I. Design Calculations

The torque required for by the robot is given by;

$T = F \times L \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..(a)$

Where T is the required torque, F is the applied force and L is the distance of the point of application of force and the pivot point.

The applied force is given by;

$F = Mg \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.(b)$

Where M is mass in kg and g is the acceleration due to gravity given as;

$g = 9.81\, m/s^2$

Force is equivalent to weight where the weight is given as;

$W = Mg \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.(c)$

Substituting (b) in (a), gives the torque required to hold a mass at a given distance from pivot is given as

$T = (Mg) \times L \dots\dots\dots\dots\dots\dots\dots\dots\dots..(1)$ L is the perpendicular length from the applied force to the pivot.

Therefore,

$\sum T = 0 = FL - T \dots\dots\dots\dots\dots\dots\dots d$

Substituting b in d, Torque can be found using torque balance as;

$L(Mg) \times L = T_A$

In order to estimate the torque at each joint, the worst case should be chosen that is L should be maximum. It should be noted that F does not change in this case.

From the above equations, we calculate the required torque to accelerate weight being held by the motors from state position as follows;

$$\sum T_N = T(holding) + T(motion)$$

$$= T \times a$$

Let A_1 and A_2 be two ends loads of a rod and W_1 be a point load acting at a point L_2m from point A_2. Let also the length of the rod be L_1 and the torque T_1 act at point A_2 of the supporting rod.

Assuming the center of mass of the rod is located at the center of the total length, then the torque T_1 is given by;

$$T_1 = L_1 A_1 + \frac{1}{2} L_1 W_1$$

A_1 is the object load and A_2 is the weight of the motor.

The torque of the motor arm will generally be given as;

$T_1 = L_1A_4 + L_2W_3 + L_3A_3 + L_4 W_2 + L_5A_2 + L_6W_1 \dots\dots\dots\dots\dots\dots..(e)$ Equation (e) solves the torque of the arm when held horizontally. However from restto move position, the arm will require an acceleration a.

$$T = I a$$

Where I is moment of inertia. Torque in this case is the sum of torque at pivot point. To calculate the extra torque required to move, we will calculate the moment of inertia from the end pivot.

$I = \frac{mr^2}{q}$ (applied at the center of the mass)

To take into consideration that moment of inertia must be calculated away from distance away from center of mass, in each joint the product of each individual mass by the square of its respective length from pivot is given as;

$$I_N = \sum_{i=1}^{N-1} \frac{m_i r_i^2}{q}$$

To calculate the required motors for mobile base;

We start with assumptions

Nominal speed is $0.3 m/s$, max speed will be dependent on other factors

Max speed range will be $0.5\ to\ 0.9 m/s$

Angle of inclination $\theta$is 20% slope,

That is;

$\theta = \arctan 0.20 = 11^0$

The acceleration time to reach nominal speed is given $0.2s$

Robot weight is $M_R = 2okg$

Maximum total load $M_L = 25kg$

Nominal speed $V_N = 0.3\ m/s$

Acceleration time $t_A = 0.5\ s$
Maximum slope $k = 20\%$ Wheel diameter $D_W = 0.13m$

Wheel traction rotation speed is given by;

$$N_T = \frac{60V_N}{\pi D_W} = \frac{60 \times 0.3}{0.13\pi} \approx 44rpm$$

The approximate torque needed to climb on maximum slop by;

$$F_T = gk(M_R + M_L)$$
$$= 9.81 \times 0.2(2o + 25)$$

$$= 88.29N$$

The traction motor power is given by;
$P_u = F_T V_N = 88.29 \times 0.3 \approx 26W$This is the power required for the entirerobot. Power for each wheel be 6W.
The torque for each traction wheel is given as;

$$T_T = \frac{1}{2}\frac{D_W}{2} \times F_T = \frac{1}{2} \times \frac{0.13}{2} \times 88.29$$

$$\approx 2.9Nm$$

<u>Selection of the motor and gearbox.</u>

Knowing

$$N_T = 44rpm$$

$$T_T = 2.9Nm$$

$$P_T = 6.6\ W$$

The speed is use hence use of more stage gearbox. With more gear box stages means less efficiency.

The gearbox efficiency is $n_G = 0.81$

The output power is given as

$$P_M = \frac{P_T}{n_G} = \frac{6.6}{0.81} = 8.19W$$

$$R_T = \frac{N_T}{N_M} = \frac{44}{4500} \approx 0.01$$

The minimum gear ration we can get from the dataset is 10.1 with 81% efficiency.

So the motor speed is ;

$$N_M = \frac{N_T}{R} = \frac{44}{10.1} = 4313rpm$$

Output torque; from datasheet, $T_M = 0.26Nm$

$$T_0 = \frac{T_M n_G}{R} = \frac{0.26 \times 0.81}{10.1} = 20\ mNm$$

<u>Calculation of Robot Stability.</u>

To calculate robot stand stability, it is important to consider force, weight and maximum reach of the robot. The mobile baseshould be designed for maximum capacity of the robot maximum load which is set to be 45kg. The maximum reach is set to be 0.8m where the mobile base must be stable at maximum load with maximum reach. Knowing that all components in the design are assumed to be rigid.

Assuming that the centre of mass occur at 50% of the robot at point at $\frac{R}{2}$. Now calculating the centre of mass for both mobile base of the arm.
Given the following parameters;
T_M: Maximum torque of the robot base joint (Nm)
m_{PL}: Maximum robot payload (kg)
m_R: Mass of robot (kg)
R : Robot reach (m)
g: Gravity (9.81 m/s^2)
S: stand centre of gravity position (m).
d_R: Distance between robot arm and mobile base with floor (m)
m_S: Mass of robot base.
L: Length of stand.

From the datasheet, with 10W power the motor speed is $N_M = 4500\ rpm$

The ideal gear ration is given as;

The following are the unknown values.

T_B: Robot base plate reaction torque (Nm).

F_2: Robot base plate force (N).

F_F: Floor front reaction Force.

Calculation of reactions.

The vertical components of the robots armweight;

$$\sum Vertical\ Force\ (V_F) = 0$$

$$F_Z = m_{PL}g + m_R g$$

Summing moments acting on the robot baseplate;

$$\sum Moments_b = 0$$

$$T_B = T_M + m_{PL}gR + m_R g \frac{R}{2}$$

Summing moments acting on the mobile base;

$$\sum Moments_M = 0$$

$$0 = m_S g S + F_Z d_R - T_B - T_F L$$

$$F_F = (m_S g S + F_Z d_R - T_B)/L$$

If F_F is greater than zero then the robot isstable.

$T_M = 82Nm$, $m_{PL} = 2.7\ kg$, $m_R = 8.7\ kg$,

$R = 0.8\ m$, $g = 9.81 m/s^2$, $F_Z = 112N$,

$T_B = 136Nm$, $m_S = 45kg$, $S = 0.36$, $d_R = 0.33m$, $L = 0.72m$,

$F_F = 82N$

Since $F_F \geq 0$; Robot is stable.