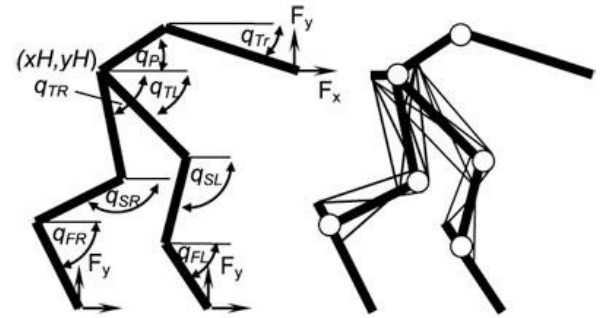


Forward Kinematics

- Denavit-Hartenberg convention

2D planar robot arm

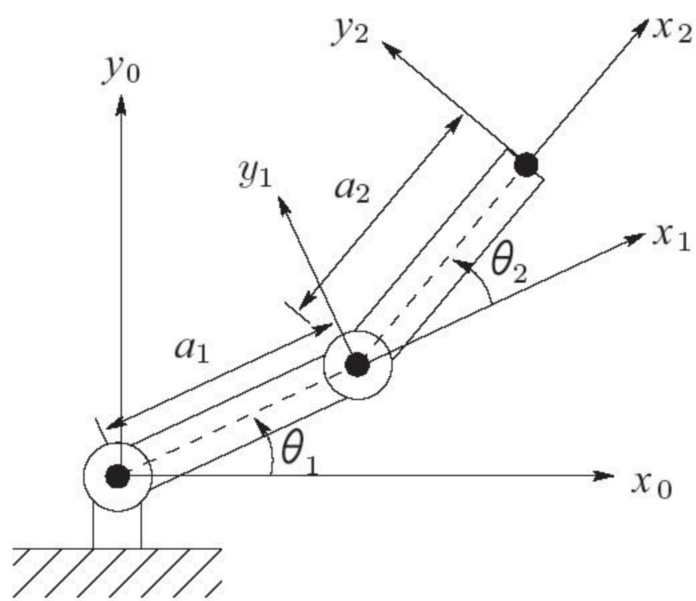
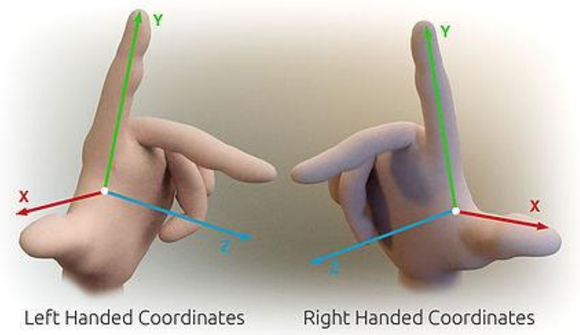


Figure 3.6: Two-link planar manipulator. The z-axes all point out of the page, and are not shown in the figure.

What's changed in 3D ?

- 6 DOF
  - 3 ”translation-units” along axes x,y and z
  - 3 ”rotation-units” about axes x,y and z
  - Eg. $R_{x,\alpha} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix}$
- Still, H = $\begin{pmatrix} R & d \\ 0 & 1 \end{pmatrix}$, but now 4x4 matrix..
- Right-handedness



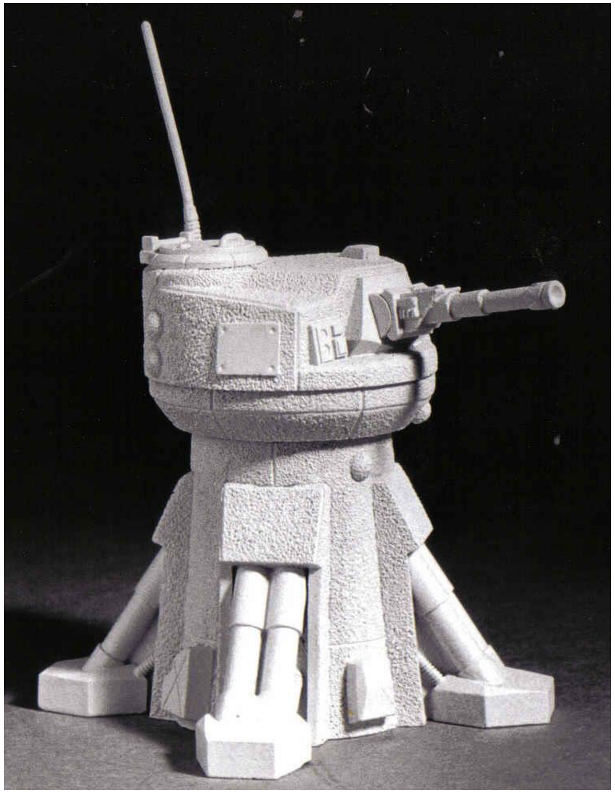
Homogeneous Transform

- Six ”elements” – 3 translation + 3 rotation
- $Trans_{x,a}$ $Trans_{y,b}$ $Trans_{z,c}$ $Rot_{x,\alpha}$ $Rot_{y,\beta}$ $Rot_{z,\theta}$

Rotation in 3D

- Euler theorem..
- Euler angles - ZYZ
- Roll-pitch-yaw (rpy) - XYZ
- Axis-angle
- Quaternion
- Rotation matrices
- ..

Singularities



Denavit-Hartenberg Convention

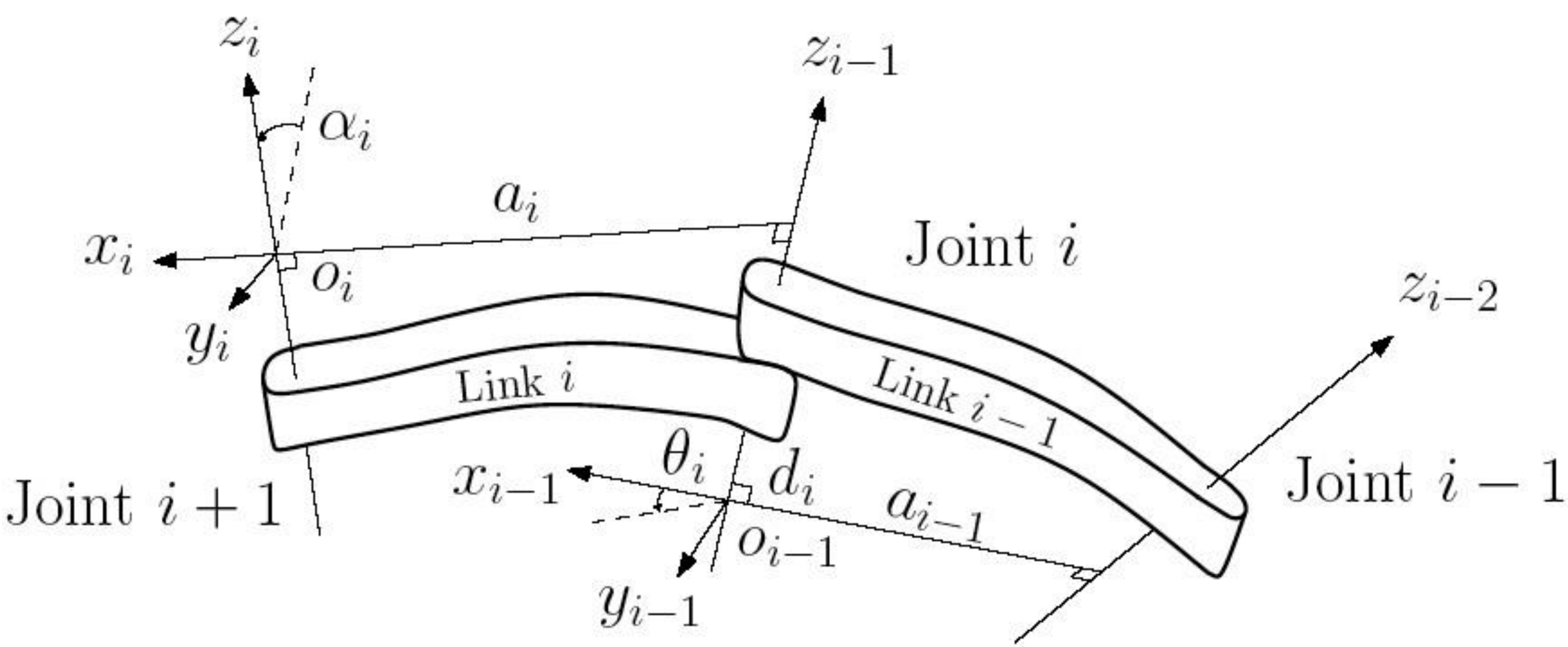


Figure 3.4: Denavit-Hartenberg frame assignment.

Denavit-Hartenberg Convention

Procedure

1. Assign coordinate frames to links
2. Extract DH parameters from kinematic model
3. Calculate transformation matrices A^i_j between frames i and j
4. Find base-to-end effector transform as

$$T^0_n = A^0_1 \, A^1_2 \, .. \, A^{n-1}_n$$

NOTE: T^0_n unique if base and end frames are the same – but intermediate A-matrices may not be..

The four DH parameters
- constrained specification

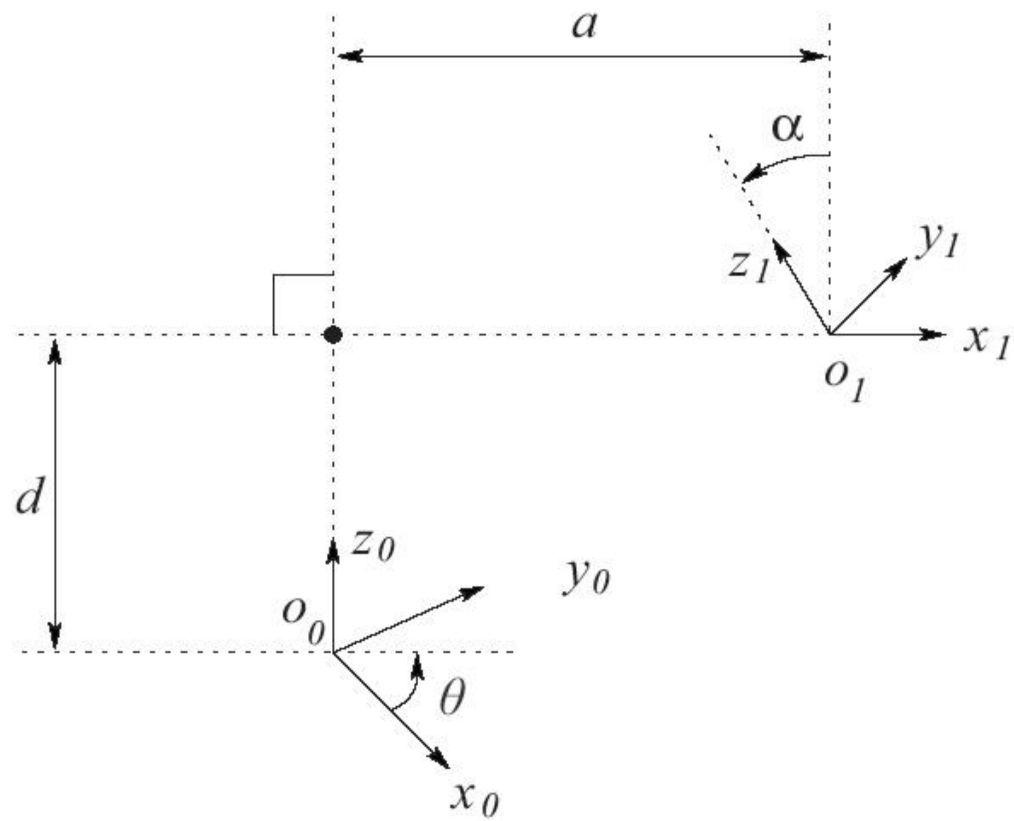


Figure 3.2: Coordinate frames satisfying assumptions DH1 and DH2.

DH1 and DH2

- DH1 :

  Axis x_1 is perpendicular to axis z_0

- DH2 :

  Axis x_1 intersects axis z_0

DH : Assign coordinate frames to links

1. Assign z-axes to directions of joint movement
2. Start with base frame o_0 – arbitrary x_0-y_0-axes
3. Assign frame i wrt. frame i-1
   - consider three different cases for z-axes
   - iterate from frame 0 to frame n-1
4. Frame n (end effector) positioned between eg. gripper fingers

DH : E.g. assign frame 1 wrt. frame 0

1) z_1 and z_0 not coplanar
   - origin is point on z_1 with <u>shortest</u> distance between z_1 and z_0
   - Here, x_1 can be chosen orthogonal to z_0 (along line from z_1 and z_0)
2) z_1 and z_0 parallel
   - origin chosen arbitrary on z_1
   - many choices for x_1
3) z_1 and z_0 intersect
   - origin is (often) point of intersection
   - x_1 chosen orthogonal to plane spanned by z_0 and z_1

NOTE: y-axis always assigned to obey right-handedness

Extract DH parameters from frames

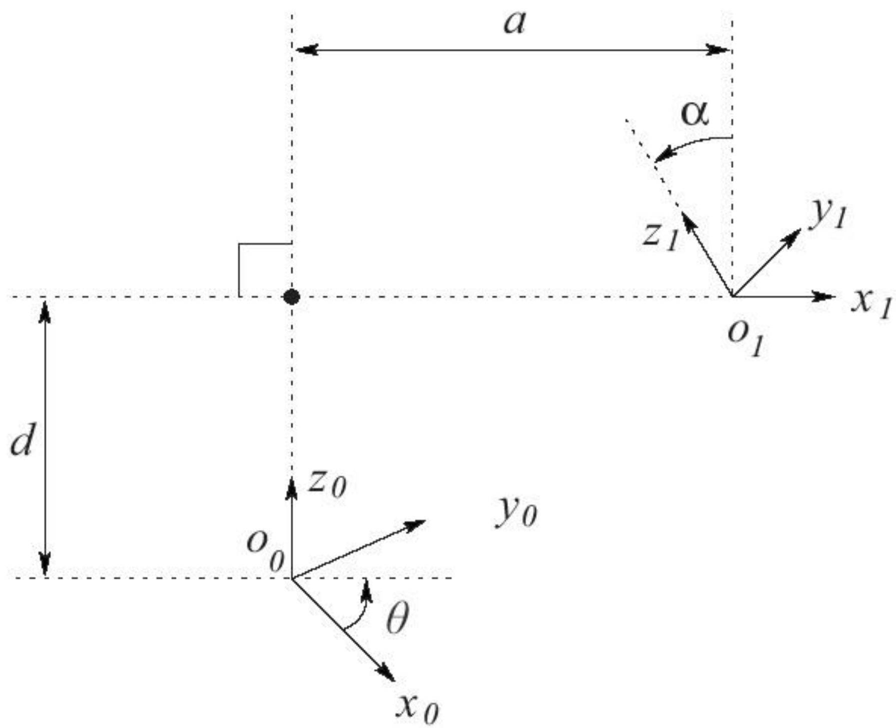


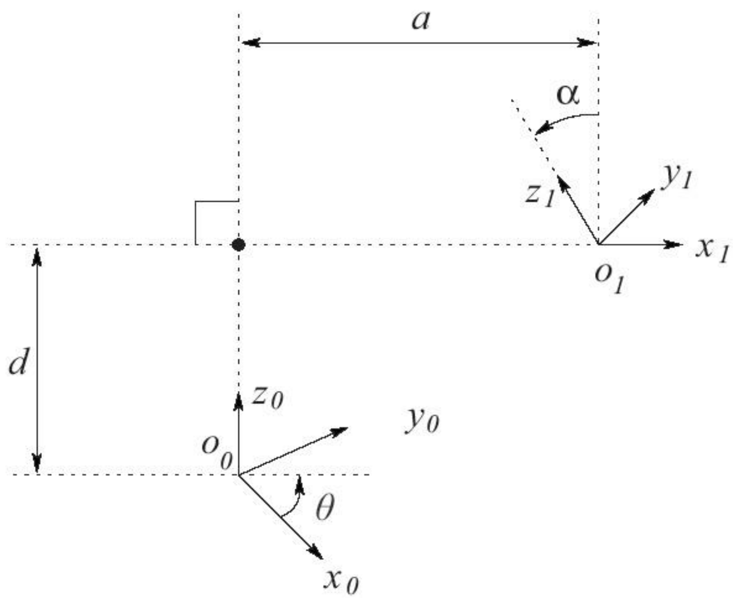
Figure 3.2: Coordinate frames satisfying assumptions DH1 and DH2.

Link	d_i	a_i	α_i	θ_i
1	0	0	-90	$\theta_1^\star$
2	d_2	0	$+90$	$\theta_2^\star$
3	$d_3^\star$	0	0	0
4	0	0	-90	$\theta_4^\star$
5	0	0	$+90$	$\theta_5^\star$
6	d_6	0	0	$\theta_6^\star$

* joint variable

Calculate A-matrices from DH parameters

- $A^{i-1}_{i} = Rot_{z,\theta}\, Trans_{z,d}\, Trans_{x,a}\, Rot_{x,\alpha}$
- One unit for each DH parameter
- Note that θ and d are the joint variables – for revolute and prismatic joints respectively.



Example : 2D planar robot arm

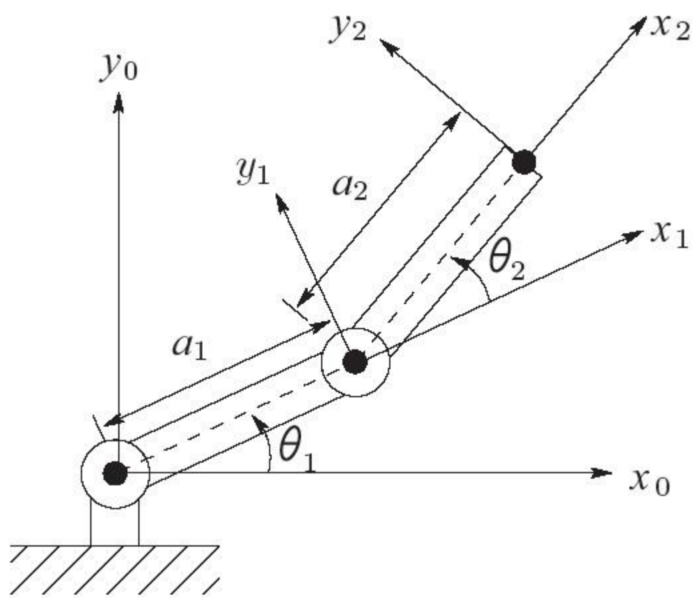
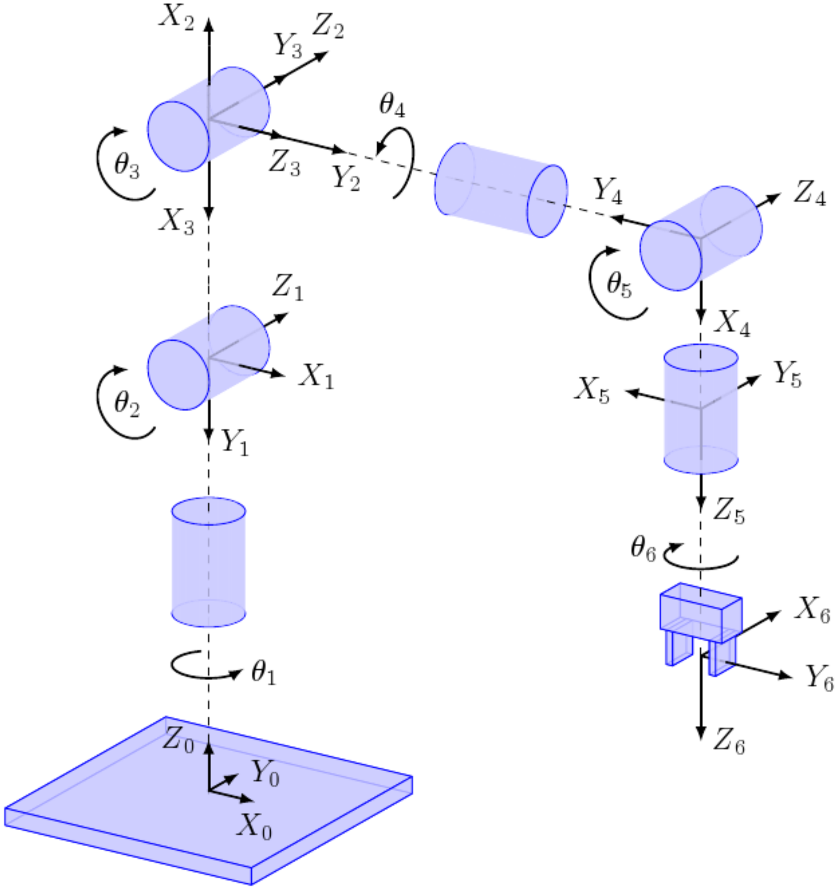
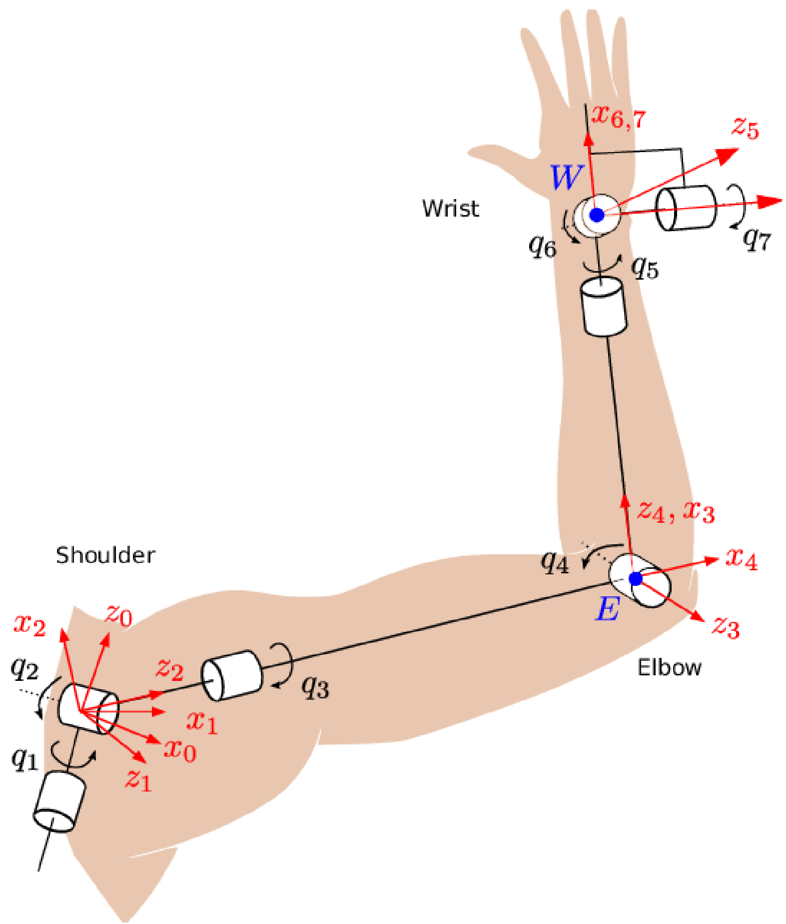


Figure 3.6: Two-link planar manipulator. The z-axes all point out of the page, and are not shown in the figure.

Example : Robotics toolbox



Example – Human Arm modelling



Mandatory exercise 2

- Denavit-Hartenberg convention
  - Forward Kinematics
  - .. On the Crustcrawler
- Assistance available next week