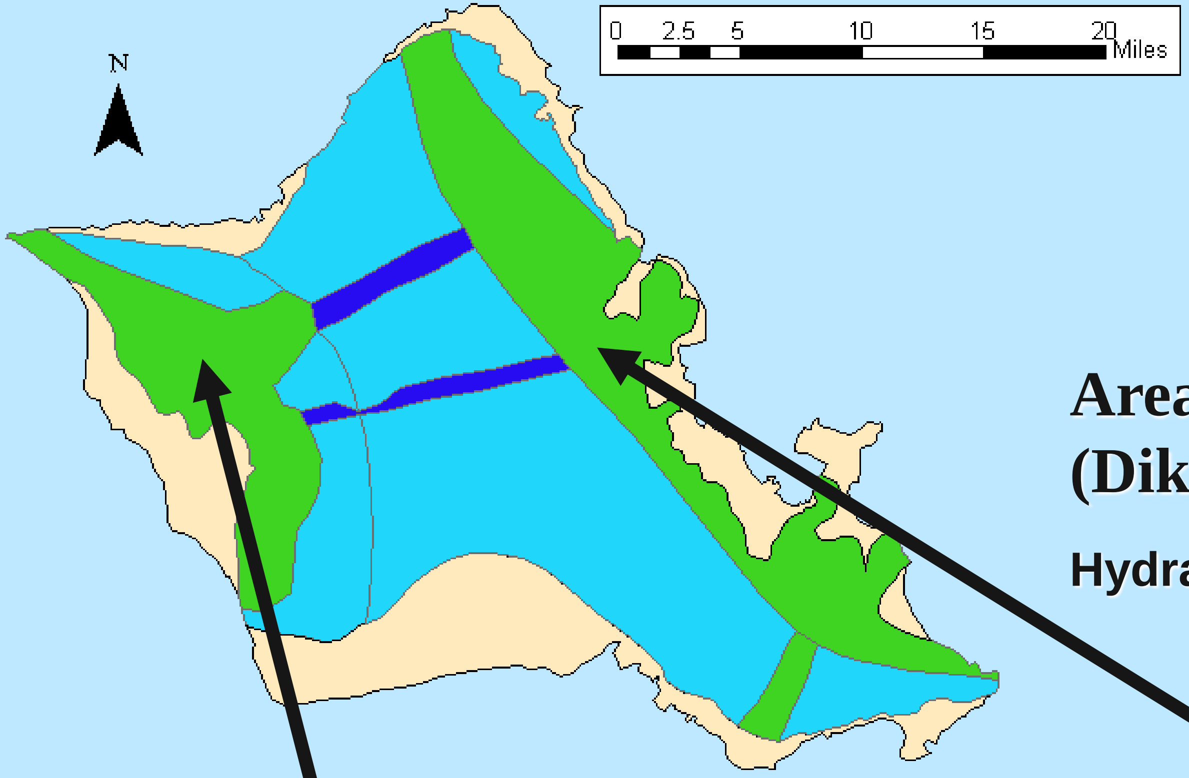
$$Q = Kb\frac{dh}{dx}\cdot 1.0$$

$$\longrightarrow T = Q \qquad for \qquad \frac{dh}{dx} = 1.0$$

Amount of water transmitted horizontally through a unit Width under unit gradient

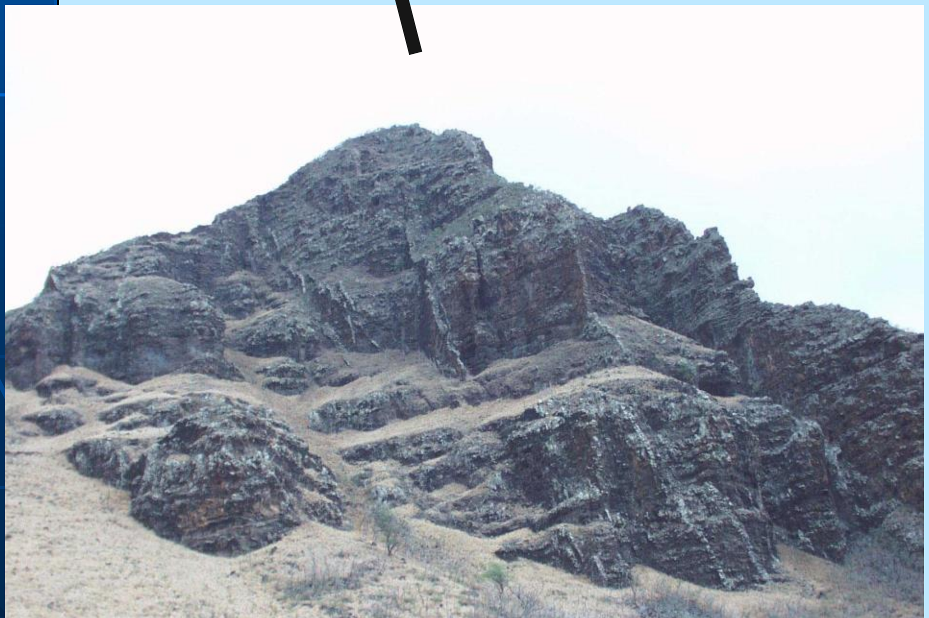
Heterogeneity and anisotropy

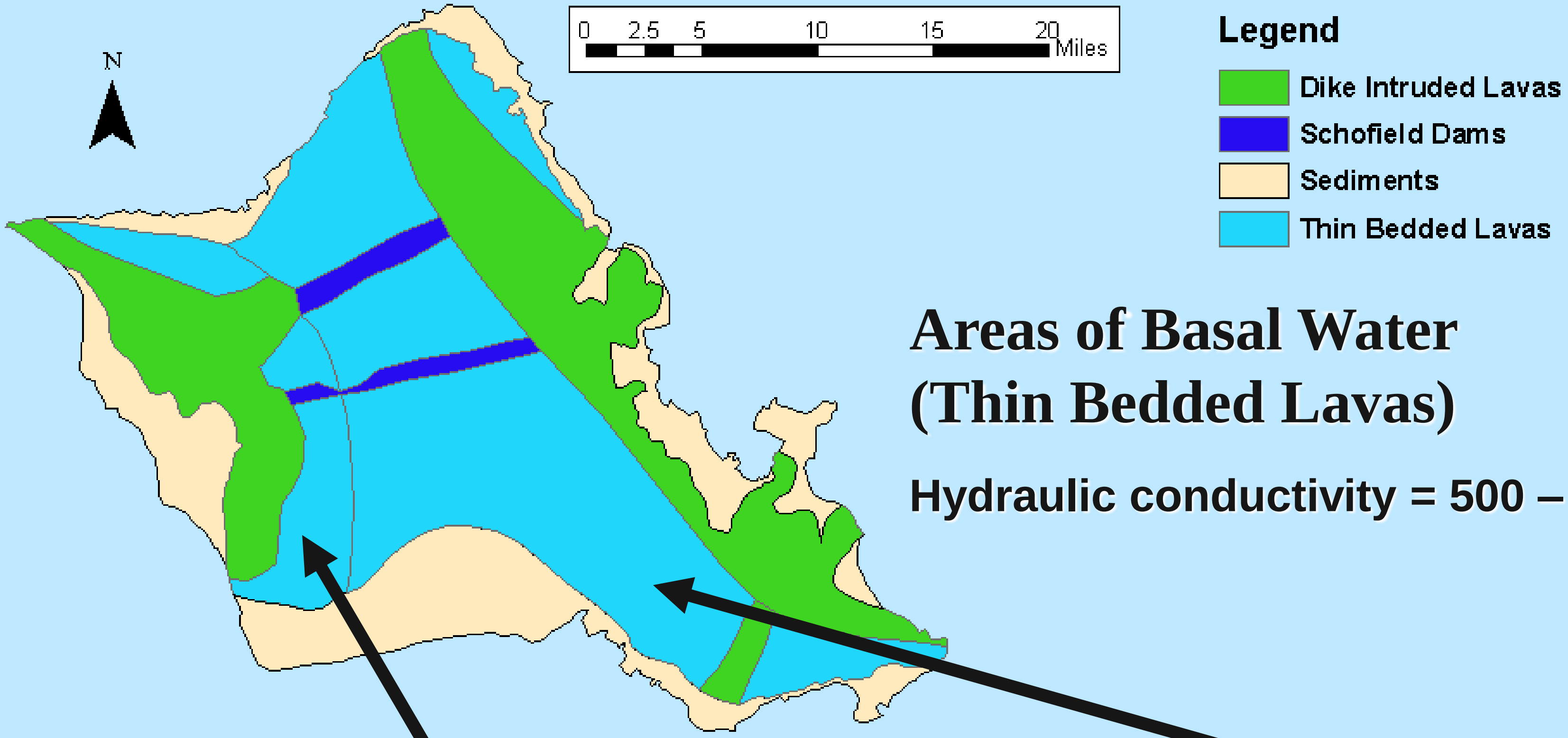




Areas of High-Level Water (Dike Intruded Lavas)

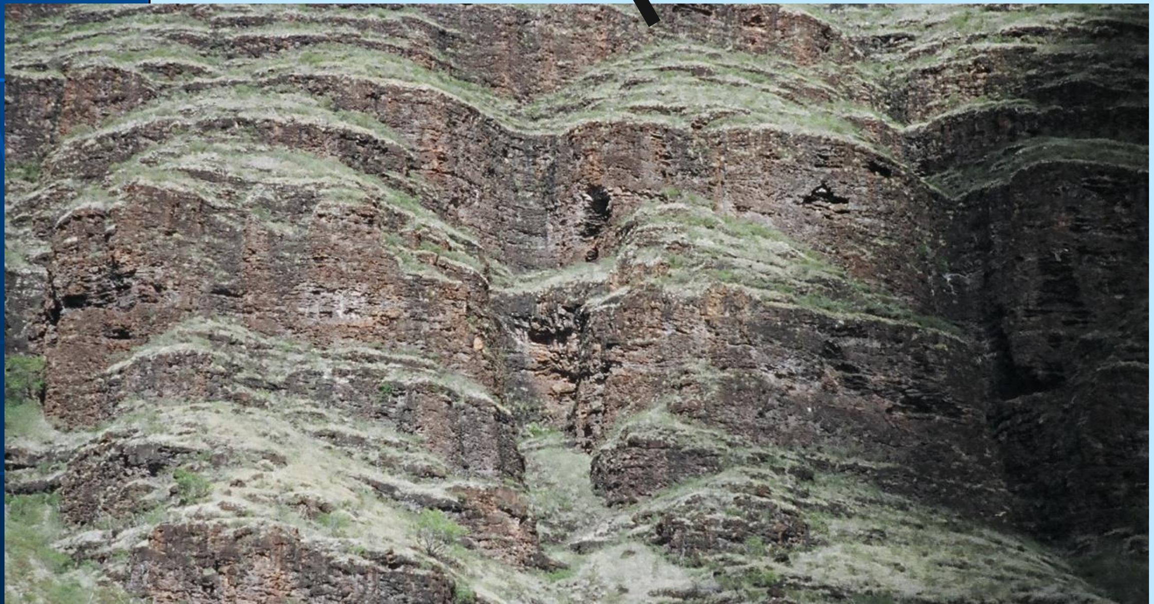
Hydraulic conductivity = 0.1 - 1 ft/d

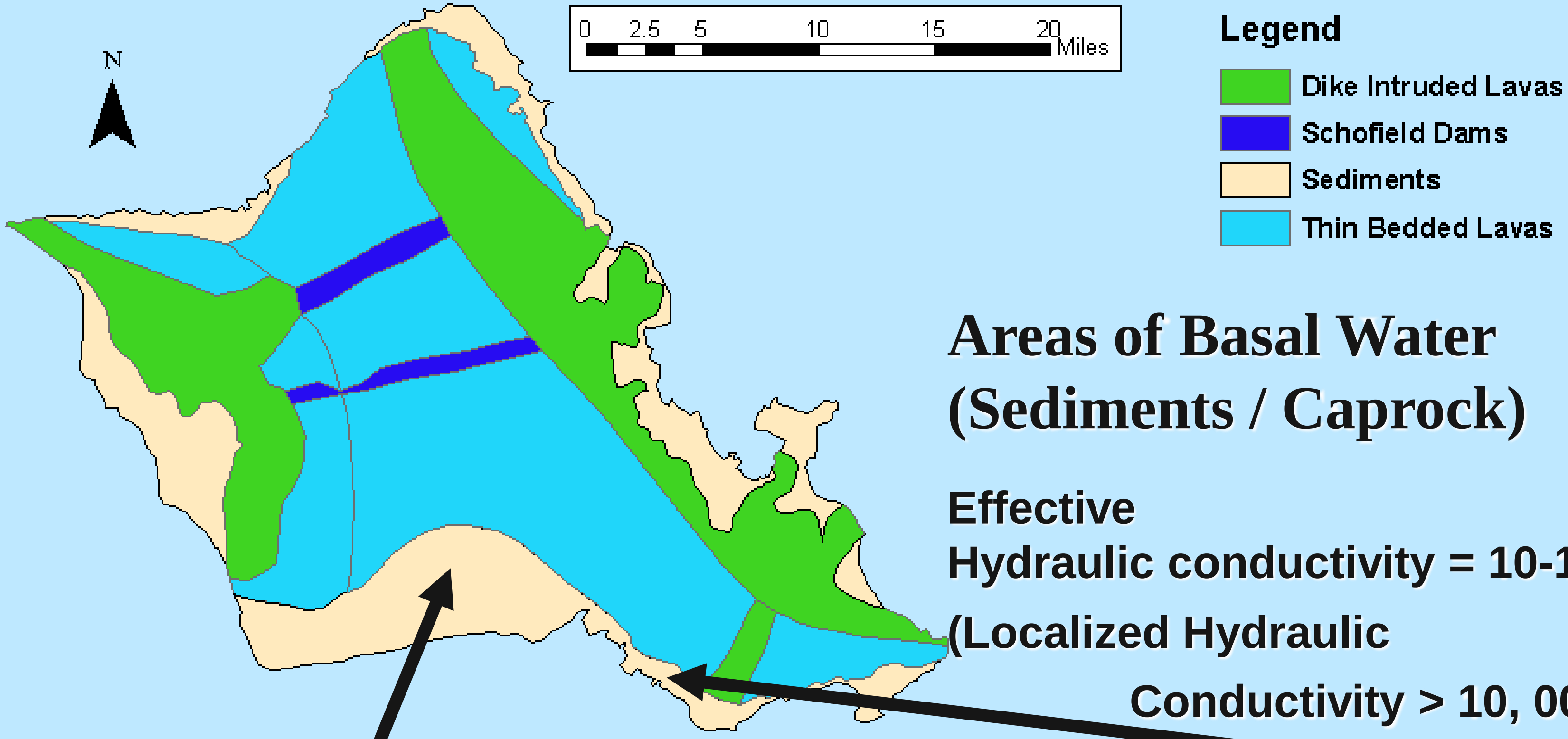




Areas of Basal Water (Thin Bedded Lavas)

Hydraulic conductivity = 500 – 5,000 ft/d



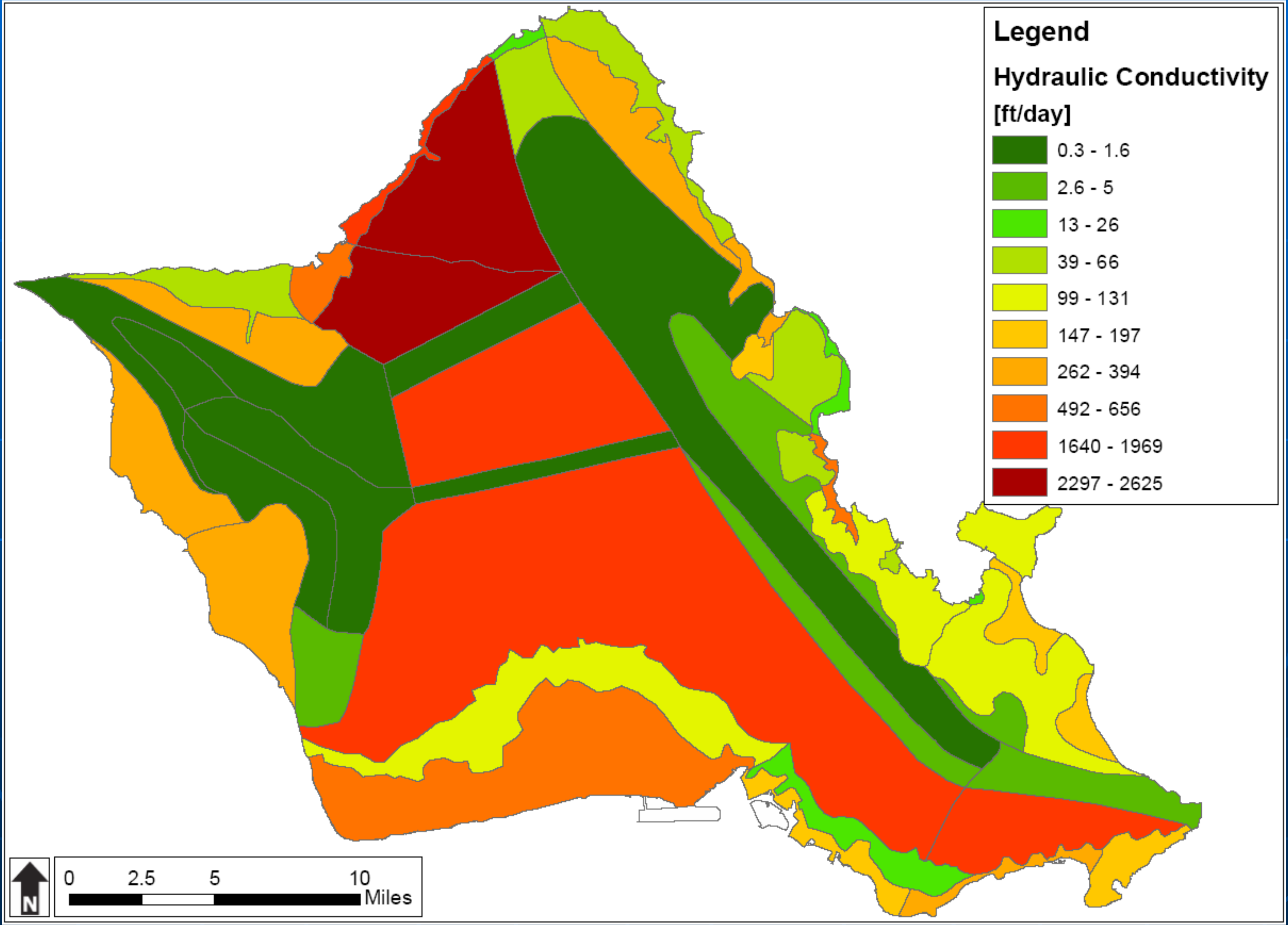


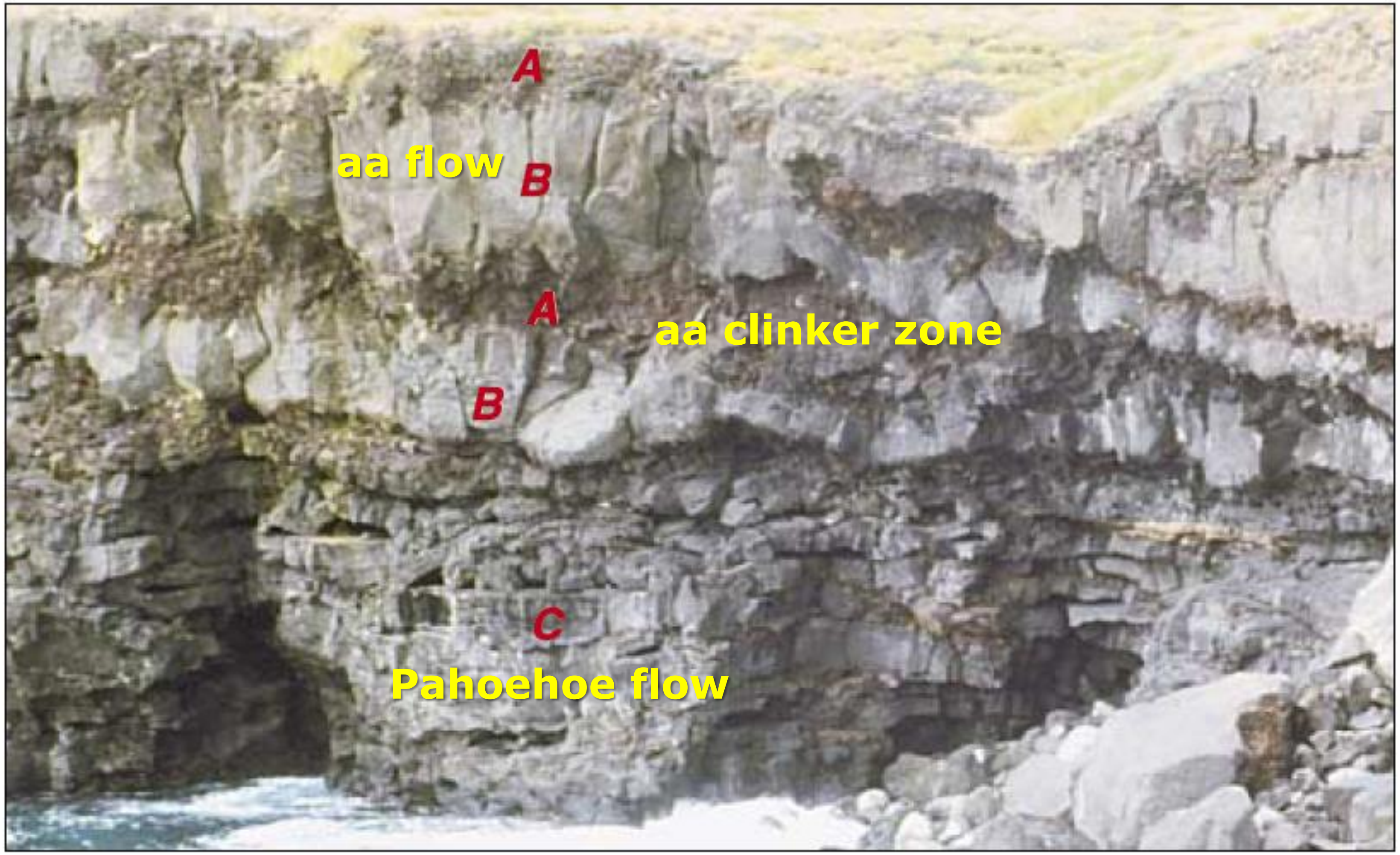
Areas of Basal Water (Sediments / Caprock)

Effective
Hydraulic conductivity = 10-100 ft/d

(Localized Hydraulic Conductivity > 10, 000 ft/d)

Hydraulic Conductivity Zones



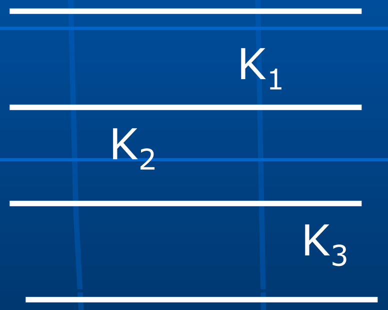


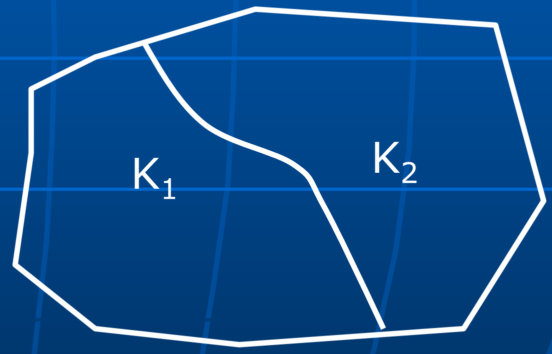
*A typical sequence of lava flows contains aa clinker zones (**A**) of relatively high permeability that occur above and below the massive central cores of aa flows (**B**), and many thin pahoehoe flows (**C**). The sequence shown is about 50 feet thick. (photo by Scot K. Izuka, USGS).*

Homogeneity

- Homogenous aquifers: same properties at all locations
- Opposite is called heterogeneous

T is heterogeneous

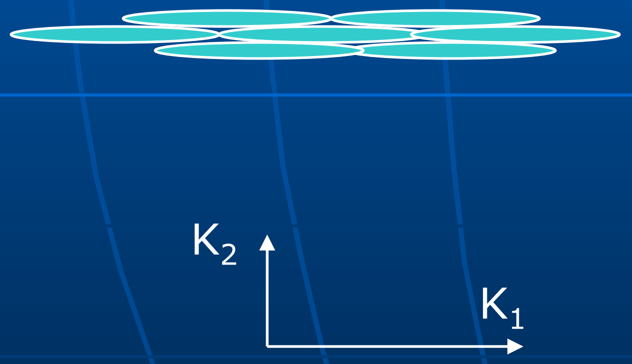




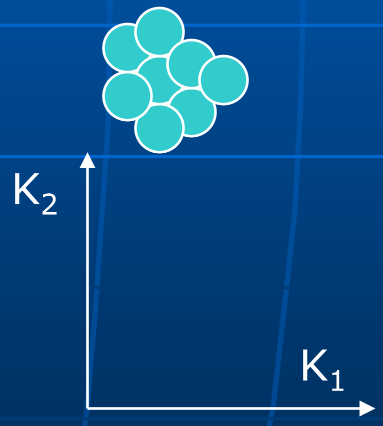
K is heterogeneous

Isotropy

- Isotropic aquifer: same conductivity in all directions
- Opposite is called anisotropic



Anisotropic



Isotropic

Hydrology is a quantitative subject

- What happens to aquifer under stresses (pumping, rainfall, etc.)?
- Are water resources sustainable?
- What is the chance of contamination?
- Which strategy works best for cleanup?
- etc.
- Integrate physics, chemistry, biology to write equations
- Solve equations to answer these questions

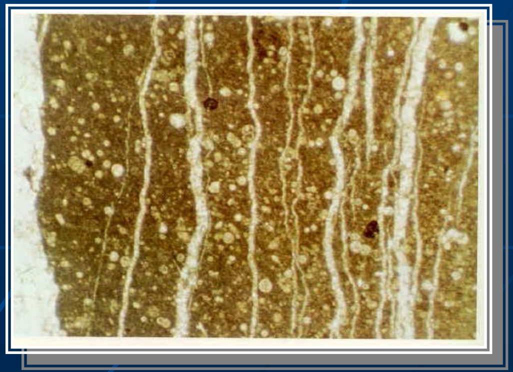
- Hydrogeology is too complicated for accurate assessment
  - Field data is needed
  - Uncertainties:
    - Equations do not describe reality
    - Data is not enough

Groundwater flow equation

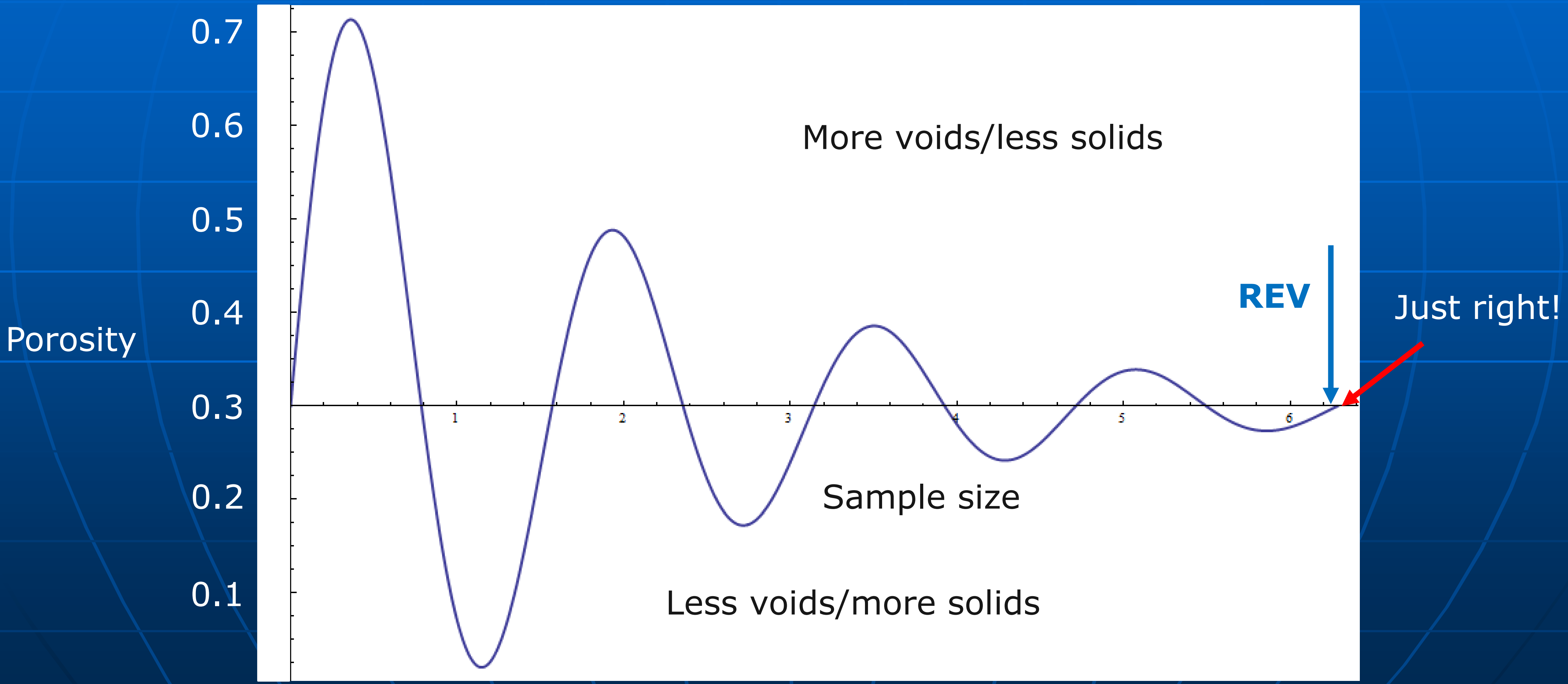
- Math Refresher

The continuum approach

- Water flow and chemical transport equations are assumed valid (applicable) at a "point"
- Medium information there should represent an aquifer of non-uniform material (solids-voids)
- Needed a minimum volume [representative elementary volume (REV)] reflecting aquifer information at this "point"
- Darcy's law is valid



The continuum approach



The continuum approach

- Darcy’s law was developed for sediments (granular material)
- Assumed valid for rocks if REV<<aquifer size
- Is not valid if REV is too large, e.g., mix of rocks and large fractures, such as lava tube
- Requires a two-media approach: Darcy’s law for the rocks and non-Darcyan (like open channel) flow for the tube with interaction between the two domains

Derivatives

- A variable h that is a function of space x and time t can be written as $h=h(x,t)$
- x and t: independent variables
- h: dependent variable.

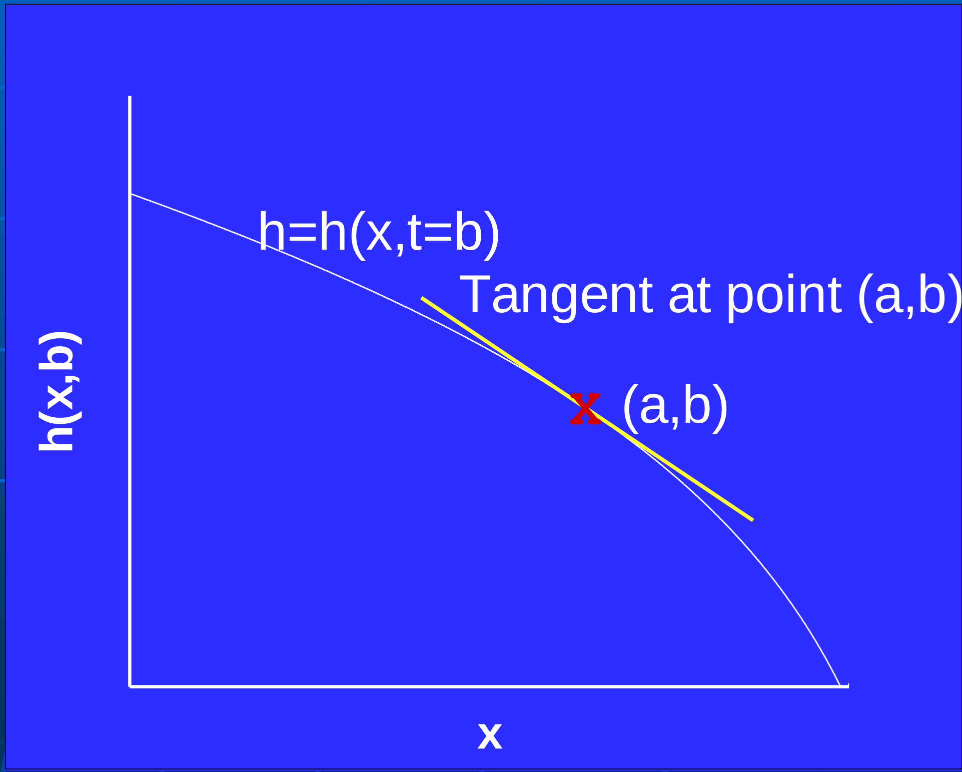
Derivatives

- Partial derivatives wrt x and t:

$$\frac{\partial h(a,b)}{\partial x} = \lim_{x \to a} \frac{h(x,b) - h(a,b)}{x - a}$$

$$\frac{\partial h(a,b)}{\partial t} = \lim_{t \to b} \frac{h(a,t) - h(a,b)}{t - b}$$

- slopes of the function h(x,t) at point x=a and t=b



Groundwater Flow Equation

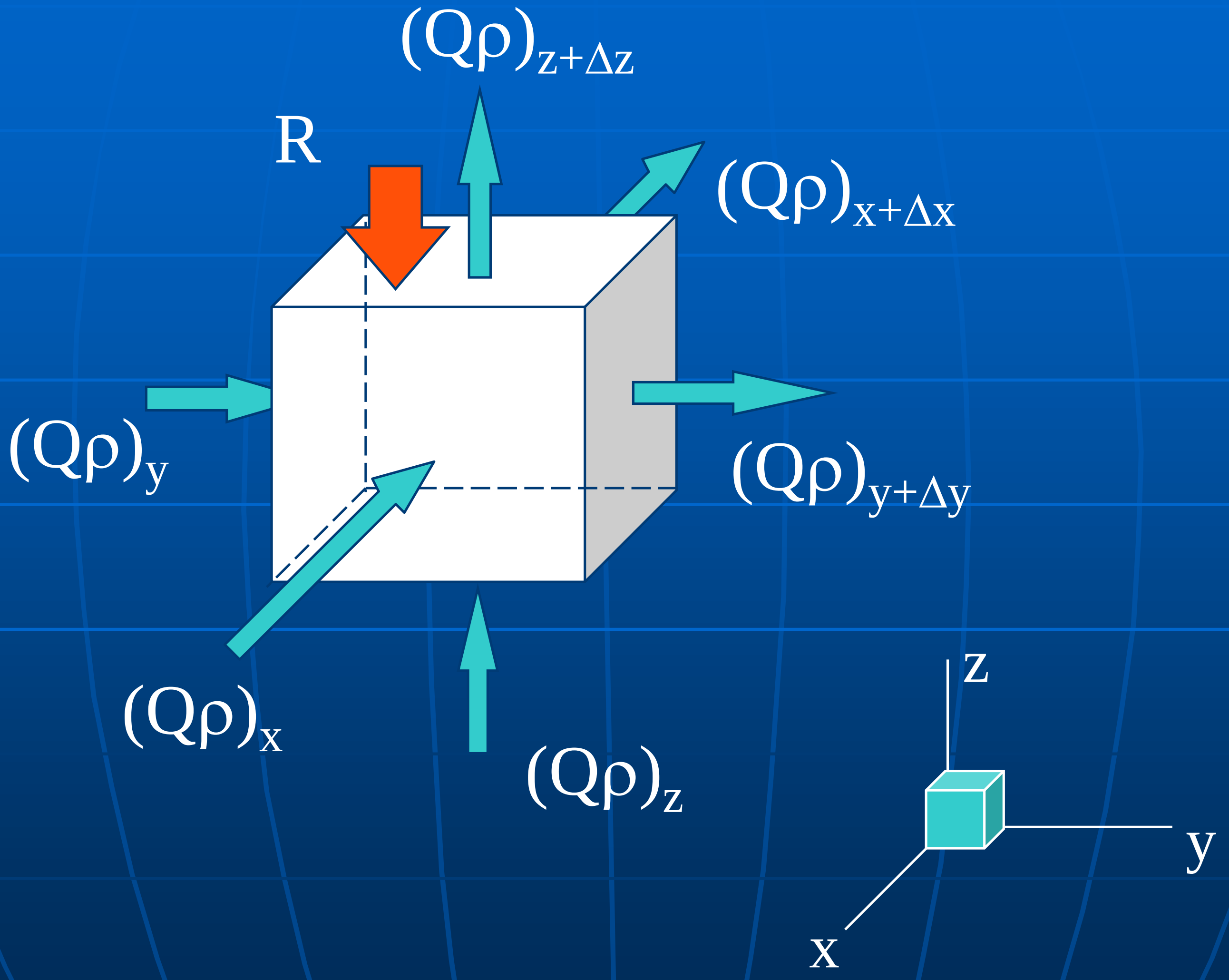
Derivation of Equation

ρ = fluid density

n = porosity

R = direct injection/discharge

Q = flux rate



$(Q\rho)_{z+\Delta z}$
R
$(Q\rho)_{x+\Delta x}$
$(Q\rho)_y$
$(Q\rho)_{y+\Delta y}$
$(Q\rho)_x$
$(Q\rho)_z$
z
y
x

(Outflow – inflow) over a time increment Δt = change of water mass in REV

$$\left\{\begin{matrix}\left[(Q\rho)_{x+\Delta x} + (Q\rho)_{y+\Delta y} + (Q\rho)_{z+\Delta z}\right] - \\ \left[(Q\rho)_{x} + (Q\rho)_{y} + (Q\rho)_{z} + \rho R \Delta x \Delta y \Delta z\right]\end{matrix}\right\}\Delta t +$$

$$\left[(n\rho)_{t+\Delta t} - (n\rho)_{t}\right]\Delta x \Delta y \Delta z = 0$$

Divide across by volume $\Delta x\Delta y\Delta z$ defining V_x as (with similar for y and z directions)

$$v_x = \frac{Q_x}{\Delta y \Delta z}$$

Insert definition of partial derivative:

$$\frac{\partial(\rho v_x)}{\partial x} = \lim_{\Delta x \to 0} \frac{(\rho v)_{x+\Delta x} - (\rho v)_x}{\Delta x}$$

$$-\frac{\partial(\rho v_x)}{\partial x} - \frac{\partial(\rho v_y)}{\partial y} - \frac{\partial(\rho v_z)}{\partial z} + \rho R = \frac{\partial(n\rho)}{\partial t}$$

Divide by ρ

$$-\frac{\partial v_x}{\partial x}-\frac{\partial v_y}{\partial y}-\frac{\partial v_z}{\partial z}+R=S_s\frac{\partial h}{\partial t}$$

Insert definition of Darcy's law

$$\boxed{\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)\overset{\pm \mathrm{R}}{\wedge}=S_s\frac{\partial h}{\partial t}}$$

Saturated 3-D equation

Where

$$S_s=\rho g(\alpha+n\beta)$$

Condensed form:

$$\nabla(\boldsymbol{K}\cdot\nabla\boldsymbol{h})+\boldsymbol{R}=\boldsymbol{S}_s\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{t}}$$

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)\pm R=S_s\frac{\partial h}{\partial t}$$

Saturated 3-D equation

- Three-dimensional
- Partial differential
- Linear
- With sources and sinks
- Transient

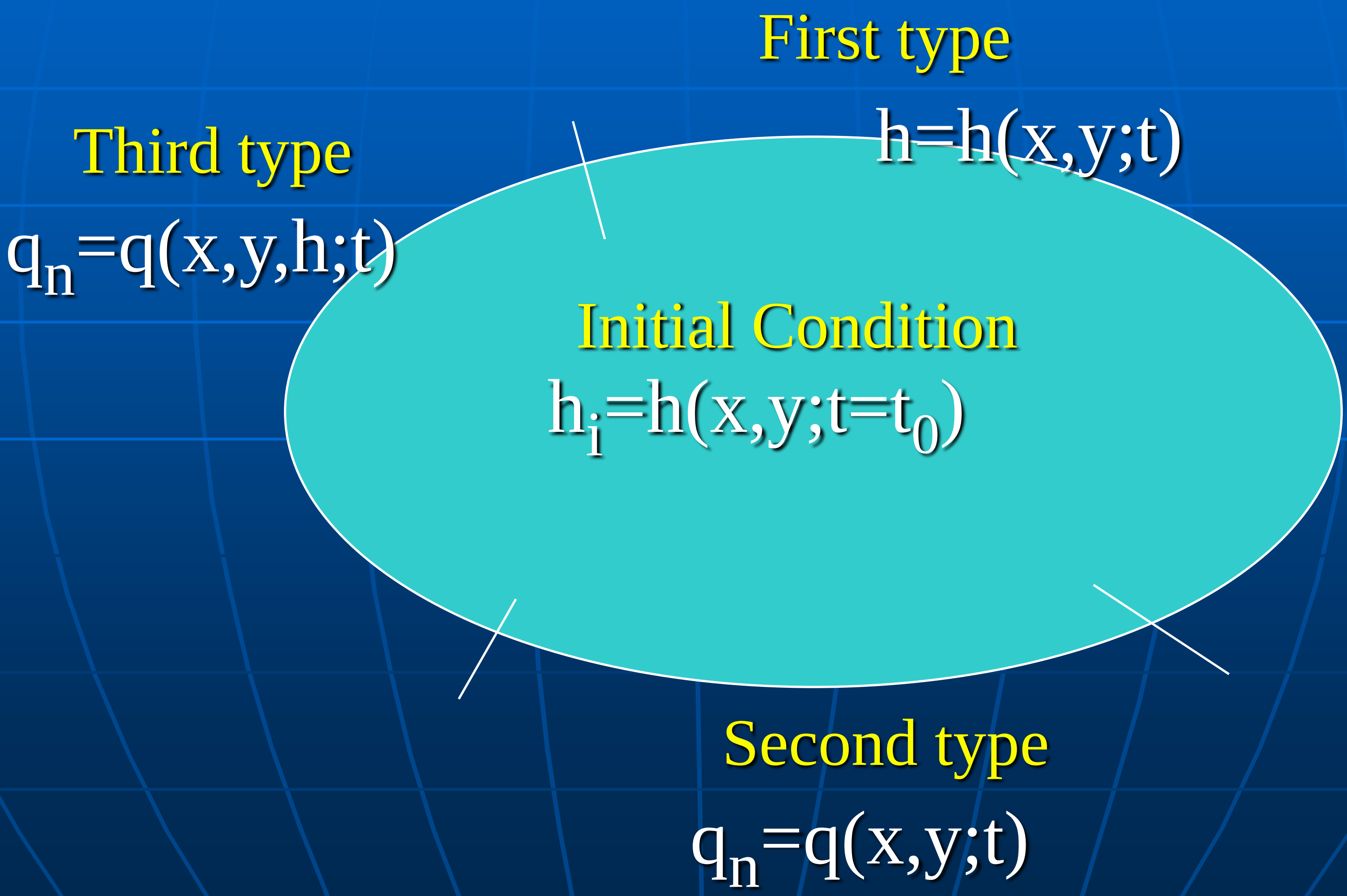
Integrating the flow over the vertical:

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right)=S\frac{\partial h}{\partial t}$$

$T = Kb$

$S = S_s b$

Boundary and Initial Conditions



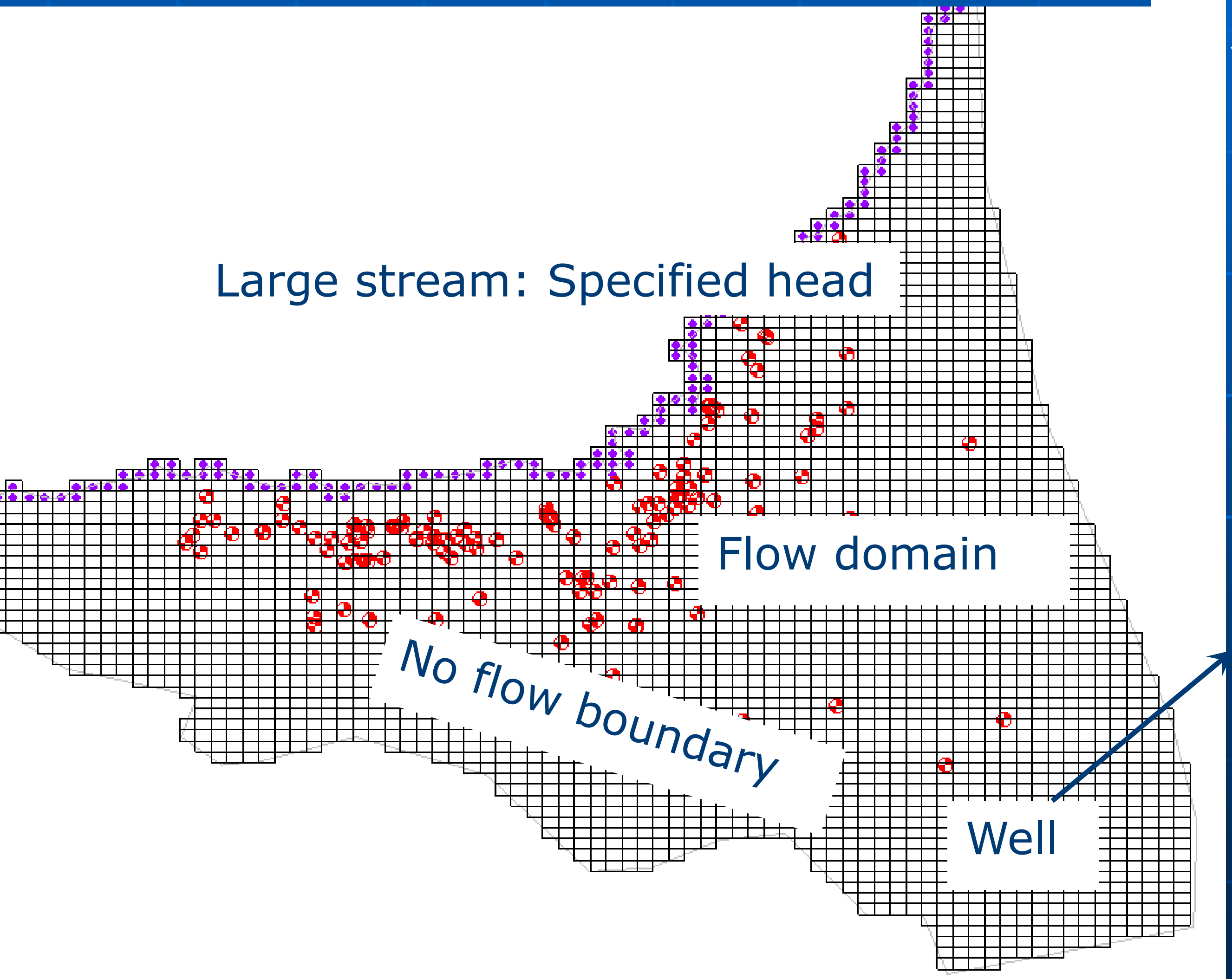
Groundwater assessment

- Problem formulation: Conceptual model

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) \pm R = S_s \cancelto{zero}{\frac{\partial h}{\partial t}}$$

Saturated 3-D equation

Example: Saturated fresh water flow for steady state
Objective: estimate water levels
Input: well fluxes, recharge boundary conditions, aquifer data, (conductivity, aquifer thickness)



No flow boundary

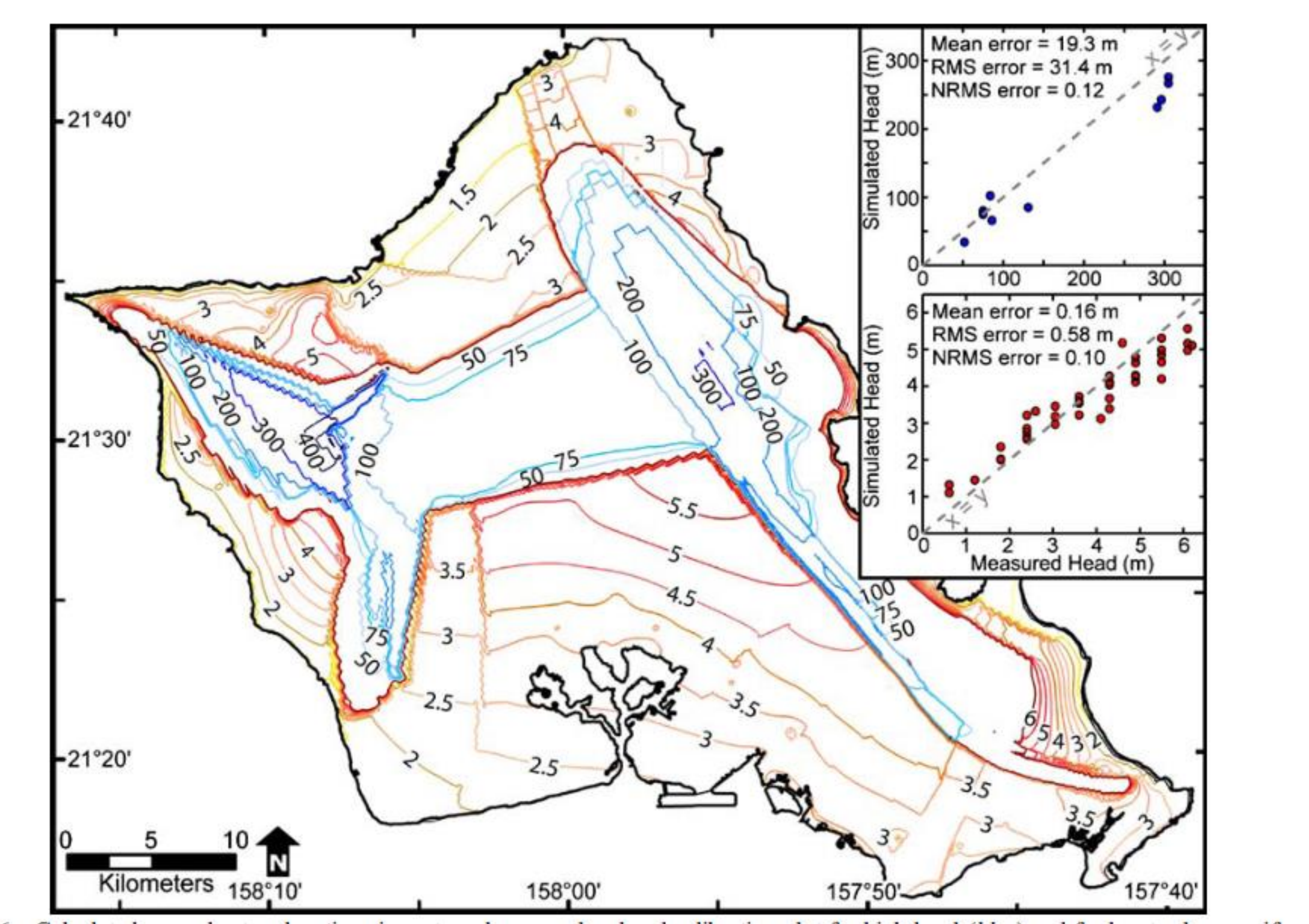
Head dependent inter-aquifer flow

Groundwater assessment

- The forward problem: estimate aquifer response (e.g., hydraulic head or water table level) to stresses (e.g., pumping)
- The inverse problem: estimate aquifer parameters (e.g., hydraulic conductivity) from hydraulic head data
- Solution can be analytical or numerical

- Example numerical (forward or inverse)

Inverse AND forward (numerical)



- Example analytical forward

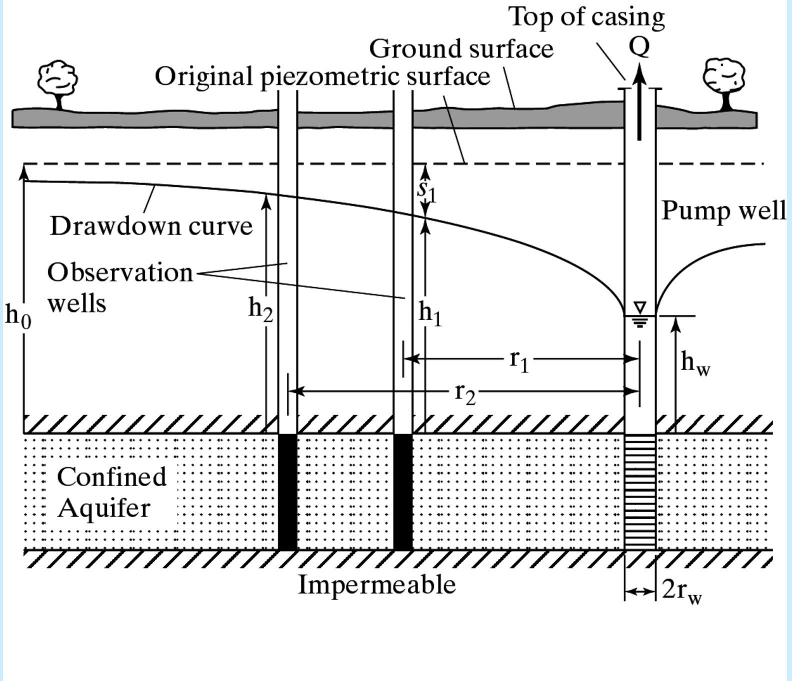
Transient condition: The Theis solution

$$s = \frac{Q}{4\pi T}\int_u^{\infty}\frac{e^{-x}}{x}dx = \frac{Q}{4\pi T}W(u)$$

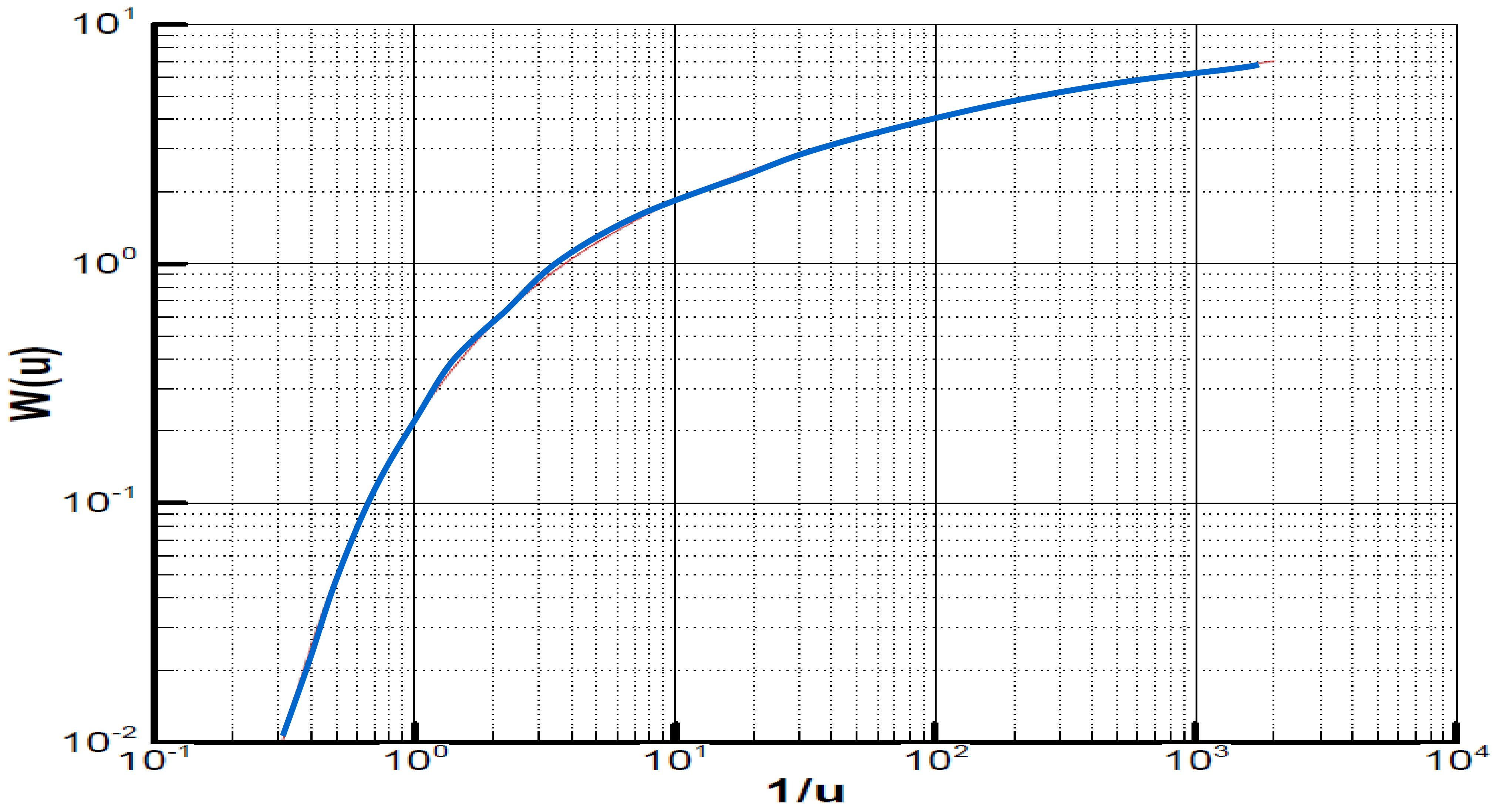
s = drawdown = h_0 – h; h_0 = initial head; h = head at time t; Q = pumping rate; T = transmissivity

$$u = \frac{Sr^2}{4Tt}$$

S = storativity (storage coefficient); r = distance from the well; t = time; W(u) = well function



Radial flow to a well penetrating an extensive confined aquifer.



10^1
10^0
10^{-1}
10^{-2}
W(u)
10^{-1}
10^0
10^1
10^2
10^3
10^4
1/u

- Example analytical inverse

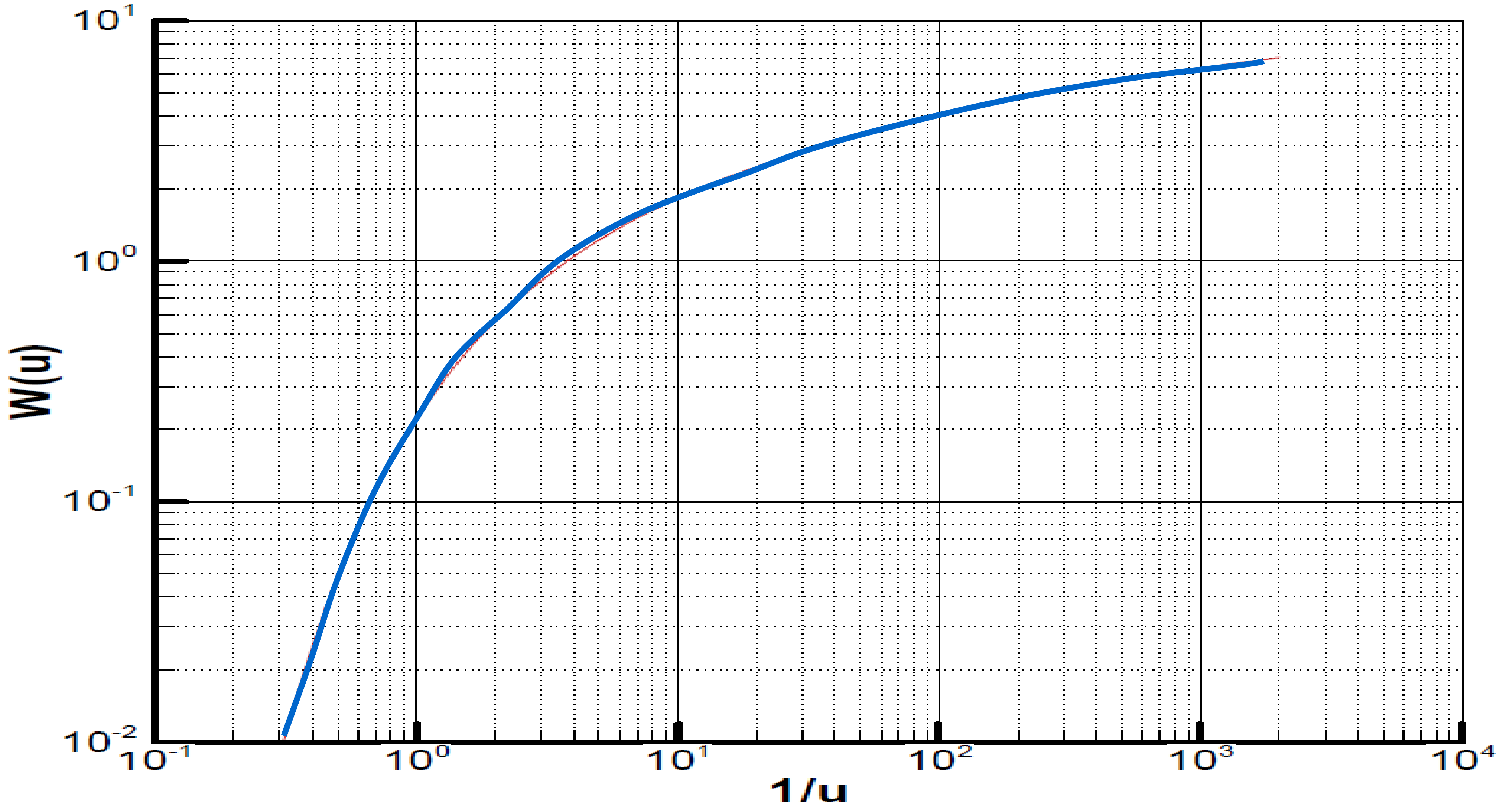
Aquifer Test Analysis: based on the Theis solution

- Objective: to determine transmissivity and storativity of aquifers
- Values are averages over large aquifer volumes in contrast to laboratory tests
- Given Q and $s = h_0 - h$ vs. t calculate T and S
- Pump tests are done during initial exploitation.
- Values are used for long-term planning

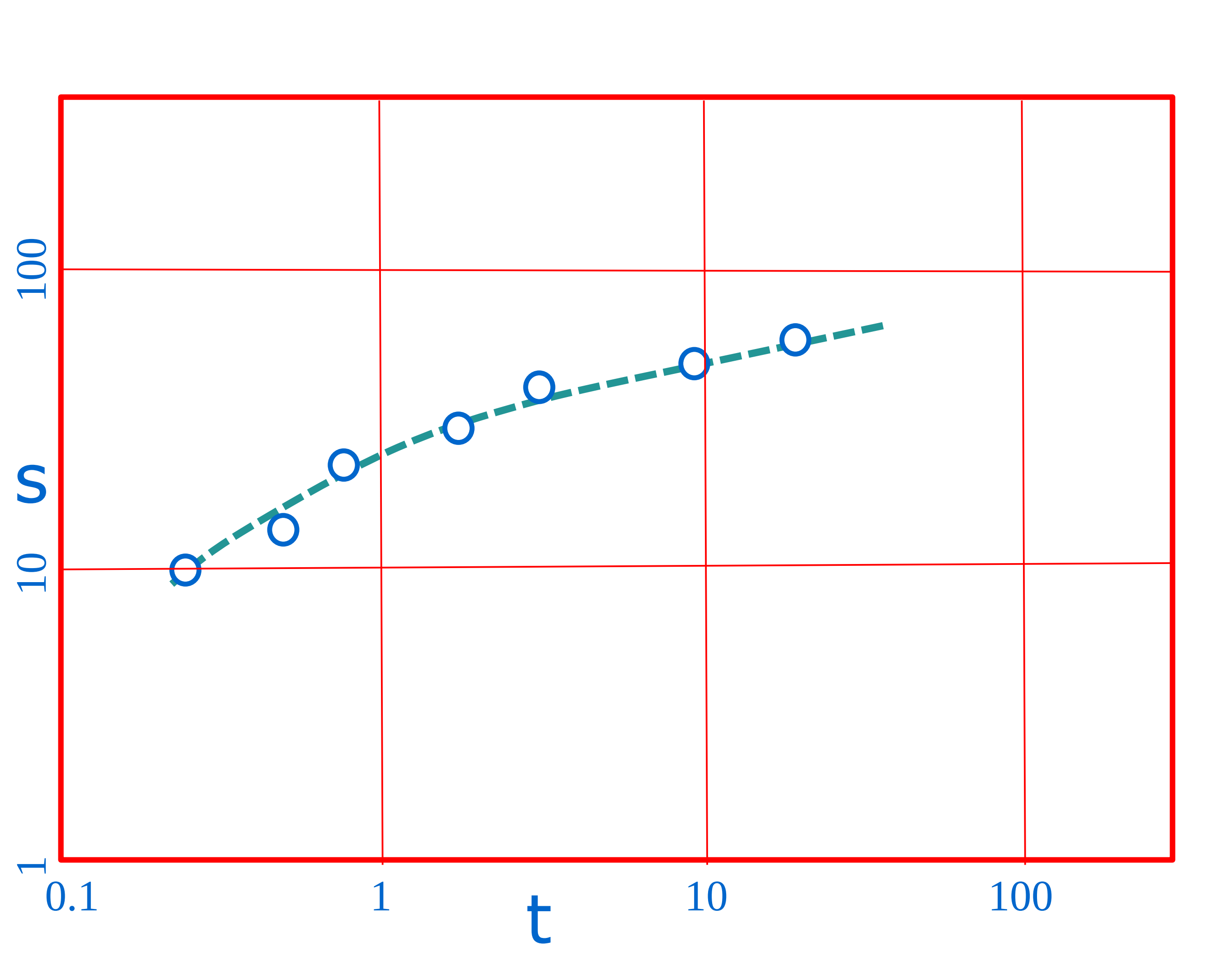
1. Log – Log Type-curve matching

- $W(u)$ vs. $\frac{1}{u}$ is similar to s vs. t

- Plot $W(u)$ vs. $1/u$ on log-log paper (type curve)
- Plot s vs. t same scale

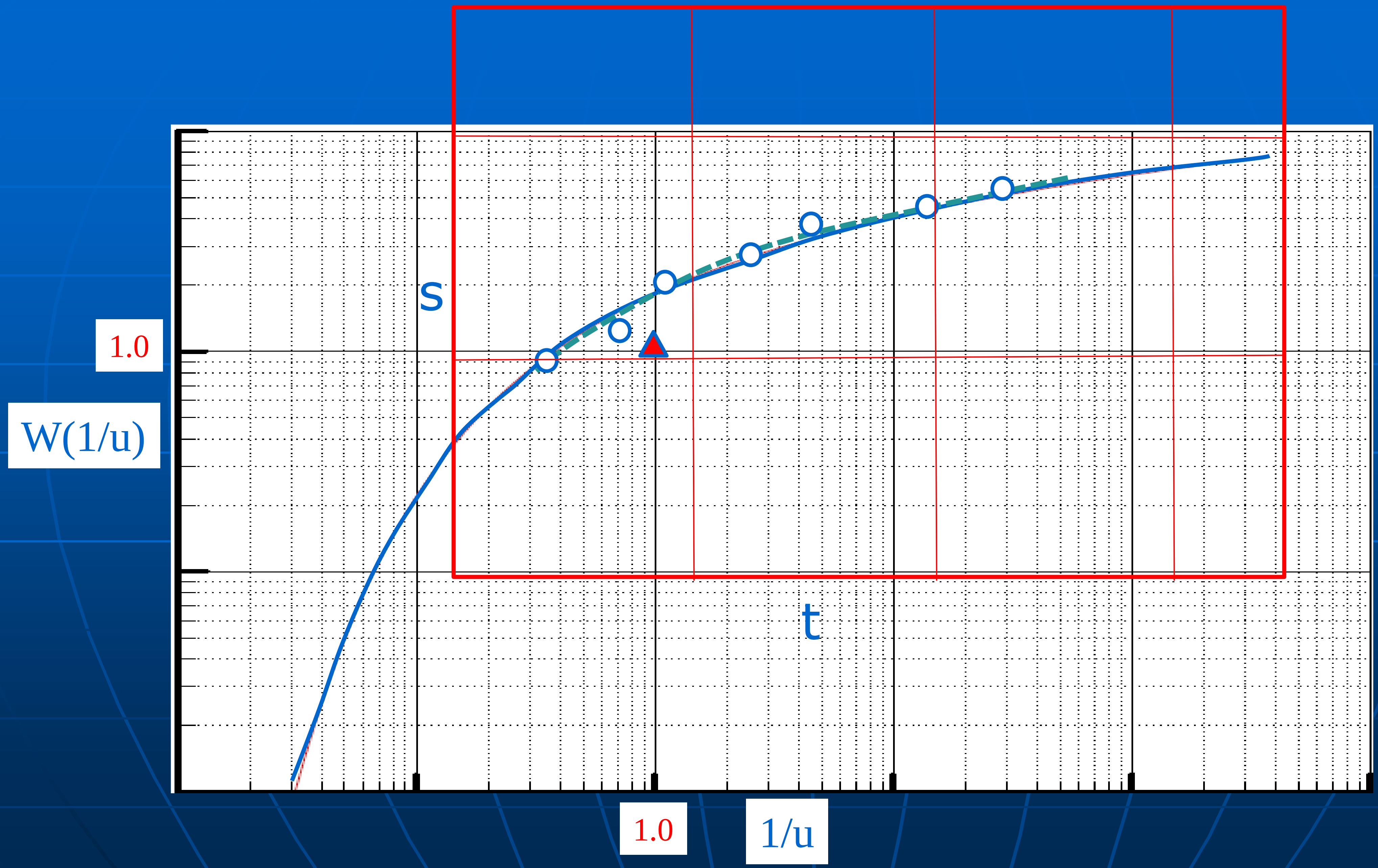


10^1
10^0
10^{-1}
10^{-2}
W(u)
10^{-1}
10^0
10^1
10^2
10^3
10^4
1/u


100
s
10
1
0.1
1
10
100
t

1. Log – Log Type-curve matching

- Superimpose
- Select any match point preferably $u = 1.0$, and W $= 1.0$
- Read respective s, t, u, and W



s
1.0
W(1/u)
t
1.0
1/u

- Use the values s, t, u, and W to calculate:

$$T = \frac{QW}{4\pi s}$$

$$S = \frac{4uTt}{r^2}$$

Use consistent units

Example

$Q = 42{,}400\ ft^3/d$

At match point

$W(u) = 1$ $u = 1$

$t = 4.1$ min $s = 2.4$ ft

$$T = \frac{QW(u)}{4\pi s} = 1400\ ft/d$$

$$S = \frac{4uTt}{r^2} = 2.4\ x\ 10^{-5}$$