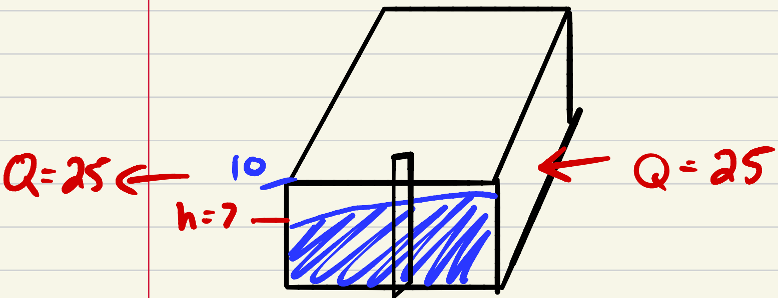
Box Model HW1

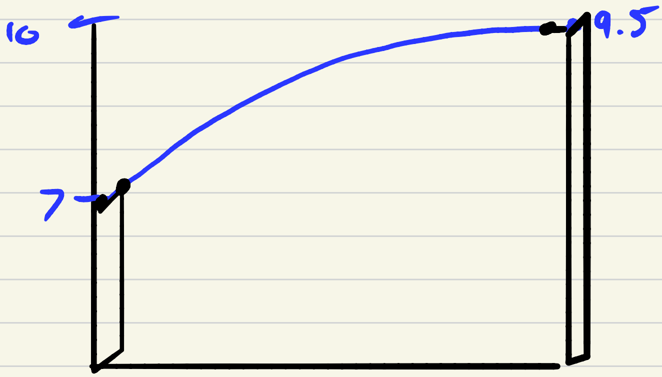
2-6-20



inflo - outflow = ΔS

Steady state
ΔS = 0

inflow = outflow



$Q = KA\, dh/dx$

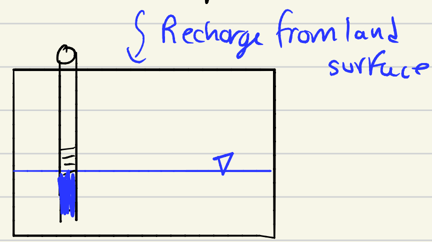
for Q to be constant

there must be an inverse relationship between A + dh/dx

Intro to Groundwater Flow

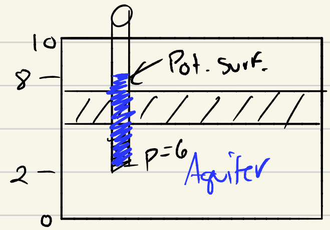
Aquifer: produces water in <u>usable</u> quantities
Aquitard: confining unit

Unconfined Aquifer



water level rises to water table

Confined Aquifer



- upper bdry is confining unit
- water will rise to potentiometric surface

Key Hyrologic parameters

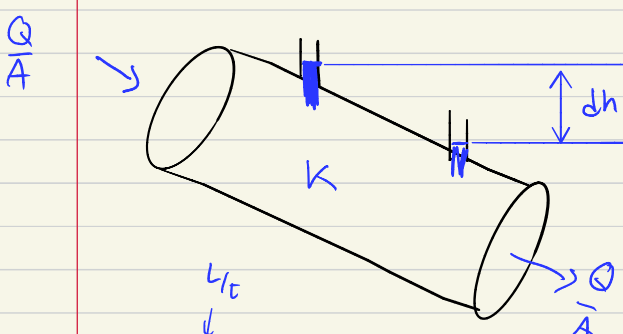
Storativity (S) [-]: The volumetric change in water storage <u>per</u> unit surface area per unit change in head

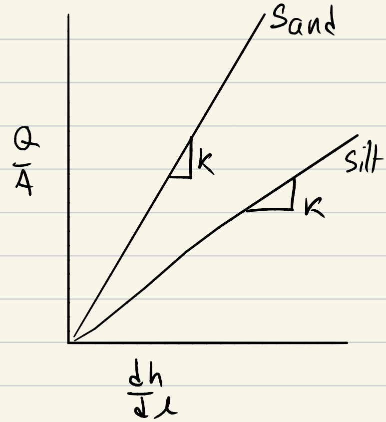
Specific yield (Sy) [-]: the change in storage caused by changes in water level

Specific storage (Ss) [1/L] = the change in storage caused by compression or expansion of mineral skeleton (By changing pressure)

Unconfined	Confined
· changes in storage by draining porespace	- Storage change from saturated storage
$S = S_y + bS_s$ ↑ Saturated thickness	- Pot surf changes but not our water levels
* Generally assume that $S_y >> S_s$	$S = bS_s$
$S = S_y$	$\sim 0.00005 - 0.005$ ↑ small S_s values mean that large press. changes produce small changes in storage
$\sim 0.07 - 0.25$	

Saturated GW Flow: Darcy's Law (1856)





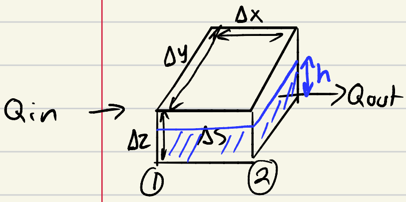
$$\boxed{Q = -KA\, dh/dL}$$

- Q: Volumetric flux L^3/t (L/t for Q/A)
- A: cross section area L^2
- dh/dL: Change in head per distance [-]

Governing eqn

1) Conservation of mass: out − in = ΔS

2) Darcy's Law: $Q = -KA\, dh/dL$



$Q_{in} = -k\, dh/dL\, \Delta z \Delta y$

$Q_{out} = Q_{in} - \frac{\partial Q_{in}}{\partial x}\Delta x$

$Q_{out} = Q_{in} - \partial/\partial x\, \underbrace{(-k\, dh/dL\, \Delta y \Delta z)}_{Q_{in}}\, \Delta x$

now apply conservation:

$Q_{out} - Q_{in} = \Delta S$

① $Q_{out} - Q_{in} = \partial/\partial x\, (k\, \partial h/\partial x\, \Delta y\, \Delta z)\Delta x = \Delta S$

② $\Delta S = S\, \Delta y \Delta x\, \partial h/\partial t$ * because $S = \frac{\Delta S}{(\Delta x \Delta y)(\partial h/\partial t)}$

change in volume; unit area; unit change in head

* Assuming Unconfined Aquifer

$\Delta S = \text{in} - \text{out} + R$

Combine eqn ① + ②

$\Delta S = \Delta S$

this is our Δz

$S_y \cancel{\Delta y}\cancel{\Delta x}\, \partial h/\partial t = \partial/\partial x\, (k\, dh/dx\, h\, \cancel{\Delta y})\cancel{\Delta x} + R\, \cancel{\Delta y}\cancel{\Delta x}$

$S_y\, \partial h/\partial t = \partial/\partial x\, (h\, \partial h/\partial x\, k) + R$

↳ $h\, \partial h/\partial x = \frac{1}{2}\frac{\partial h^2}{\partial x}$ because calculus

$S_y\, \partial h/\partial t = \frac{1}{2}\, \partial/\partial x\, (\partial h^2/\partial x)\, k + R$

$$\boxed{\frac{2 S_y}{k}\, \partial h/\partial t = \frac{\partial^2 h^2}{\partial x^2} + \frac{2R}{k}}$$ ← 1d flow, homogeneous unconfined aquifer

$$\boxed{\frac{2 S_y}{k}\frac{dh}{dt} = \frac{d^2 h^2}{dx^2} + \frac{d^2 h^2}{dy^2} + \frac{d^2 h^2}{dz^2} + \frac{2R}{k}}$$ ← 3D flow