Module 1

Targets/Objectives

At the end of the lesson, students should be able to:

- Round numbers to the nearest ten, hundred and thousand.
- Write numbers into correct scientific notation.
- Apply knowledge of significant figures to scientific calculations.
- Convert metric units of length, mass, volume and temperature
- Compute problems involving units of length, mass, volume and temperature

Use dimensional analysis in converting Metric to English Systems of measurements

Lecture Guide

Rounding Off Numbers

Rounding off is a type of estimation where the number is made simpler without changing the value. It is done for whole numbers and decimals at various decimal place values.

General Rounding Rules

1. If the number you are rounding is followed by a digit less than 5 (e.g. 0, 1, 2, 3, or 4), round the number by dropping the digits and replace by zeroes.

Example1. Round off 16.5832 to the nearest hundredths

Since the digit to be rounded is 8 followed by 3 which is less than 5, drop the digits and replace by zeroes. Thus, the answer is 16.58<u>00.</u>

2. If the number you are rounding is followed by a digit equal to or greater than 5 (e.g. 6,7,8,9), round the number by adding 1 to the digit.

Example 2. Round off 12.74589 to the nearest thousandths

The digit to be rounded is 5 followed by 8 which is greater than 5, add 1 to 5 and drop the other digits. Thus, the answer is 12.746

Scientific Notation

A standardized way of representing any number as the product of a real number and a power of 10.

$$ax10^{b}$$

where a is called the coefficient and b is the exponent.

Writing correct scientific notation is important to engineers and physicists.

Example is the charge of a single electron, the "C" stands for Coulomb, the unit of electrical charge.

$$q_e = 0.000000000000000000162\ C$$

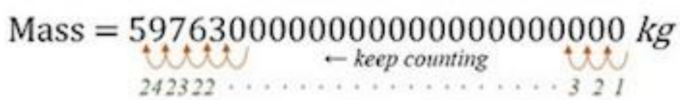
To write in correct scientific notation, move the decimal point 19 digits going to the right until the first non-zero digit: in this case, 1 is isolated. That means the coefficient will be 1.62 and the exponent will be -19. This results in the form:

$$q_e = 1.62x10^{-19}$$

Another example is the mass of the Earth in kilograms.

Mass of the Earth= 5976300000000000000000000 kg

It would be convenient to use scientific notation in expressing the mass. To write in correct scientific notation, move the decimal point to the left until there is one number to the left of the decimal point.



Mass = $5.9763x10^{24}kg$ *Note:*

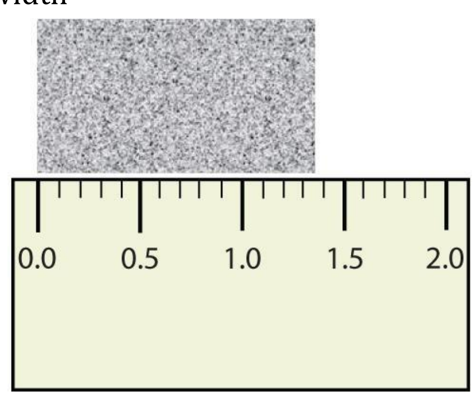
- If the number is greater than 1, move the decimal point going to the left, the coefficient is positive.
- If the number is less than 1, move the decimal point going to the right, the coefficient is negative.

Significant Figures

Significant figures of a number are digits which contribute to the precision of that number.

Consider using a ruler to measure the width of an object as shown below.

Expressing width

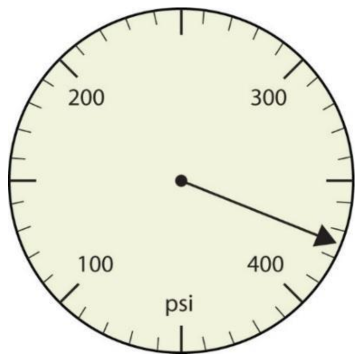


Based from the tick marks in the ruler, the object is more than 1 cm long, so we are assured that the first digit is 1. By counting the tick marks on the ruler, the object is at least three tick after 1, and each tick is 0.1 cm, so the object is at least 1.3 cm wide. But the ruler does not have any more ticks between 0.3 and 0.4 marks, so we don't know exactly how much the next decimal digit is. We can only estimate it as about six-tenths of the way between 0.3 and 0.4 marks. So, we estimate the hundredths place to be 6, so the measurement is 1.36 cm wide.

What is the proper way to express the width of this object?

Since the hundredths place is an estimate only, it will be useless to do an estimate of the thousandths place unless a different instrument is used like the Vernier caliper or Micrometer. Our best measurement is until the hundredths place, so we express the width to be 1.36 cm.

Consider the reading in a pressure gauge as shown below.



Based on the position of the arrow, it passes the 70th mark, so our initial reading is 370 psi. We are assured of the first and second digits. Since the arrow stops between 7th and 8th tick mark, we can estimate it as about 5 psi between 70 and 80 psi. So, the pressure reading is 375 psi.

This concept of reporting the proper number of digits in a measurement or a calculation is called significant figures. Significant figures (sometimes called significant digits) represent the limits of what values of a measurement or a calculation we are sure of. The convention for a measurement is that the quantity reported should be all known values and the first estimated value.

Rules for Determining if a Number is Significant or Not

1. All non-zero digits are considered significant.
   Example: 56 has two significant figures (5 and 6) while 456.897 has six significant figures (4,5,6,8,9 and 7)
2. Zeros appearing between two non-zero digits are significant. Example: 205.113 has six significant figures (2,0,5,1,1 and 3)
3. Leading zeros or zeros before non-zero numbers are not significant. Example: 0.0078 has two significant figures (7 and 8)
4. Trailing zeros or zeros after non-zero numbers without a decimal are generally not significant.

Example: 5000 has one significant figure (5)

5. Trailing zeros in a number containing a decimal point are significant. Example: 153.68000 has eight significant figures (1,5,3,6,8,0,0,0). The decimal number 0.000145600 has six significant figures (1,4,5,6,0,0)
6. The number 0 has one significant figure. Therefore, any zeros after the decimal point are also significant.
   Example: 0.00 has three significant figures
7. Any numbers in scientific notation are considered significant. Example: $2.300x10^{-5}$ has four significant figures

Conversion of Units

Standard Units (SI Units)

The International System of Units (abbreviated SI) is the metric system used in science, industry, and medicine.

Units of the SI System

There are seven base units in the SI system:

the kilogram (kg), for mass the second (s), for time the kelvin (K), for temperature the ampere (A), for electric current the mole (mol), for the amount of a substance the candela (cd), for luminous intensity the meter (m), for distance

English System

The English system of measurement is a system of measure that uses body parts and familiar objects. An example is ground distances that uses human foot for short distance and paces for longer distances. This system allows discrepancies between measurements obtained by different individuals. A standard was eventually set to ensure that all measurements represented the same amount for everyone.

Some conversion factors for English System

Length	Weight	Capacity
1inch (in)=2.54 centimeters (cm)	1 pound (lb)= 16 ounces (oz)	1 cup (c)=16 tablespoons
1 foot (ft) = 12 inches (in)	1 ton = 2000 lbs	1 cup = 8 fluid ounces
1 yard (yd) = 3 feet	1 lb=0.4536 kilograms	1 pint(pt)=2 cups
1 mile (mi) = 5280 feet		1 quart (qt)= 2 pints
1 mile = 1760 yards		1 gallon(gal)= 4 quarts

Factor Label Method

A conversion ratio or unit factor is a ratio equal to one. This conversion ratio is based on the concept of equivalent values. It can be used for conversion within English system and metric system or between the systems. In the example below one foot is substituted for its equivalent measure of 12 inches.

$$\left.\frac{12\ inches}{12\ inches} \quad \frac{1\ foot}{12\ inches}\right\} \qquad = \qquad \text{conversion ratios always equal to 1}$$

This is the usual way of converting units that most students are comfortable with or just using the conversion factor directly.

In the factor label technique, the conversion factors are obtained from the unit equivalents and the two conversion factors are reciprocals of each other, and both of them are equal to one. Thus, when multiplying a measurement by a conversion factor only the units are changing and not the value.

Unit Equivalents	Conversion Factors (longer to shorter units of measurement)	Conversion factors (shorter to longer units of measurement)
1 inch (in)= 2.54 centimeters (cm)	$\frac{2.54\ cm}{1\ in}$	$\frac{1\ in}{2.54\ cm}$
1 foot(ft) = 12 inches(in)	$\frac{12\ in}{1\ ft}$	$\frac{1\ ft}{12\ in}$
1 yard(yd) =3 feet (ft)	$\frac{3\ ft}{1\ yd}$	$\frac{1\ yd}{3\ ft}$

Example 1. Convert 3.5 inches to feet.

Use the conversion ratio and start with the given unit going to the required unit.

Note:

$$\frac{12\ inches}{1\ foot} = \frac{1\ foot}{12\ inches}$$

- The unit in the numerator of the proper conversion factor should match the unit you are trying to find.
- Conversion to a bigger unit produces a smaller number, and vice versa.

what is required

what is given

$$3.5\ inches\ x\ \frac{1\ foot}{12\ inches} = 0.292\ foot$$

Example 2. Convert 10 tons to pounds (lb)

Using the conversion ratio

$$\frac{1\ ton}{2000\ lbs} \quad \frac{2000\ lbs}{1\ ton} \rightarrow \boxed{\text{use this ratio}}$$

Starting with the given 10 tons equation will be

$$10\ ton\ x\ \frac{2000\ lbs}{1\ ton} = 20{,}000\ lbs$$

Metric System-Moving the Decimal Point

In the metric system, each basic type of measurement (length, weight, capacity) has one basic unit of measure (meter, gram, liter). Conversions are quickly made by multiplying or dividing by factors of 10. It is as simple as moving the decimal point to the right (for smaller prefixes) or to the left (for larger prefixes).

The names of metric units are formed by adding a prefix to the basic unit of measurement. To tell how large or small a unit is, you look at the prefix. To tell whether the unit is measuring length, mass, or volume, you look at the base.

Prefixes in the Metric System						
kilo-	**hecto-**	**deka-**	**meter gram liter**	**deci-**	**centi-**	**milli-**
1,000 times larger	100 times larger	10 times larger	base units	10 times smaller	100 times smaller	1,000 times smaller
than base unit	than base unit	than base unit		than base unit	than base unit	than base unit

Using this table as a reference, you can see the following:

- a kilogram is 1,000 times larger than one gram (so 1 kilogram = 1,000 grams).
- a centimeter is 100 times smaller than one meter (so 1 meter = 100 centimeters).
- a dekaliter is 10 times larger than one liter (so 1 dekaliter = 10 liters).

A similar table that just shows the metric units of measurement for mass, along with their size relative to 1 gram (the base unit).

Measuring Mass in the Metric System						
kilogram (kg)	hectogram (hg)	dekagram (Dag)	gram (g)	decigram (dg)	centigram (cg)	milligram (mg)
1,000 grams	100 grams	10 grams	gram	0.1 gram	0.01 gram	0.001 gram

Since the prefixes remain constant through the metric system, you could create similar charts for length and volume. The prefixes have the same meanings whether they are attached to the units of length (meter), mass (gram), or volume (liter).

Example 1. How many milligrams are in one decigram?

Recreate the order of the metric units as shown below.

$$kg \quad hg \quad dag \quad g \quad dg \underbrace{\qquad}_{1} cg \underbrace{\qquad}_{2} mg$$

The question asks you to start with 1 decigram and convert that to milligrams. As shown above, milligram is two places to the right of decigrams. You can just move the decimal point two places to the right to convert decigrams to milligrams:

$$1dg = 1\underbrace{0}_{1}\underbrace{0}_{2}.\ mg$$

The answer is 100 mg

Example 2. Convert 1 cm to km.

Recreating the order of the metric units as in example 1.

$$km \underbrace{\qquad}_{5} hm \underbrace{\qquad}_{4} dam \underbrace{\qquad}_{3} m \underbrace{\qquad}_{2} dm \underbrace{\qquad}_{1} cm \quad mm$$

Note that instead of moving to the right, you are now moving to the left—so the decimal point must do the same:

$$1cm = 0.\underbrace{0}_{5}\underbrace{0}_{4}\underbrace{0}_{3}\underbrace{0}_{2}\underbrace{1}_{1}\,km$$

The answer is 0.00001 km

Temperature

Temperature is the only measurement that is asked to convert between instead of within systems. The most common temperature scales are Celcius (°C), Fahrenheit (°F), and Kelvin (K).

Common comparison points:

$0°\mathrm{C} - 32°\mathrm{C} - 273.15K \rightarrow$ water freezes

$100°\mathrm{C} - 212°\mathrm{F} - 373.15K \rightarrow$ water boils

$22°\mathrm{C} - 72°\mathrm{F} - 295.15K \rightarrow$ room temperature

Conversion of Temperature Between Celsius and Kelvin

- temperature conversion formula from Celsius to Kelvin: $K = °\mathrm{C} + 273.15$
- temperature conversion formula from Kelvin to Celcius:

  $°\mathrm{C} = K - 273.15$

Conversion of Temperature Between Fahrenheit and Celsius

- temperature conversion formula from Fahrenheit to Celsius $°\mathrm{C} = \frac{°\mathrm{F} - 32}{1.8}$

- temperature Conversion Formula from Celsius to Fahrenheit: $°F = 1.8°C + 32$

Conversion of Temperature Between Fahrenheit and Kelvin

- temperature Conversion Formula from Fahrenheit to Kelvin: $K = \frac{°F-32}{1.8} + 273.15$
- temperature Conversion Formula from Kelvin to Fahrenheit: $°F = (K - 273.15)x1.8 + 32$

Example 1: Convert 60°C to °F.

Using the formula for converting Celcius to Fahrenheit.

$$°F = 1.8°C + 32$$
$$= 1.8\ (60) + 32$$
$$= 140\ °F$$

Example 2. Convert 610 K to °F

Using the formula for converting Kelvin to Fahrenheit

$$°F = (K - 273.15)x1.8 + 32$$
$$= (610 - 273.15)x\ 1.8 + 32$$
$$= 638.33$$

Time

Both the Metric and English systems use the same units of measurement for time. Same with most conversions, the conversion table for units of time should familiarized.

The most common conversion factors of time are:

$1\ hr = 60\ mins$

$1\ \text{min} = 60\ sec$

Example 1. Convert 400 seconds to mins

Using the factor label method, we use the conversion factor $1\ \text{min} = 60\ sec$.

$$400\ \text{sec}\ x \frac{1\ min}{60\ sec} = 6.667\ mins$$

Note:

As a general rule in this subject, we round off our answers to three decimal places.

Dimensional Analysis in Chemistry

Dimensional analysis is a way chemists and other scientists convert units of measurement. With a known conversion factor, it is possible to convert any unit to another unit of the same dimension. This method is used from both simple to complex conversions between different sets of units. In order to compute from one unit to another, we need to know how those units are related to help set up and solve calculations. Dimensional analysis is also called factor label method.

Example 1. Convert 5.5 ft to cm.

We first convert $ft \rightarrow in$ then $in \rightarrow cm$ or $ft \rightarrow in \rightarrow cm$ using the conversion factors 1ft =12 in and 1in=2.54 cm.

Set up the equation. Start with the given unit going to the required unit. The unit in the numerator of the conversion factor is the unit you are trying to find.

$$5.5\ ft\ x \frac{12\ in}{1 ft} \frac{2.54\ cm}{1\ in} x = 167.64\ cm$$

Another way of solving this problem is to place the units first then the conversion factors starting from the given unit going to the required unit.

$$ft \rightarrow in \rightarrow cm$$

$$5.5\ ft\ x \frac{?\ in}{?ft} \frac{?\ cm}{?\ in} x$$

Make sure that the unit you are trying to find is in the numerator. If you try to simplify the units, what is left is the required unit.

$$5.5\ \cancel{ft}\ x \frac{?\ \cancel{in}}{?\cancel{ft}} x \frac{?\ cm}{?\ \cancel{in}} = ?\ cm$$

Place the conversion factors.

$$5.5\ ft\ x \frac{12\ in}{1\ ft} \frac{2.54\ cm}{1\ in} x = 167.64\ cm$$

Example 2. Convert 350mm to ft.

Set up the equation. Start with the given unit going to the required unit. The flow of units will be $mm \rightarrow cm \rightarrow in \rightarrow ft$.

$$350\ mm\ x \frac{1\ cm}{10\ mm} \frac{1\ in}{2.54\ cm} \frac{1\ ft}{12\ in} x x \frac{350x1x1x1x1}{10x2.54x12} == 1.148\ ft$$

Derived Units

When units are squared or cube as with the area and volume, the conversion factors themselves must also be squared or cubed. The most common volume units are milliliters (ml), Liters (L), cubic meter (m^3) and cubic centimeters (cm^3), which is $1m^3 = 100^3 cm^3$. The conversion factor $1\ cm^3 = 1\ ml$ is very useful in converting metric system to English system of volumes.

Example 3. Convert 1 L to m^3.

Set up the equation. The flow of conversion will be from $L \rightarrow ml \rightarrow cm^3 \rightarrow m^3$, taking into account that we are to cube the conversion factors.

$$1L \, x \, \frac{1000 \, ml}{1 \, L} \, x \, \frac{1 \, cm^3}{1 \, ml} \, x \, \frac{1 \, m^3}{100^3 cm^3} = 0.001 \; or \; 1.0x10^{-3} m^3$$