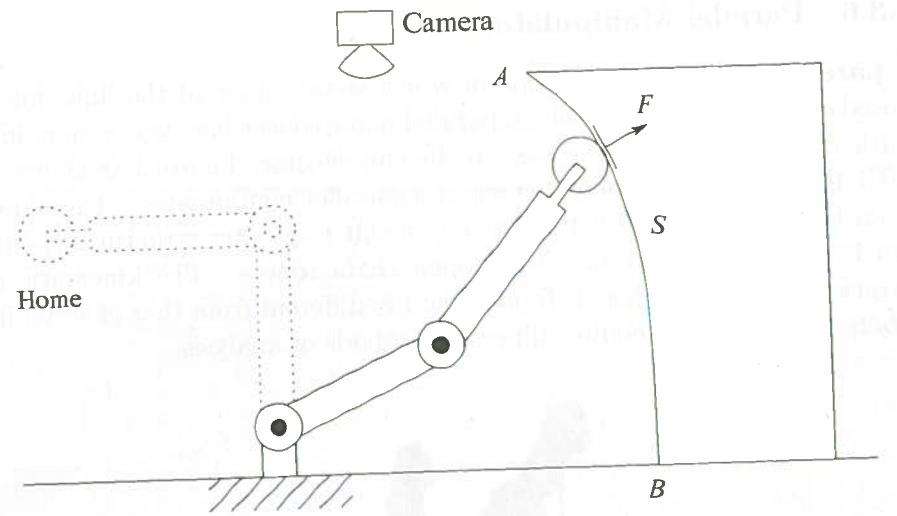

Figure 1.19: Two-link planar robot example. Each chapter of the text discusses a fundamental concept applicable to the task shown.

use the simple two-link planar mechanism to illustrate some of the major issues involved and to preview the topics covered in this text.

Suppose we wish to move the manipulator from its **home** position to position A, from which point the robot is to follow the contour of the surface S to the point B, at constant velocity, while maintaining a prescribed force F normal to the surface. In so doing the robot will cut or grind the surface according to a predetermined specification. To accomplish this and even more general tasks, we must solve a number of problems. Below we give examples of these problems, all of which will be treated in more detail in the remainder of the text.

Forward Kinematics

The first problem encountered is to describe both the position of the tool and the locations A and B (and most likely the entire surface S) with respect to a common coordinate system. In Chapter 2 we describe representations of coordinate systems and transformations among various coordinate systems.

Typically, the manipulator will be able to sense its own position in some manner using internal sensors (position encoders located at joints 1 and 2) that can measure directly the joint angles θ_1 and θ_2. We also need therefore to express the positions A and B in terms of these joint angles. This leads to the **forward kinematics problem** studied in Chapter 3, which is to determine the position and orientation of the end effector or tool in terms

of the joint variables.

It is customary to establish a fixed coordinate system, called the **world** or **base** frame to which all objects including the manipulator are referenced. In this case we establish the base coordinate frame $o_0x_0y_0$ at the base of the robot, as shown in Figure 1.20.

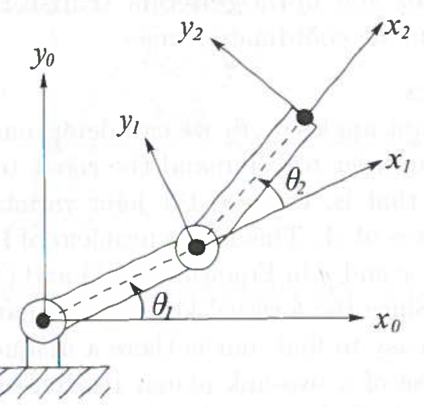

Figure 1.20: Coordinate frames attached to the links of a two-link planar robot. Each coordinate frame moves as the corresponding link moves. The mathematical description of the robot motion is thus reduced to a mathematical description of moving coordinate frames.

The coordinates (x, y) of the tool are expressed in this coordinate frame as

$$x = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2) \tag{1.1}$$

$$y = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2) \tag{1.2}$$

in which a_1 and a_2 are the lengths of the two links, respectively. Also the **orientation of the tool frame** relative to the base frame is given by the direction cosines of the x_2 and y_2 axes relative to the x_0 and y_0 axes, that is,

$$\begin{array}{ll} x_2 \cdot x_0 = \cos(\theta_1+\theta_2)\,; & y_2 \cdot x_0 = -\sin(\theta_1+\theta_2) \\ x_2 \cdot y_0 = \sin(\theta_1+\theta_2)\,; & y_2 \cdot y_0 = \cos(\theta_1+\theta_2) \end{array} \tag{1.3}$$

which we may combine into a **rotation matrix**

$$\begin{bmatrix} x_2 \cdot x_0 & y_2 \cdot x_0 \\ x_2 \cdot y_0 & y_2 \cdot y_0 \end{bmatrix} = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix} \tag{1.4}$$

Equations (1.1), (1.2), and (1.4) are called the **forward kinematic equations** for this arm. For a six-DOF robot these equations are quite