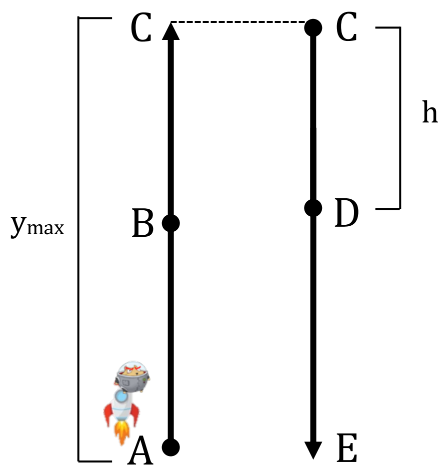
Ryan Mechery October 8th, 2021 Section: R

Über Rocket: Calculus Problem

Description:
One calm afternoon Calculus Cam decides to launch Hamster Huey into the air using a model rocket. The rocket is launched straight up off the ground, from rest. The rocket engine is designed to burn for specified time while producing non-constant net acceleration given by the equations below. After the engine stops the rocket continues upward in free-fall. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens, assume the rocket instantly stops, and then increases speed to a terminal velocity given by the equation below. Assume the air resistance affects the rocket only during the parachute stage.

Diagram:



Givens:

$$a_{BC} = -9.8\ m/s^2$$
$$t_E = \Delta t_{AB} = 4.2s$$
$$a_y[t]_{AB} = -1.3t^2 + 19$$
$$v_A = 0\,m/s$$
$$y_A = 0m$$

$$a_{CD} = -9.8\ m/s^2$$
$$y_C = 0m$$
$$v_C = 0\,m/s$$
$$h = \Delta y_{CD} = 100m$$
$$v_p[t] = v_{DE}[t] = -12\left(1 - e^{-\frac{t}{8}}\right) m/s$$

Strategy:
To solve Über Rocket, I split the problem into 5 letter points with 4 stages. In stage AB, the acceleration is variable and represented by an equation. Since the object starts at rest, I knew both initial position and velocity were zero, meaning that I could derive both Δpos. and Δvel. functions in terms of time. Since time was already given, I didn't need to calculate time but I needed to find the final position and velocity at point B to proceed. In stage BC, acceleration was a constant g and using the both the pos. and vel. I previously found I could calculate Δt_{BC} at max height which is when velocity was equal to zero. In stage CD, acceleration was also constant g and I set Δpos equal to h to find Δt_{CD}. In the final stage DE, the rocket *instantly* changes velocity from neg. to 0 and velocity is calculated by the function $v_p[t]$. I found the integral of that function and set it equal position at point D to solve for Δt_{CD} when the rocket returned to ground. The final step was just to add up the four Δt values and **bam**, I got the total flight time.

Stage AB: Lift Off
Found the equation for $\Delta v_{AB}[t]$ and used that equation to find $\Delta y_{AB}[t]$. Calculated $\Delta y_{AB}[\Delta t_{AB}]$ to get y_B.

$$\underline{\Delta t_{AB} = 4.2s}$$
$$y_A = 0m$$
$$v_A = 0\,\frac{m}{s}$$
$$a_y[t]_{AB} = -1.3t^2 + 19$$
$$\Delta v_y[t]_{AB} = \int_0^t a_y[t]_{AB}dt + \cancel{v_A}\,0$$
$$\Delta v_y[t]_{AB} = \int_0^t -1.3t^2 + 19dt$$
$$\underline{\Delta v_y[t]_{AB} = -\tfrac{1.3}{3}t^3 + 19t}$$
$$\Delta y[t]_{AB} = \int_0^t v_y[t]_{AB}dt + \cancel{x_A}\,0$$
$$\Delta y[t]_{AB} = \int_0^t -\tfrac{1.3}{3}t^3 + 19tdt$$
$$\underline{\Delta y[t]_{AB} = -\tfrac{1.3}{12}t^4 + \tfrac{19}{2}t^2}$$
$$\Delta y_B = y[4.2]_{AB} = -\tfrac{1.3}{12}(4.2)^4 + \tfrac{19}{2}(4.2)^2$$
$$\Delta y_B = \underline{y_B = 133.86996m}$$

Stage BC: Engine Switch-Off
Since object is in free fall, acceleration is at a constant g or $-9.8\ m/s^2$. Calculated v_B using eq. from Stage AB. Found Δt_{BC} using Fund. EQ. 2. Found y_C using Fund. EQ. 3.

$$v_B = ?$$
$$t_B = 4.2s$$
$$a_B = -9.8\,m/s^2$$
$$\Delta v_y[t]_{AB} = -\tfrac{1.3}{3}t^3 + 19t$$
$$\Delta v_B = \Delta v_y[4.2]_{AB} = -\tfrac{1.3}{3}(4.2)^3 + 19(4.2)$$
$$\underline{\Delta v_B = v_B = 47.6952m/s}$$

Rocket will fall when velocity is 0 so…

$$v_C = 0\ m/s$$
$$(EQ\ 2)\ \cancel{v_C}\,0 = a\Delta t_{BC} + v_B$$
$$0 = -9.8\Delta t_{BC} + 47.6952$$
$$9.8\Delta t_{BC} = 47.6952$$
$$\underline{\Delta t_{BC} = 4.866857s}$$

$$y_C = ?$$
$$(EQ\ 3)\ y_C = \frac{1}{2}a\Delta t^2 + v_B\Delta t + y_B$$
$$y_C = \frac{1}{2}(-9.8)(4.8669)^2 + (47.6952)(4.8669) + 133.86996$$
$$\underline{y_C = 249.93282m}$$

Stage CD: Free-Fall

Since object is in free fall, acceleration is also at a constant g or $-9.8\ m/s^2$. Calculated Δy_{CD} from h and found y_D. Used $v_C = 0\ m/s$ and Δy_{CD} to calculate v_D using Fund. EQ. 4. Found Δt_{CD} using Fund. EQ. 2.

$$y_C = 249.93282m$$
$$h = 100m$$

Object is falling so change in height is negative.

$$\underline{\Delta y_{CD} = -100m}$$
$$\Delta y_{CD} = y_D - y_C$$
$$-100 = y_D - 249.93282$$
$$\underline{y_D = 149.93282m}$$

$$v_D = ?$$
$$(EQ\ 4)\ v_D^2 = \cancel{v_C^2}\,0 + 2a\Delta y_{CD}$$
$$v_D^2 = 2(-9.8)(-100)$$
$$\sqrt{v_D^2} = \sqrt{1960}$$

Since object is falling, velocity will be negative.

$$\underline{v_{D^-} = -44.271887\ m/s}$$

$$\Delta t_{CD} = ?$$
$$(EQ\ 2)\ v_D = a\Delta t + \cancel{v_C}\,0$$
$$-44.272 = (-9.8)\Delta t_{CD}$$
$$\underline{\Delta t_{CD} = 4.517540s}$$

Stage DE: Parachute

Although v_D was previously calculated, careful to note the exp. -. This means that the velocity from the left *changes* at the right. The reason is because the speed during this stage is *instantly* affected by function $v_{DE}[t]$. Next, I found the integral $v_{DE}[t]$ to get indefinite $y_{DE}[t]$ function, set that function to height at point D which was 149.93m, found C which was 245.93, and then solved for Δt_{DE} when the function was equal to zero or when the rocket returned to ground. You only take the positive value since time can only return at a positive time in this instance.

$$v_p[t] = v_{DE}[t] = -12\left(1 - e^{-\frac{t}{8}}\right) m/s$$
$$\underline{v_{DE}[t] = -12 + 12e^{-\frac{t}{8}}\ m/s}$$

Since object instantly changes velocity when par. opens,

$$\underline{v_{D^+} = 0\ m/s}$$

$$\Delta y_{DE} = |y_E - y_D|$$
$$\Delta y_{DE} = |0 - 149.93282|$$
$$\underline{\Delta y_{DE} = 149.93282m}$$

$$\Delta y_{DE}[t] = \int v_{DE}[t]dt$$
$$\Delta y_{DE}[t] = \int -12 + 12e^{-\frac{t}{8}}\,dt$$
$$\Delta y_{DE}[t] = -12t + 12/-\tfrac{1}{8}\ e^{-\frac{t}{8}} + C$$
$$\Delta y_{DE}[t] = -12t - 96\ e^{-\frac{t}{8}} + C$$
$$149.93282 = -12(0) - 96e^{-0} + C$$
$$C = 245.93282m$$
$$\underline{\Delta y_{DE}[t] = -12t - 96\ e^{-\frac{t}{8}} + 245.93282}$$
$$-12t - 96\ e^{-\frac{t}{8}} + 245.93282 = 0\ (eval.\ graphically)$$
$$\cancel{\Delta t_{DE} = -10.95031s}\ or\ \underline{\Delta t_{DE} = 19.823s}$$

Final:

Just add up the 4 Δt values and 

you get the total time.

$$t_{tot} = \Delta t_{AB} + \Delta t_{BC} + \Delta t_{CD} + \Delta t_{DE}$$
$$t_{tot} = 4.2 + 4.8669 + 4.5175 + 19.823$$
$$t_{tot} = 1s$$
$$\boxed{t_{tot} = 33.41s}$$

t (s)	y (m)	v (m/s)	a (m/s²)
0.0	0.0	0.0	19.0
1.0	9.4	18.6	17.7
2.0	36.3	34.5	13.8
3.0	76.7	45.3	7.3
4.2	133.9	47.7	-3.9
4.2	133.9	47.7	-9.8
5.0	168.9	39.9	-9.8
6.0	203.8	30.1	-9.8
7.0	229.0	20.3	-9.8
8.0	244.4	10.5	-9.8
9.1	249.9	0.0	-9.8
9.1	249.9	0.0	-9.8
10.0	245.7	-9.1	-9.8
11.0	231.6	-18.9	-9.8
12.0	207.8	-28.7	-9.8
13.6	149.9	-44.3	-9.8
13.6	149.9	0.0	-1.5
14.0	149.8	-0.6	-1.4
15.0	148.5	-1.9	-1.3
17.0	142.3	-4.2	-1.0
19.0	132.2	-5.9	-0.8
21.0	119.0	-7.2	-0.6
23.0	103.4	-8.3	-0.5
25.0	86.0	-9.1	-0.4
27.0	67.1	-9.8	-0.3
29.0	47.0	-10.3	-0.2
31.0	26.1	-10.6	-0.2
33.4	0.1	-11.0	-0.1