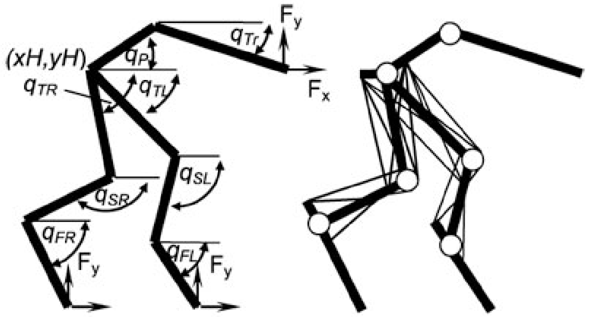




# Forward Kinematics

## - Denavit-Hartenberg convention

# 2D planar robot arm

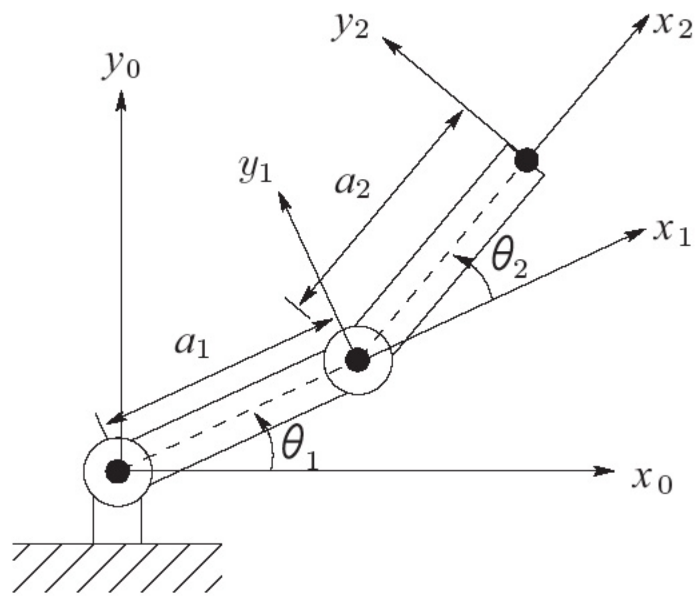
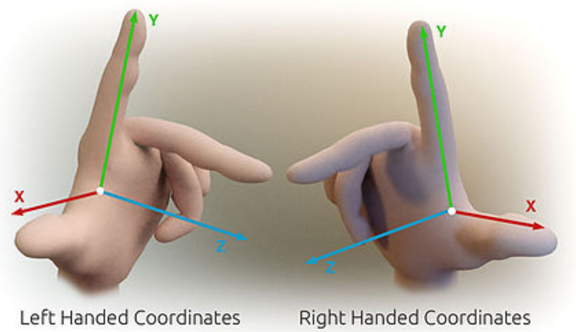



Figure 3.6: Two-link planar manipulator. The $z$-axes all point out of the page, and are not shown in the figure.

# What's changed in 3D ?

- 6 DOF
  - 3 "translation-units" along axes x,y and z
  - 3 "rotation-units" about axes x,y and z
  - Eg. $R_{x,\alpha} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix}$
- Still, H = $\begin{pmatrix} R & d \\ 0 & 1 \end{pmatrix}$, but now 4x4 matrix..
- Right-handedness



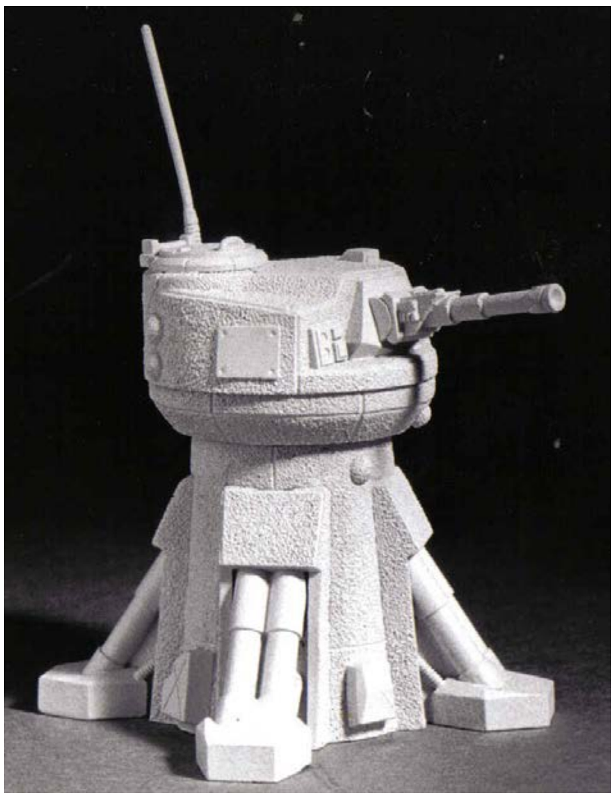
# Homogeneous Transform

- Six ”elements” – 3 translation + 3 rotation
- $\text{Trans}_{x,a}$ $\text{Trans}_{y,b}$ $\text{Trans}_{z,c}$ $\text{Rot}_{x,\alpha}$ $\text{Rot}_{y,\beta}$ $\text{Rot}_{z,\theta}$

# Rotation in 3D

- Euler theorem..
- Euler angles - ZYZ
- Roll-pitch-yaw (rpy) - XYZ
- Axis-angle
- Quaternion
- Rotation matrices
- ..

# Singularities



# Denavit-Hartenberg Convention

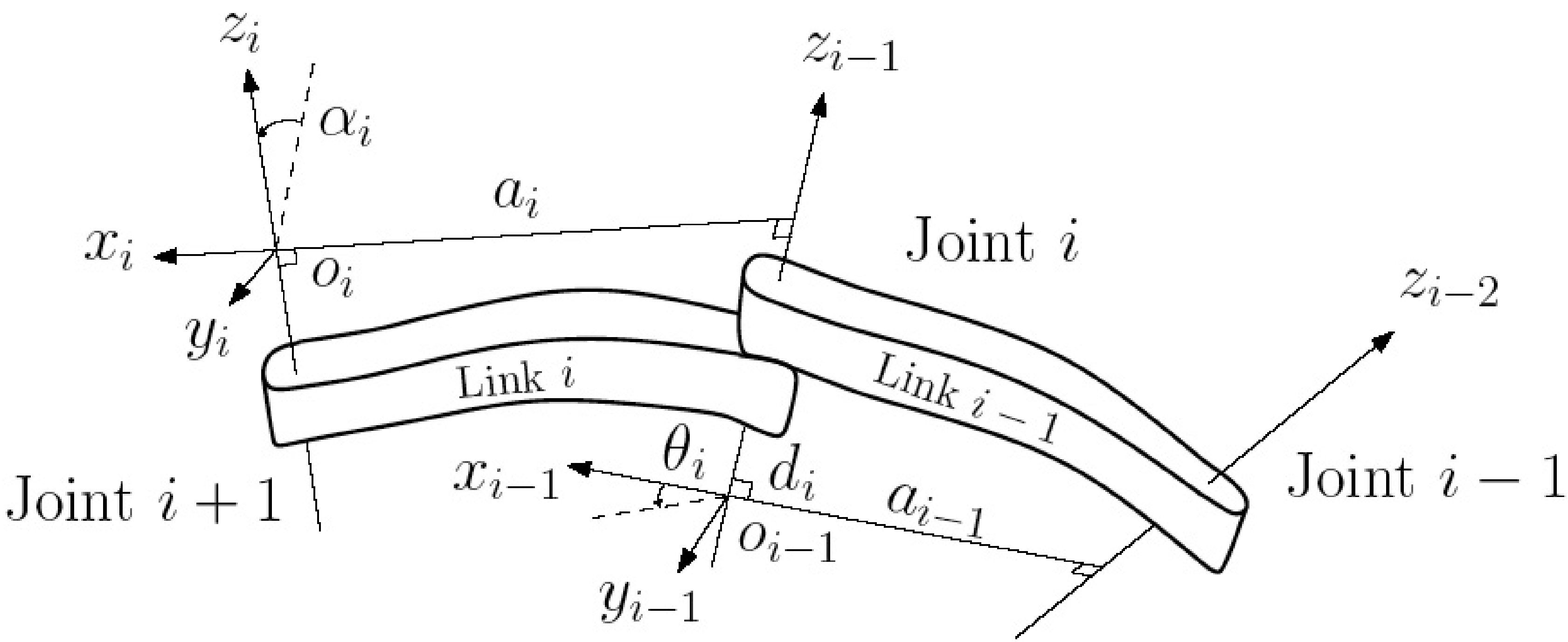



Figure 3.4: Denavit-Hartenberg frame assignment.

# Denavit-Hartenberg Convention

## Procedure

1. Assign coordinate frames to links
2. Extract DH parameters from kinematic model
3. Calculate transformation matrices $A^i_j$ between frames i and j
4. Find base-to-end effector transform as

$$T^0_n = A^0_1 \, A^1_2 \, .. \, A^{n-1}_n$$

NOTE: $T^0_n$ unique if base and end frames are the same – but intermediate A-matrices may not be..

# The four DH parameters
## - constrained specification

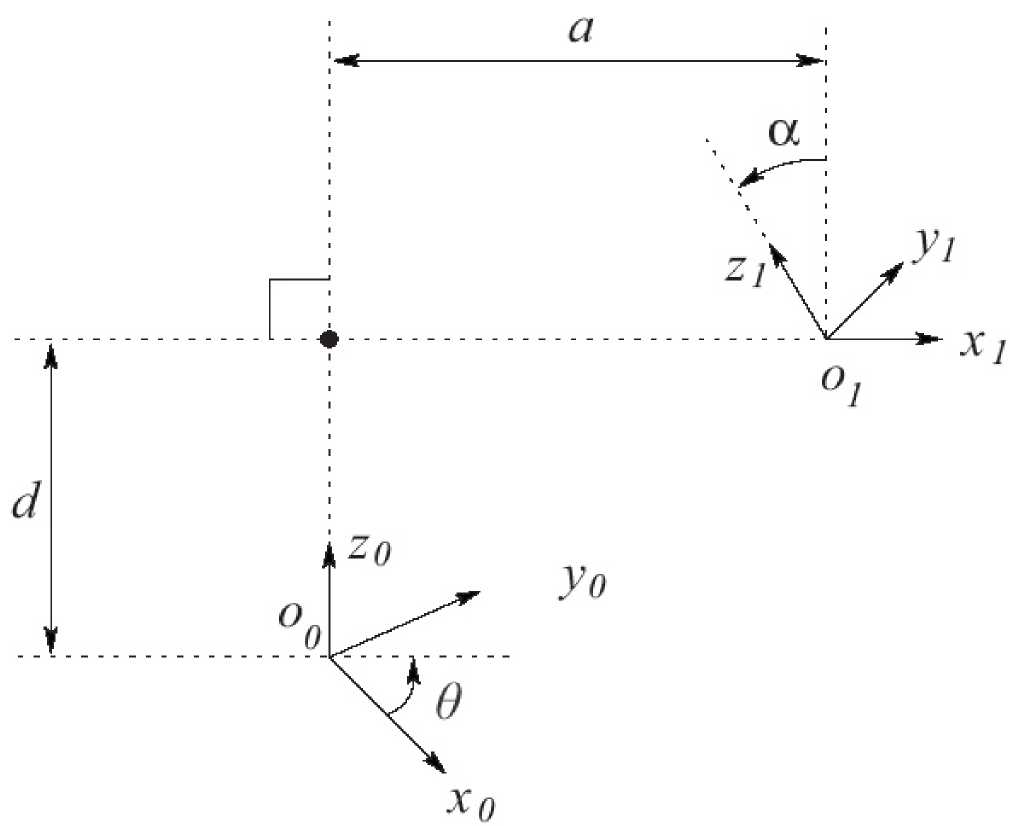



Figure 3.2: Coordinate frames satisfying assumptions DH1 and DH2.

# DH1 and DH2

- DH1 :

  Axis $x_1$ is perpendicular to axis $z_0$

- DH2 :

  Axis $x_1$ intersects axis $z_0$

# DH : Assign coordinate frames to links

1. Assign z-axes to directions of joint movement
2. Start with base frame $o_0$ – arbitrary $x_0$-$y_0$-axes
3. Assign frame i wrt. frame i-1
   - consider three different cases for z-axes
   - iterate from frame 0 to frame n-1
4. Frame n (end effector) positioned between eg. gripper fingers

# DH : E.g. assign frame 1 wrt. frame 0

1) $z_1$ and $z_0$ not coplanar
   - origin is point on $z_1$ with <u>shortest</u> distance between $z_1$ and $z_0$
   - Here, $x_1$ can be chosen orthogonal to $z_0$ (along line from $z_1$ and $z_0$)
2) $z_1$ and $z_0$ parallel
   - origin chosen arbitrary on $z_1$
   - many choices for $x_1$
3) $z_1$ and $z_0$ intersect
   - origin is (often) point of intersection
   - $x_1$ chosen orthogonal to plane spanned by $z_0$ and $z_1$

NOTE: y-axis always assigned to obey right-handedness

# Extract DH parameters from frames

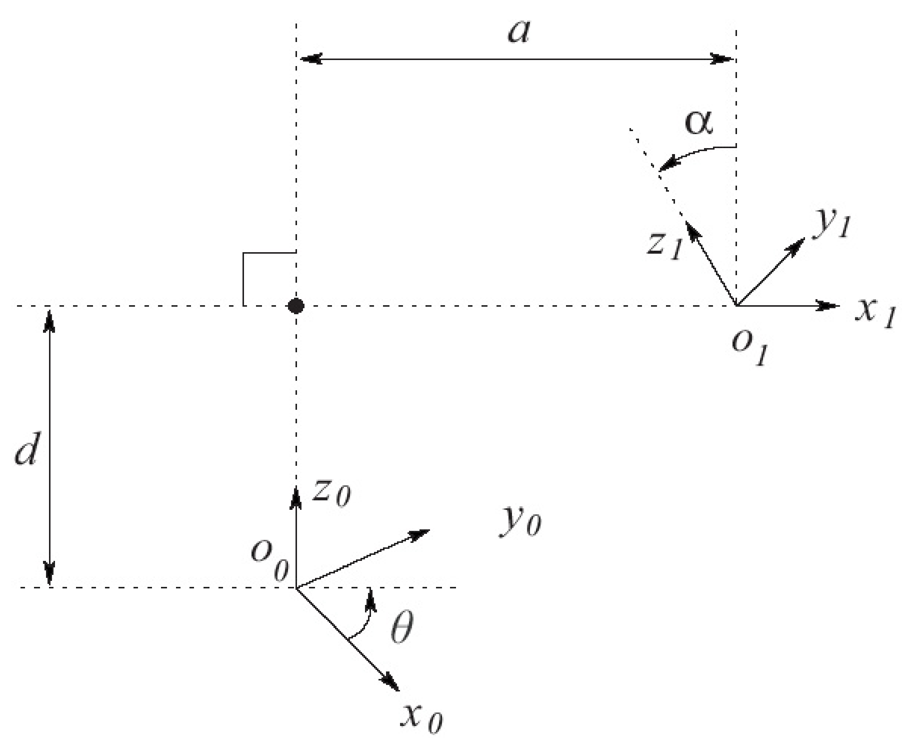



Figure 3.2: Coordinate frames satisfying assumptions DH1 and DH2.

| Link | $d_i$ | $a_i$ | $\alpha_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 0 | $-90$ | $\theta_1^\star$ |
| 2 | $d_2$ | 0 | $+90$ | $\theta_2^\star$ |
| 3 | $d_3^\star$ | 0 | 0 | 0 |
| 4 | 0 | 0 | $-90$ | $\theta_4^\star$ |
| 5 | 0 | 0 | $+90$ | $\theta_5^\star$ |
| 6 | $d_6$ | 0 | 0 | $\theta_6^\star$ |

* joint variable

# Calculate A-matrices from DH parameters

- $A^{i-1}_{i} = Rot_{z,\theta}\, Trans_{z,d}\, Trans_{x,a}\, Rot_{x,\alpha}$
- One unit for each DH parameter
- Note that θ and d are the joint variables – for revolute and prismatic joints respectively.

$$
{}^{j-1}\boldsymbol{A}_j = \begin{pmatrix} \cos\theta_j & -\sin\theta_j \cos\alpha_j & \sin\theta_j \sin\alpha_j & a_j \cos\theta_j \\ \sin\theta_j & \cos\theta_j \cos\alpha_j & -\cos\theta_j \sin\alpha_j & \alpha_j \sin\theta_i \\ 0 & \sin\alpha_j & \cos\alpha_j & d_j \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

# Example : 2D planar robot arm

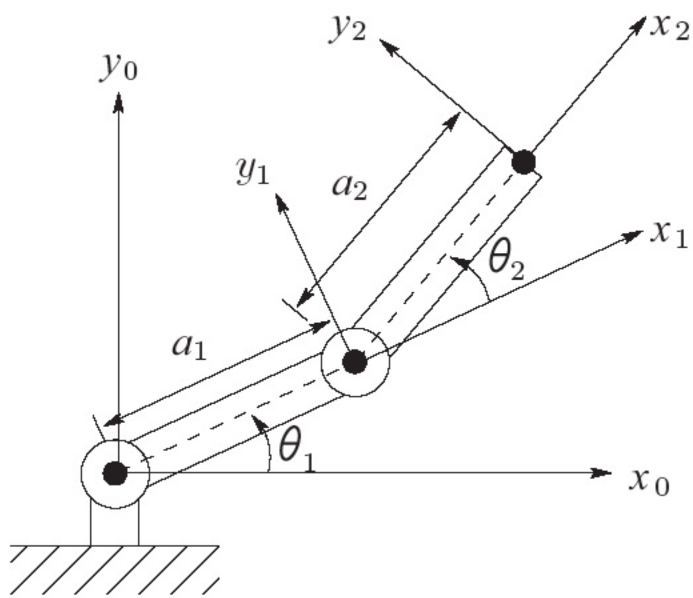
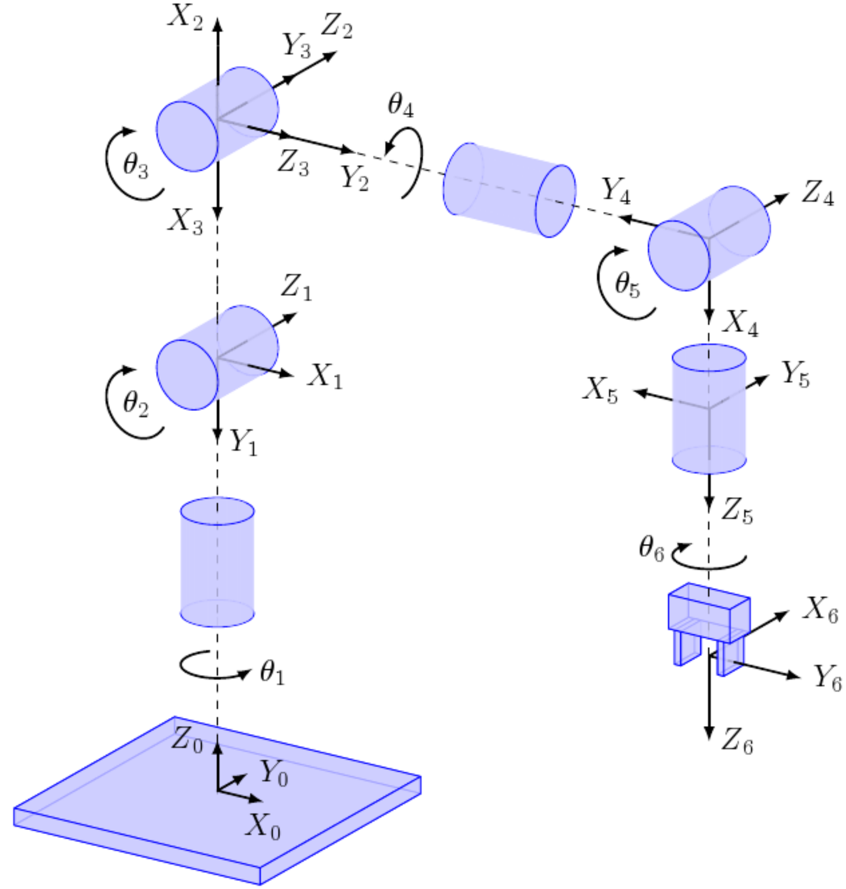
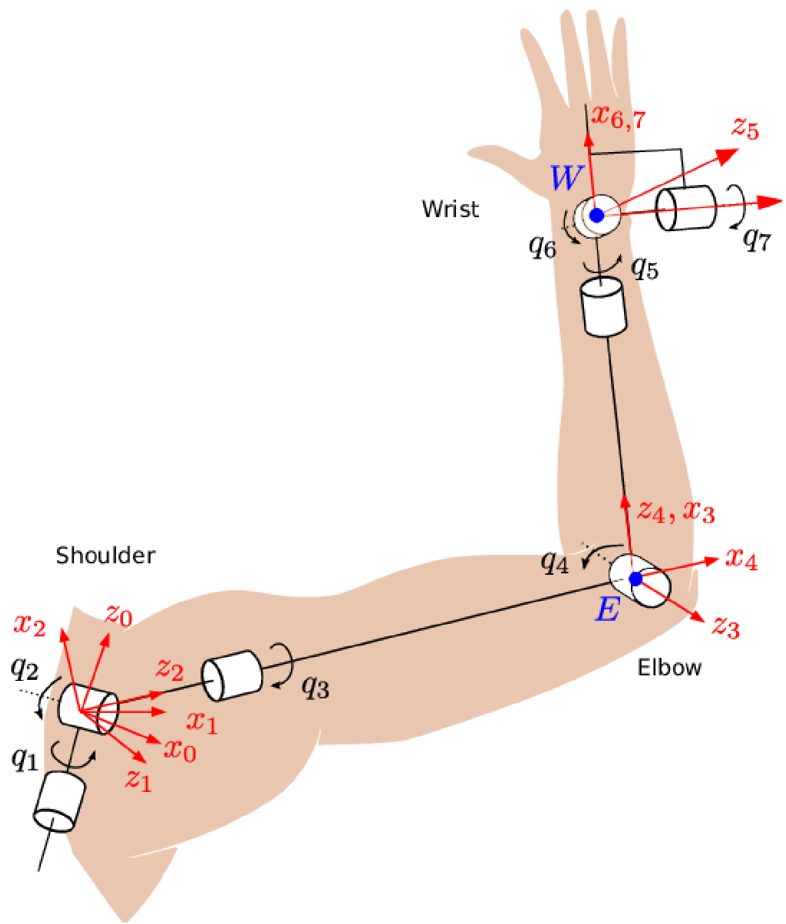



Figure 3.6: Two-link planar manipulator. The $z$-axes all point out of the page, and are not shown in the figure.

# Example : Robotics toolbox



# Example – Human Arm modelling



# Mandatory exercise 2

- Denavit-Hartenberg convention
  - Forward Kinematics
  - .. On the Crustcrawler
- Assistance available next week