

There are two approaches to the two-sensor track fusion problem. Bar-Shalom and Campo [4] recently presented the state vector fusion method which combines the filtered state vectors from the two sensors to form a new estimate while taking into account the correlated process noise. The measurement fusion method or data compression [5] combines the measurements from the two sensors first and then uses this fused measurement to estimate the state vector. The two methods are compared and an example shows the amount of improvement in the uncertainty of the resulting estimate of the state vector with the measurement fusion method.

### I. INTRODUCTION

In many situations targets are tracked by a variety of sensors. The decision process involved in associating tracks belonging to the same target is a correlation problem which has been previously examined for various situations [1, 2]. Once the targets are correlated an algorithm is needed to provide a single target track which has less uncertainty than that of the individual tracks themselves. This process is often referred to as track fusion.

Track fusion was examined by Singer and Kanyuck [1] under the assumption that the process noise between the sensors is independent. Bar-Shalom [3] showed that this noise is actually not independent, because whenever the target maneuvers or deviates from the process model, the deviation is modeled by the process noise which is the same for both sensors. Bar-Shalom and Campo [4] showed that when this dependence is taken into account the area of the error ellipse for an  $\alpha$ - $\beta$  filter is reduced by 70 percent instead of being cut in half as would be the case if the independent noise assumption were correct.

The purpose of this correspondence is to compare these new results from Bar-Shalom and Campo with the measurement fusion method [5]. If the measurements from the sensors are fused and then tracking is done on these fused measurements, the error in the filtered state vector is reduced more than that presented in [4]. Section II describes the two different methods of track fusion and Section III compares the performances of the two with an example.

### II. TRACK FUSION

Target motion can be modeled by the process equation for sensors  $i$  and  $j$  as

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$$x_{k+1}^m = \Phi_k x_k^m + \Gamma_k v_k, \quad m = i, j$$

where  $x_k$  is the state vector at time  $k$ , and  $v_k$  is the process noise with

$$E[v_k] = 0; \quad E[v_k v_l] = q_k \delta_{kl}.$$

The measurement equation is given by

$$z_k^m = H_k^m x_k^m + w_k^m, \quad m = i, j$$

where  $z_k$  is the measurement at time  $k$  and  $w_k$  is the measurement noise with

$$E[w_k^m] = 0; \quad E[w_k^m w_l^m] = r_k^m \delta_{kl};$$

$$E[w_k^i w_l^j] = 0. \quad (4)$$

Fusion of these tracks can now take place at either the state vector or measurement level.

#### A. State Vector Fusion

Track fusion can be performed by fusing the filtered state vectors ( $x_{k/k}^i$  and  $x_{k/k}^j$ ) into a new estimate of the state vector [4]. The new estimate of the state vector ( $\hat{x}_{k/k}$ ) is given by the following fusion equation

$$\hat{x}_{k/k} = x_{k/k}^i + P_{xz} P_{zz}^{-1} (x_{k/k}^j - x_{k/k}^i)$$

$$P_{xz} = P_{k/k}^i - P_{k/k}^{ij}$$

$$P_{zz} = P_{k/k}^i + P_{k/k}^j - P_{k/k}^{ij} - P_{k/k}^{ji}$$

where  $x_{k/k}^i$  is the  $i$ th filtered state vector,  $P_{k/k}^i$  is the covariance matrix for  $x_{k/k}^i$ , and  $P_{k/k}^{ij}$  is the cross covariance matrix between  $x_{k/k}^i$  and  $x_{k/k}^j$ . The cross covariance matrix is given by the recursive equation [3]

$$P_{k/k}^{ij} = (I - K_k^i H_k^i) \Phi_{k-1} P_{k-1/k-1}^{ij} \Phi_{k-1}^t (I - K_k^j H_k^j)^t + (I - K_k^i H_k^i) \Gamma_{k-1} q_{k-1} \Gamma_{k-1}^t (I - K_k^j H_k^j)^t \quad (8)$$

where  $K_k^i$  is the Kalman filter gain matrix for sensor  $i$  at time  $k$ . The covariance matrix of the fused estimate is then given by

$$P_{k/k} = P_{k/k}^i - (P_{k/k}^i - P_{k/k}^{ij})(P_{k/k}^i + P_{k/k}^j - P_{k/k}^{ij} - P_{k/k}^{ji})^{-1} (P_{k/k}^i - P_{k/k}^{ji}). \quad (9)$$

This method of combining tracks has been shown by Willsky et. al [6] to be in general suboptimal. The advantage to using state vector fusion is a reduced computational load on the central processor or it can be implemented without a central processor where each local processor computes its own estimate of the state from the state of all other processors in a distributed system [7].

#### B. Measurement Fusion

The second approach to track fusion is to fuse the measurements from the sensors and then track those fused

- (1) measurements to obtain an estimate of the state vector [5]. Since the measurement noise is independent for sensors  $i$  and  $j$  the equation for fusing the measurement vectors  $z_k^i$  and  $z_k^j$ , in recursive form, to obtain the minimum mean square estimate ( $\bar{z}_k$ ) is given by

$$\bar{z}_k = z_k^i + R_k^i (R_k^i + R_k^j)^{-1} (z_k^j - z_k^i) \quad (10)$$

- (3) where  $R_k^i$  is the covariance matrix of the measurement vector  $z_k^i$ . This filtered measurement then has a covariance matrix given by

$$\bar{R}_k = [(R_k^i)^{-1} + (R_k^j)^{-1}]^{-1}. \quad (11)$$

These filtered measurements can then be tracked to obtain the estimate of the state vector  $\hat{x}_{k/k}$ .

### III. COMPARISON OF FUSION METHODS

To illustrate the improvement achieved using measurement fusion over state vector fusion the example of [4] is reproduced using both methods of track fusion. The tracking algorithm considered is the  $\alpha$ - $\beta$  tracker. The process equation for one dimension is

$$x_{k+1}^m = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x_k^m + \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} v_k \quad (12)$$

- (6) with sampling time  $T = 1$  and noise variance  $q$ . The measurement equation of the two sensors is

$$z_k^m = [1 \ 0] x_k^m + w_k^m \quad (13)$$

where the measurement noise is independent with variance  $r_k = 1$ .

The steady state covariance and gain for the filter is given in [8, 9]. The steady state cross covariance matrix  $P^{ij}$  can be calculated by substituting the steady state gain into (8), letting  $P_{k/k}^{ij} = P_{k-1/k-1}^{ij}$  and solving for the components of the cross covariance matrix. Fig. 1 shows

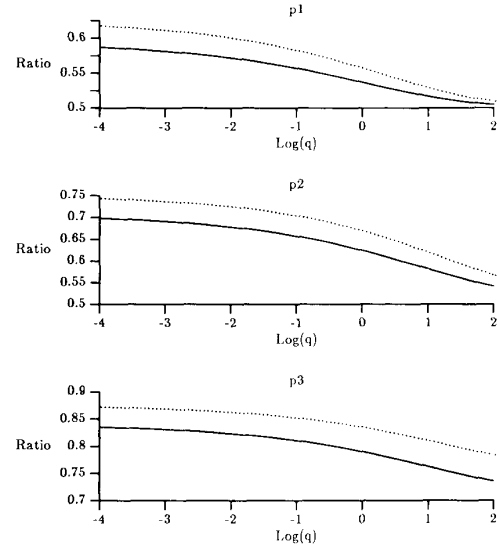


Fig. 1. Ratio of components of covariance matrix for two-sensor fused estimate to one sensor. Dotted lines are state vector fusion and solid lines are measurement fusion method.

the reduction in the components of  $\mathbf{P}_{k/k}$  where

$$\mathbf{P}_{k/k} = \begin{bmatrix} p1 & p2 \\ p2 & p3 \end{bmatrix} \quad (14)$$

for the two-sensor fusion over the single-sensor case for a wide range of process noise  $q$ . The dotted lines are the reduction using the state vector fusion method described in [4] and expressed by (9). The solid lines are the reduction in  $\mathbf{P}_{k/k}$  using the method of measurement fusion. In this method the measurements are fused by (10) and the resulting measurement variance in equation (11) is  $\bar{r}_k = 1/2$  if  $r_k^i = r_k^j = 1$ . Fig. 2 shows the

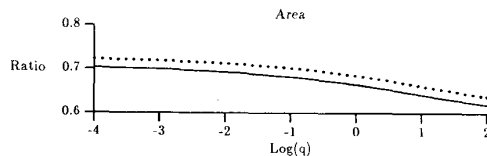


Fig. 2. Ratio of area of error ellipses for fused state vector to one-sensor state vector. Dotted line is state vector fusion and solid line is measurement fusion method.

reduction in the area of the error ellipses corresponding to the covariance matrices whose components are shown in Fig. 1.

#### IV. SUMMARY

Two methods for fusing the tracks of two different sensors have been considered. The first presented by Bar-Shalom and Campo [4] fuses the filtered state vectors taking into account the common process noise. The second [5] takes the measurements corresponding to the same target, fuses them together and then tracks the fused measurements. This correspondence shows the reduction achieved in the covariance of the filtered state vector by utilizing measurement fusion.

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## An Anticipative Adaptive Array for Frequency-Hopping Communications

**For the full exploitation of the theoretical processing gain achievable when an adaptive array and frequency hopping are combined, frequency compensation is required. This paper examines improved versions of an anticipative adaptive array that provide efficient compensation by adapting the complex weights at each antenna element to the appropriate values for a carrier frequency before that frequency is received. The underlying adaptive algorithm is the Maximin algorithm, which has major advantages compared with other algorithms. Computer simulation results are used to compare the different versions of anticipative processing. These results show that an appropriate version can ensure the rapid convergence of weights to values that provide wideband nulling of the interference and noise.**

#### I. INTRODUCTION

In designing an adaptive antenna system for wideband frequency-hopping communications, one must select both an adaptive algorithm and some method of compensation for frequency-dependent fluctuations in the suppression of interference. The Maximin algorithm [1, 2] is an adaptive

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