Rumor Spreading in local communication networks

Life Data Epidemiology A.Y. 2023/24

Barbiero - Bordin - Pitteri

Rumor Spreading

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Introduction

According to the Cambridge dictionary a Rumor is

“an unofficial, interesting story or piece of news that might be true or invented, and that is communicated quickly from person to person”

Introduction

To observe the spreading of a rumor, we need three key components:

1. A network of people that talk to each other
2. A rumor to be spread
3. A model to depict the communication process

The Network

The Network

Overview

The dataset used in this study is the

Interaction data from the Copenhagen Networks Study

Published in 2019 by Sapiezynski et al, it features data collected on more than 700 freshmen at the Technical University of Denmark regarding:

1. Phone calls
2. Messages
3. In person contacts

The Network

Data cleanup and preprocessing

We decided to focus only on calls and SMS data, since in-person interactions would have required a separate modeling, outside the scope of this research. Data is presented as

`calls.csv`

- timestamp of interaction
- id of sender
- id of receiver
- duration

`sms.csv`

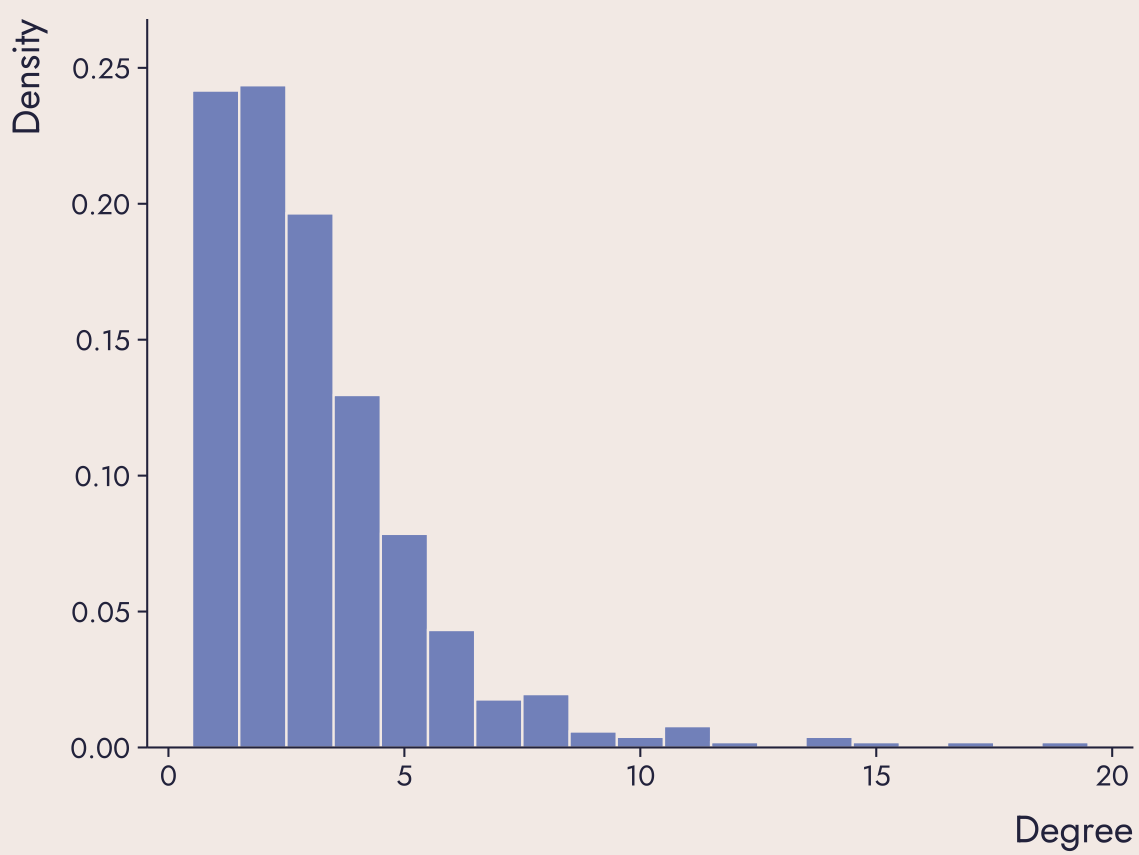
- timestamp of interaction
- id of sender
- id of receiver

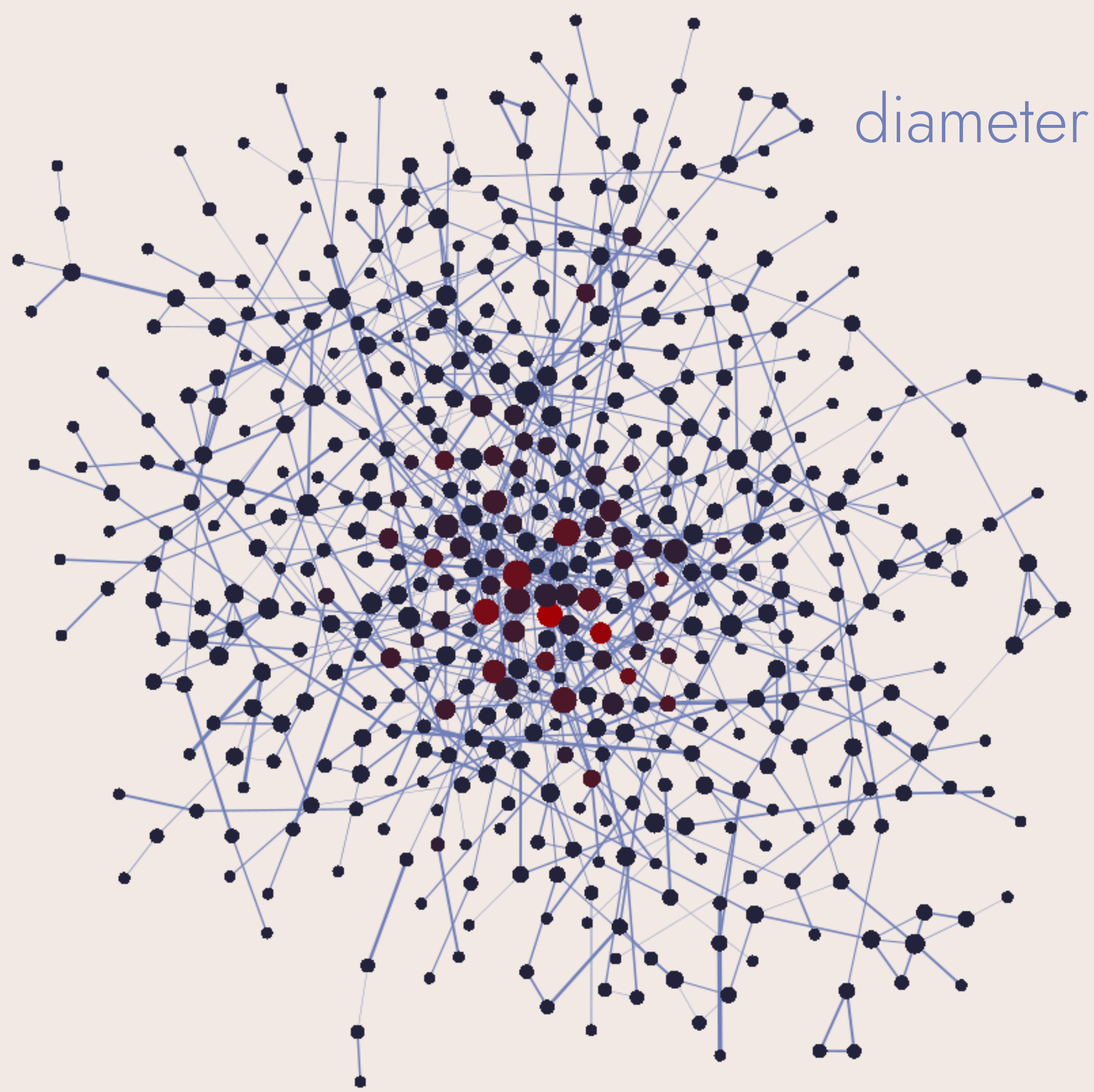
The Network

Data cleanup and preprocessing

The final network is comprised of all the nodes which had at least one interaction with any other node, being it via call or via SMS

This network features 509 nodes





diameter: 19

The Network

Data cleanup and preprocessing

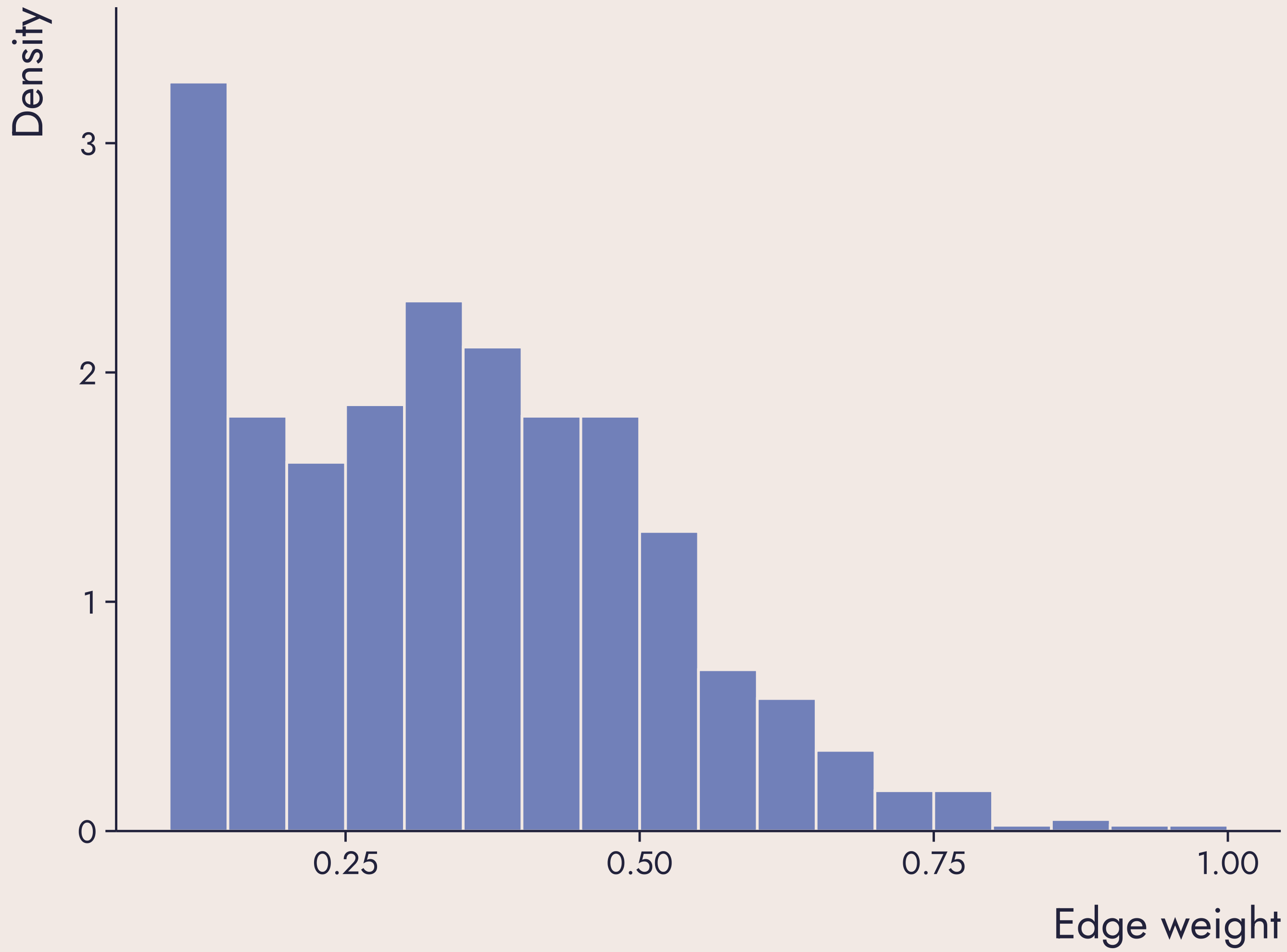
In order to capture the different magnitudes of communication between nodes the network is a weighted network with

$$w_{ij} = 1 + \ln(N_{\text{calls}} + N_{\text{text}})$$

and then rescaled in [0, 1]

This way the diameter is reduced

diameter: 6.92



The Models

The Models

Two ways

Two models were analyzed

Simple contagion

Developed initially by

Daley and Kendall

1964-1965

Complex contagion

Developed as

part of the project

The Models

Simple contagion

Both models are compartmental models with three possible states:

S → susceptible: ignorant to the rumor and recipient to it

I → infected: knows and spreads the rumor

R → stifler: not interested in the rumor anymore

The Models

Simple contagion - Base

The rate equations for the Daley and Kendall model are

$$I + S \xrightarrow{\lambda} I + I$$

$$I + I \xrightarrow{\alpha} R + R$$

$\rightarrow$ The rumor loses its appeal

$$R + I \xrightarrow{\alpha} R + R$$

The Models

Simple contagion - Refinements

We decided to add two modifications already explored in the literature

Add the probability p that a susceptible does not believe or is not interested in the rumor[1]

$$I + S \xrightarrow{\lambda} I + I \quad \rightarrow \quad \begin{cases} I + S \xrightarrow{\lambda(1-p)} I + I \\ I + S \xrightarrow{\lambda p} I + R \end{cases}$$

Change the recovery process to[2]

$$I + I \xrightarrow{\alpha} R + R \quad \rightarrow \quad I + I \xrightarrow{\alpha} I + R$$

1: Borge-Holtoefer et al. (2013)   2: Maki and Thompson (1973)

The Models

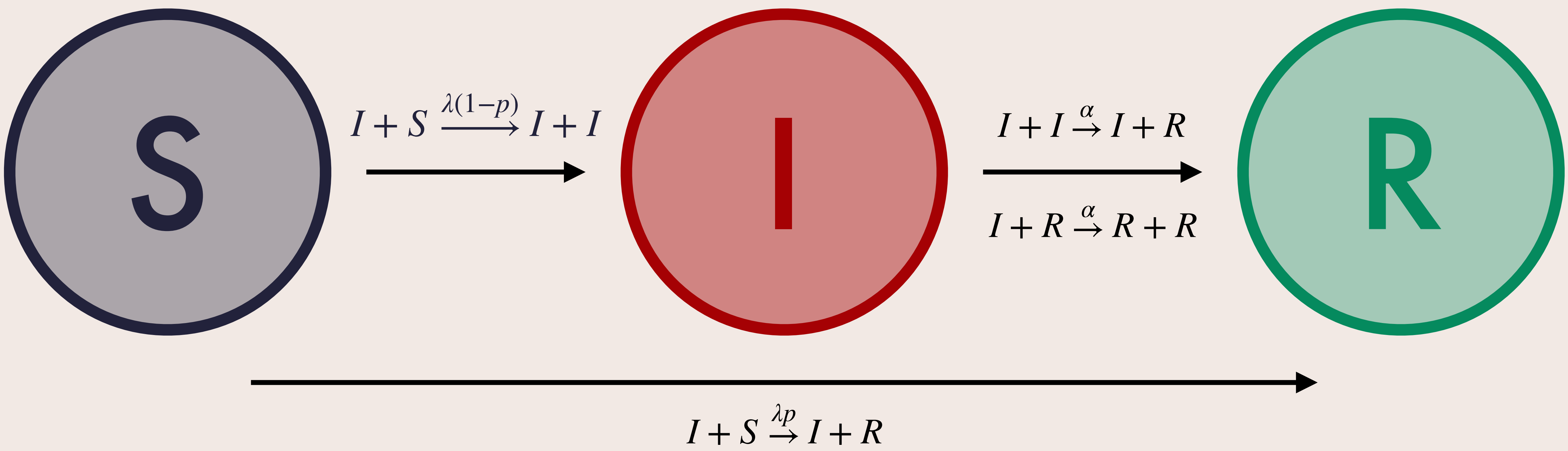
Simple contagion - Refinements

Finally, the parameters λ, α were rescaled to take weights into account

- $\lambda \to \lambda \mathrm{w}_{ij}$
- $\alpha \to \alpha \mathrm{w}_{ij}$

The Models

Simple contagion - Refinements



The Models

Simple contagion - Well mixed population

In the homogeneous mixing approximation

$$\begin{cases} \dfrac{\mathrm{d}s}{\mathrm{d}t} = -\lambda si \\ \dfrac{\mathrm{d}i}{\mathrm{d}t} = \lambda(1-p)si - \alpha i^2 - \alpha ir \\ \dfrac{\mathrm{d}r}{\mathrm{d}t} = \lambda psi + \alpha i^2 + \alpha ir \end{cases}$$

The Models

Simple contagion - Well mixed population

At the beginning of the "epidemic"

$s \approx 1,\ i = i_0 \ll 1,\ r = 0$ so

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \lambda(1-p)si - \alpha i^2 - \alpha ir = i[\lambda(1-p)(1-i) - \alpha i]$$

$$\left.\frac{\mathrm{d}i}{\mathrm{d}t}\right|_{t=0} > 0 \iff \lambda(1-p)(1-i_0) - \alpha i_0 > 0 \iff I_0 < \frac{N\,\lambda(1-p)}{\alpha + \lambda(1-p)}$$

conversely, for $I_0 = 1 \quad N > 1 + \dfrac{\alpha}{\lambda(1-p)}$

Always satisfied for reasonably large N

The Models

Simple contagion - Well mixed population

$$\begin{cases} \frac{\mathrm{d}s}{\mathrm{d}t} = -\lambda s(1-s-r) \\ \frac{\mathrm{d}r}{\mathrm{d}t} = \lambda p s(1-s-r) + \alpha(1-s-r)^2 + \alpha(1-s-r)r \end{cases}$$

Self-consistent equation for the final attack rate

$$\frac{\mathrm{d}r}{\mathrm{d}s} = -\frac{\lambda p s + \alpha r + \alpha(1-s-r)}{\lambda s}$$

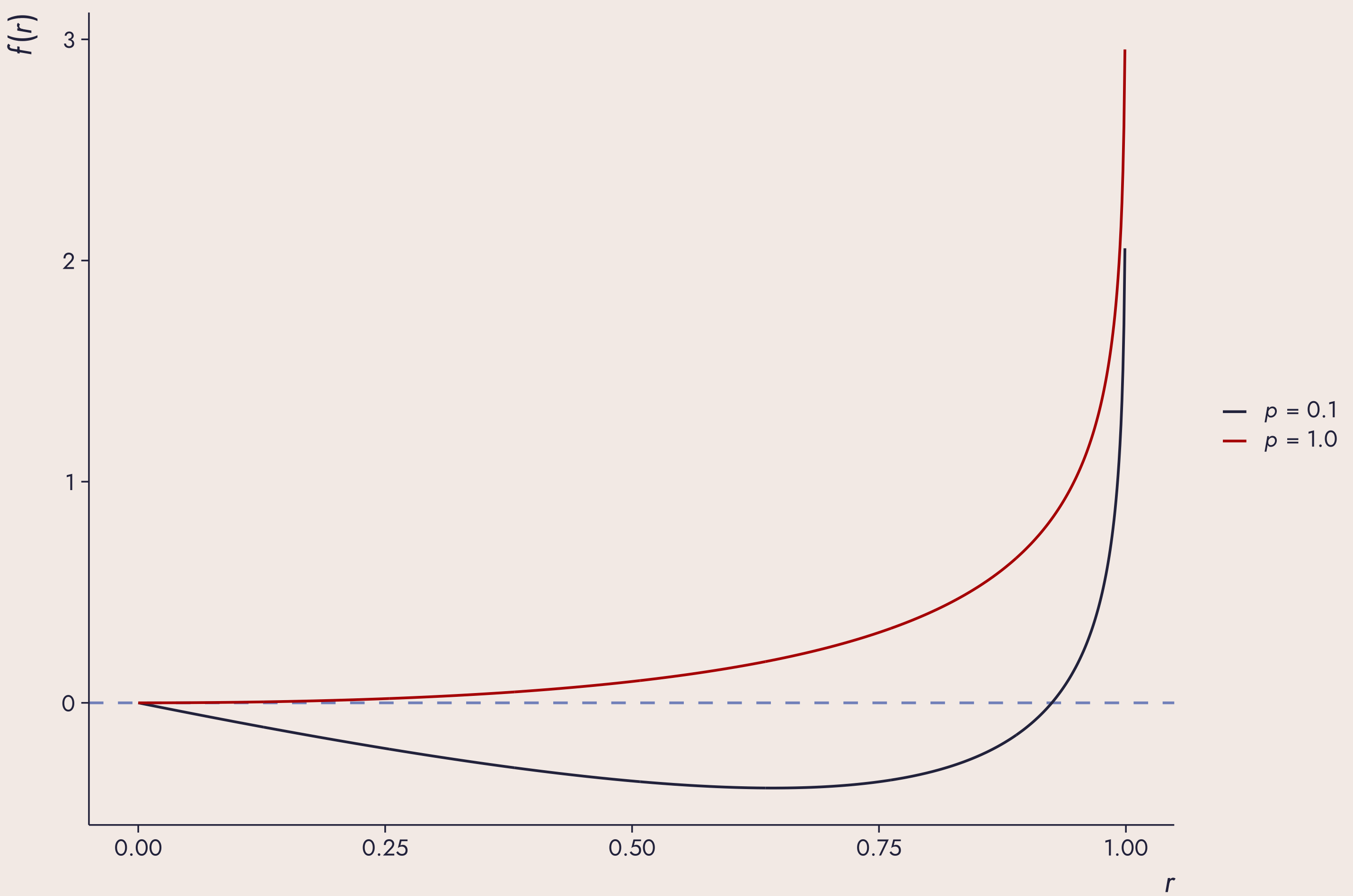
$$r_\infty = \left(\frac{\alpha}{\lambda} - p\right)(1-r_\infty) - \frac{\alpha}{\lambda}\ln(1-r_\infty) + p - \frac{\alpha}{\lambda}$$

The Models

Simple contagion - Well mixed population

Looking for the solution of the equation

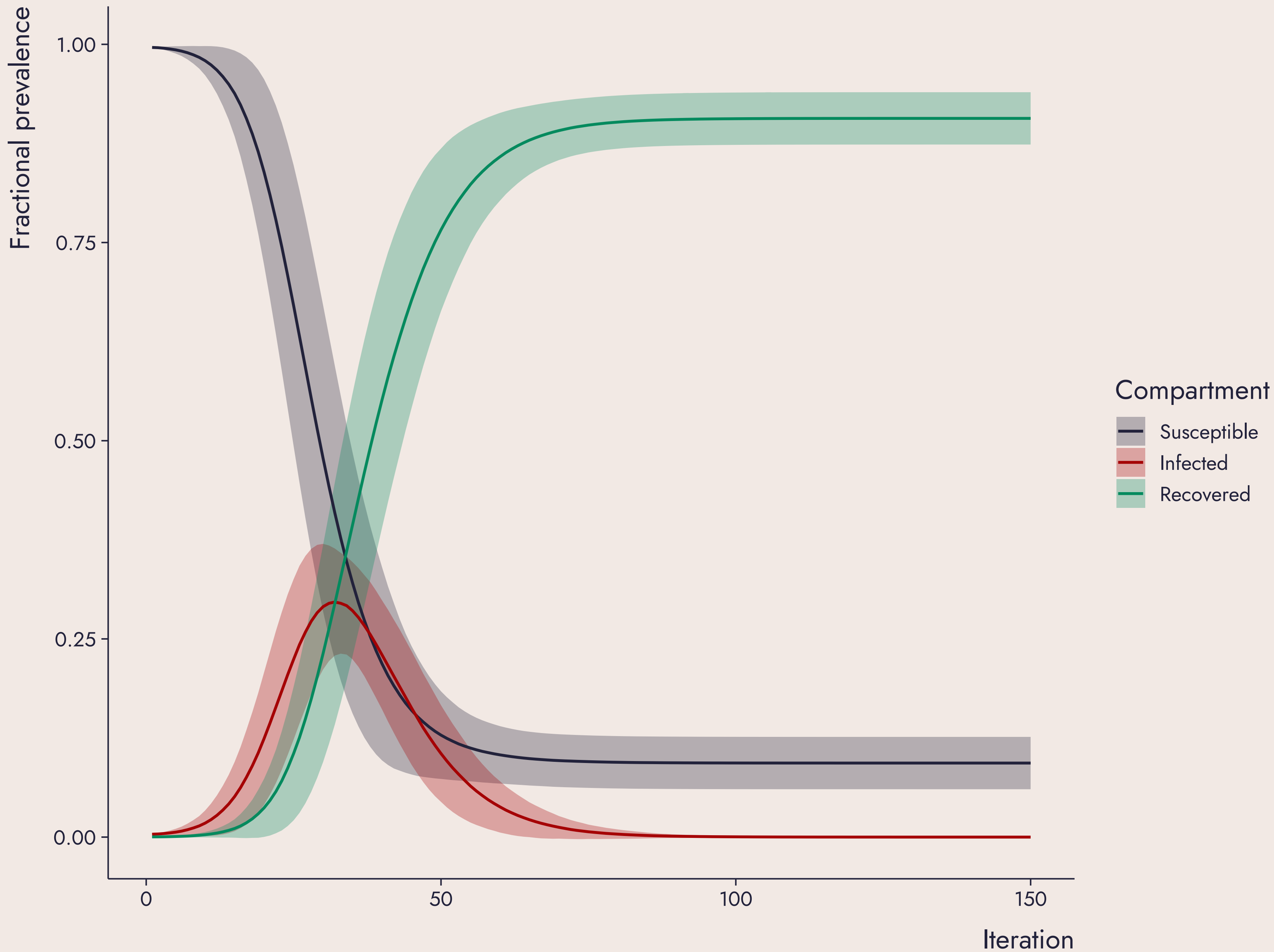
$$f(r_\infty) = \left(\frac{\alpha}{\lambda} - p\right)(1 - r_\infty) - \frac{\alpha}{\lambda}\ln(1 - r_\infty) + p - \frac{\alpha}{\lambda} - r_\infty$$



The Models

Simple contagion - Well mixed population

Stochastic homogeneous mixing



1000 simulations

$\alpha = 0.15$

$\lambda = 0.3$

$p = 0.1$

The Models

Complex contagion

The complex contagion model aims to repurpose the equations in a dose-threshold framework by implementing:

1. Doses mechanics
2. Threshold mechanics

The Models

Doses

Doses have been interpreted in the following way:

- An interaction in favor of the rumor adds one dose of infection
- Any interaction against a rumor results in the loss of one dose

In both cases the gain/loss of doses are applied to the recipient

$I + S \xrightarrow{\lambda(1-p)} I + I$		$I + S \xrightarrow{\lambda(1-p)}$ **+1 dose to** S
$I + S \overset{\lambda p}{\to} I + R$	$\longrightarrow$	$I + S \overset{\lambda p}{\to}$ **-1 dose to** S
$I + I \overset{\alpha}{\to} R + I$		$I_1 + I_2 \overset{\alpha}{\to}$ **-1 dose to** I_1
$I + R \overset{\alpha}{\to} R + R$		$I + R \overset{\alpha}{\to}$ **-1 dose to** I

The Models

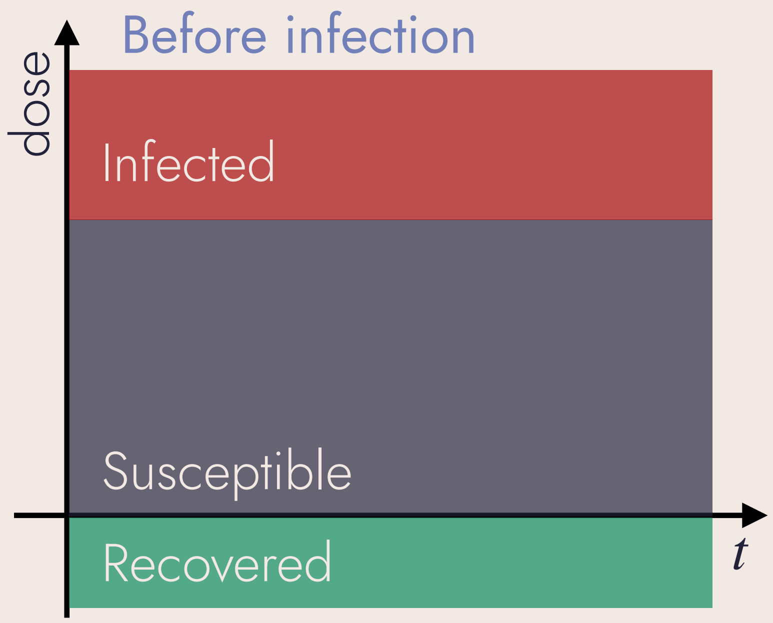
Thresholds

Two thresholds have been defined

1. $T_S = 0$ below this threshold both infected and susceptible recover

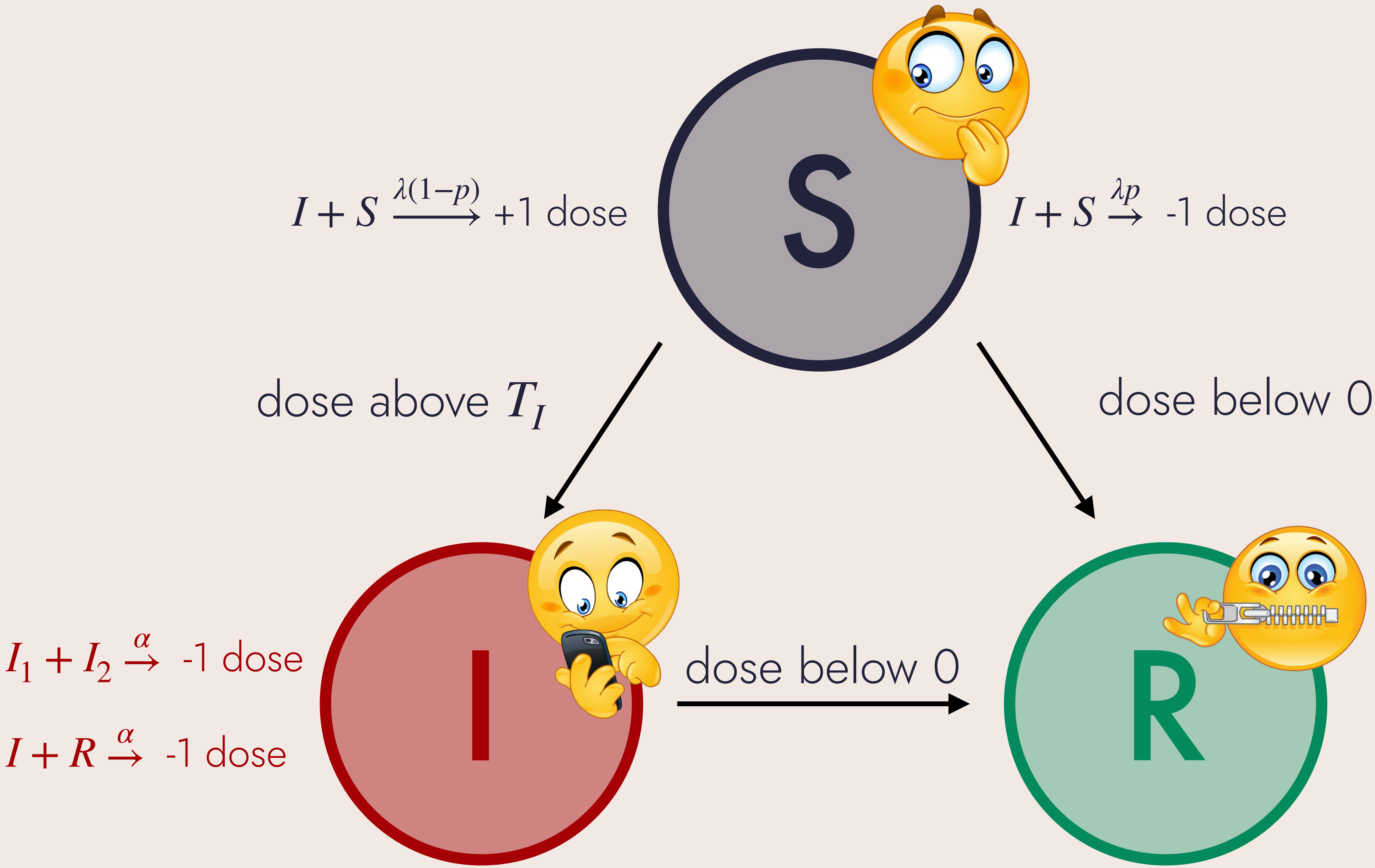
2. T_I is a parameter of the model and quantifies the does of infection a susceptible needs to absorb before becoming infected

All nodes have initial dose 0 except for infected nodes that start at dose T_I



The Models

Thresholds

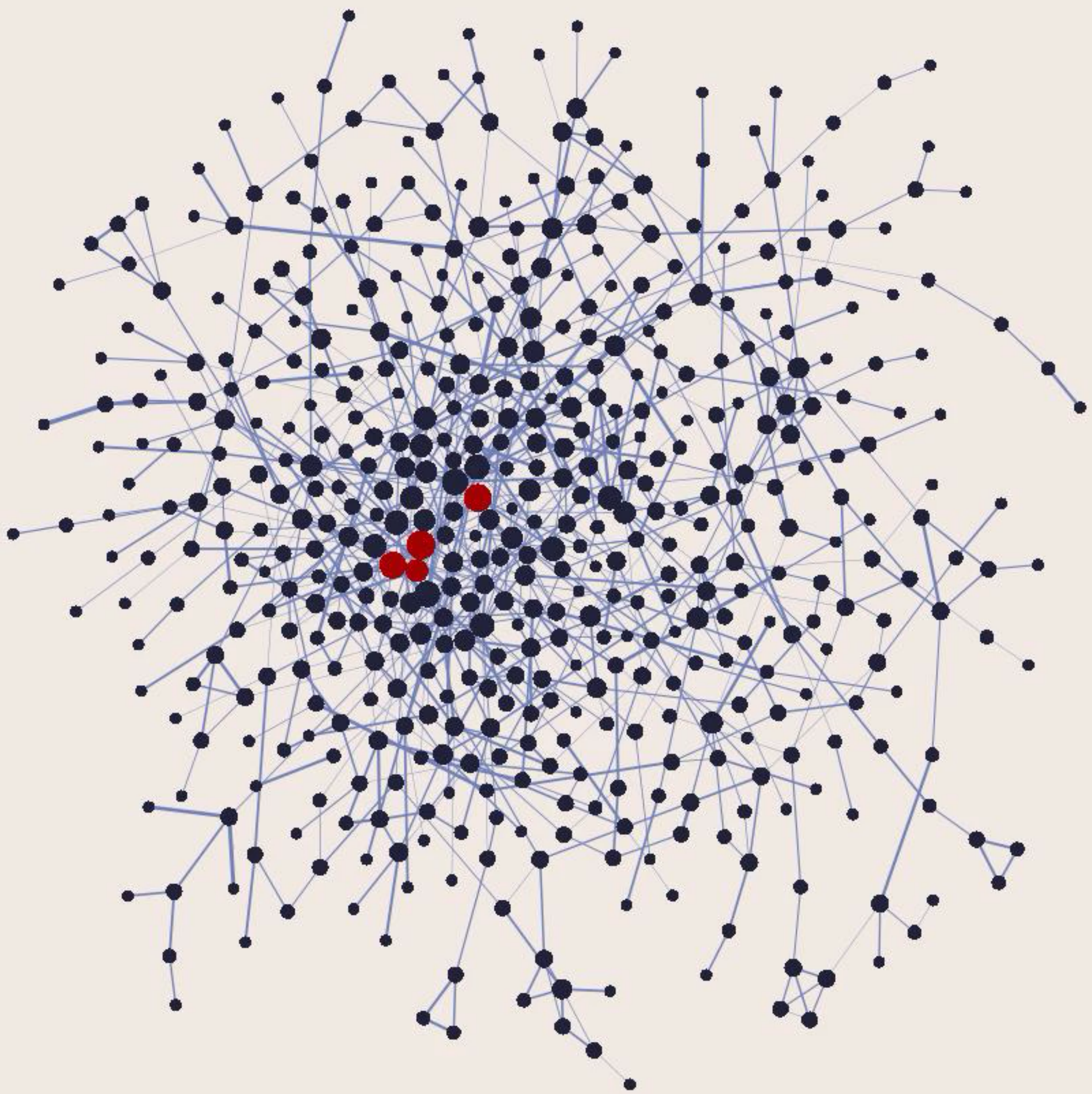


The Results

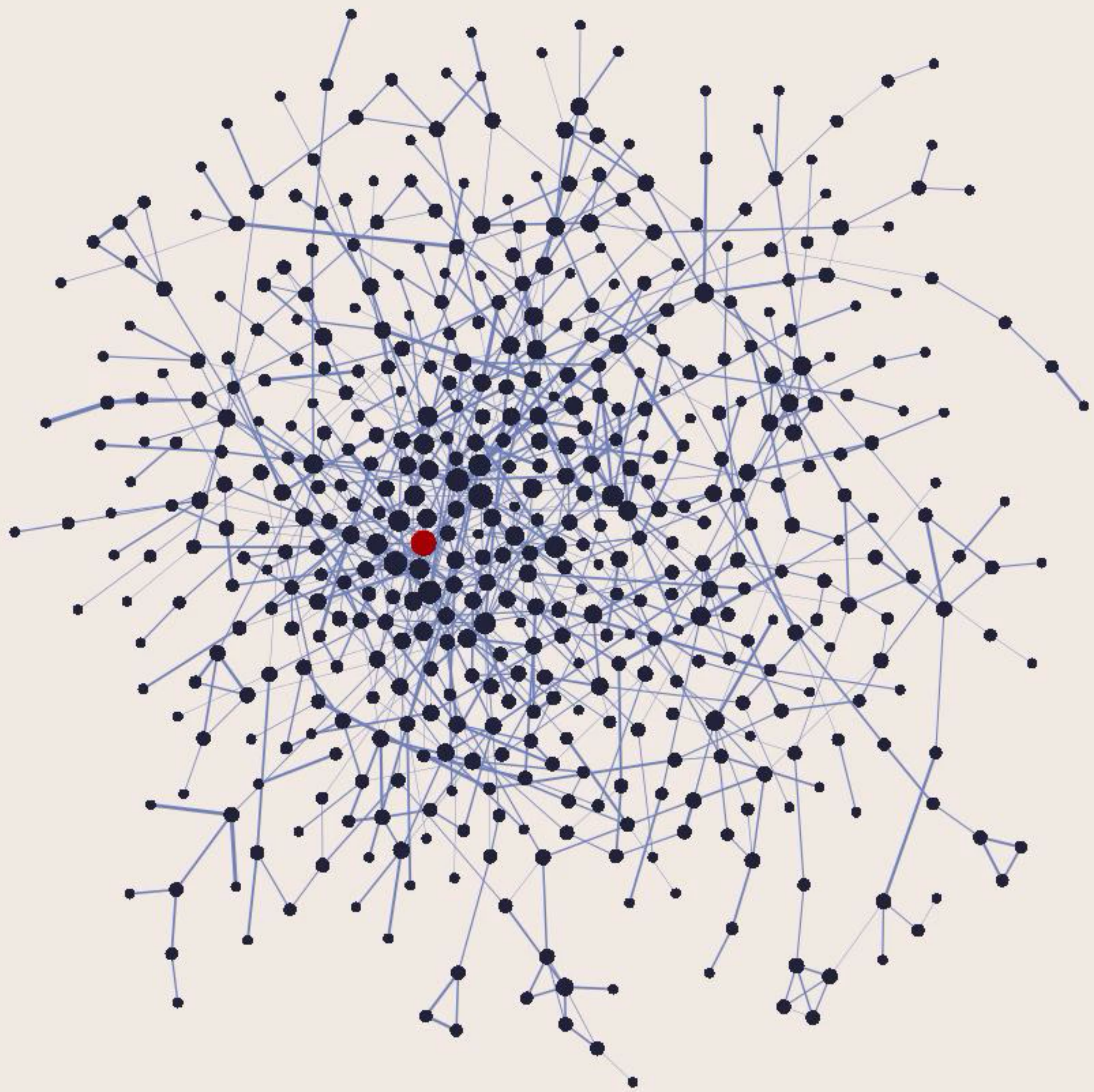
The Results

Fancy animation

simple contagion

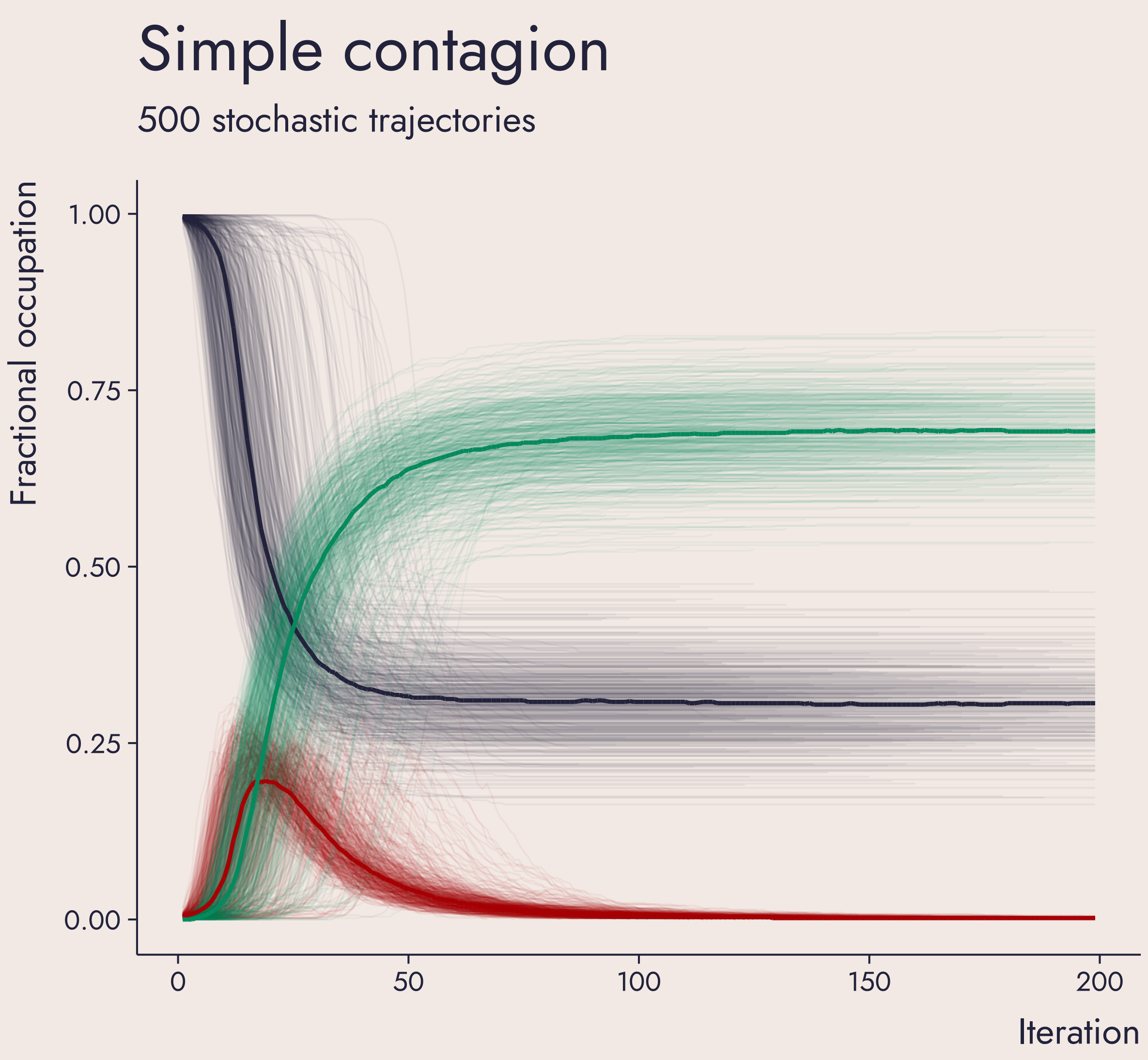


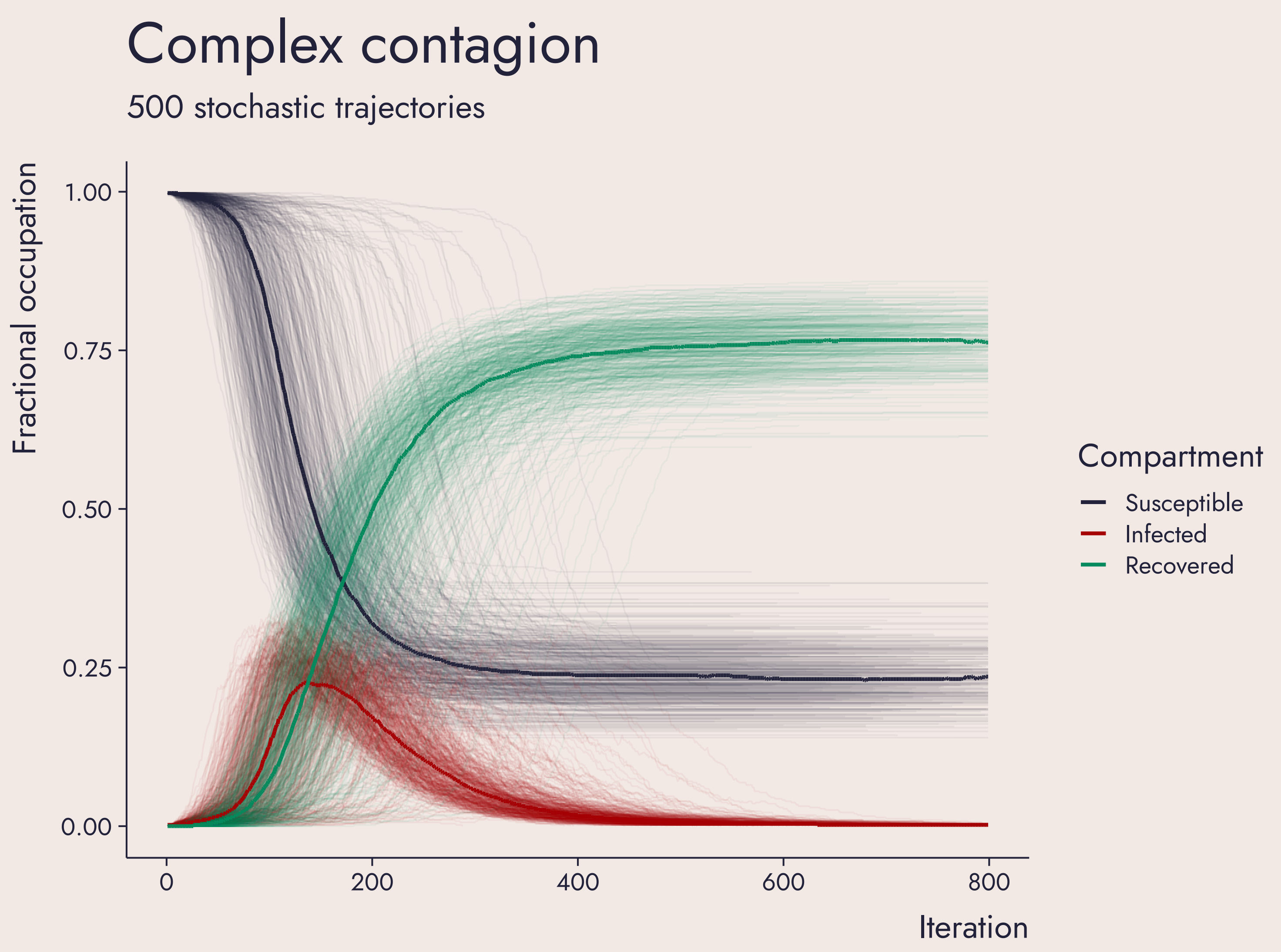
complex contagion



The Results

Qualitative results

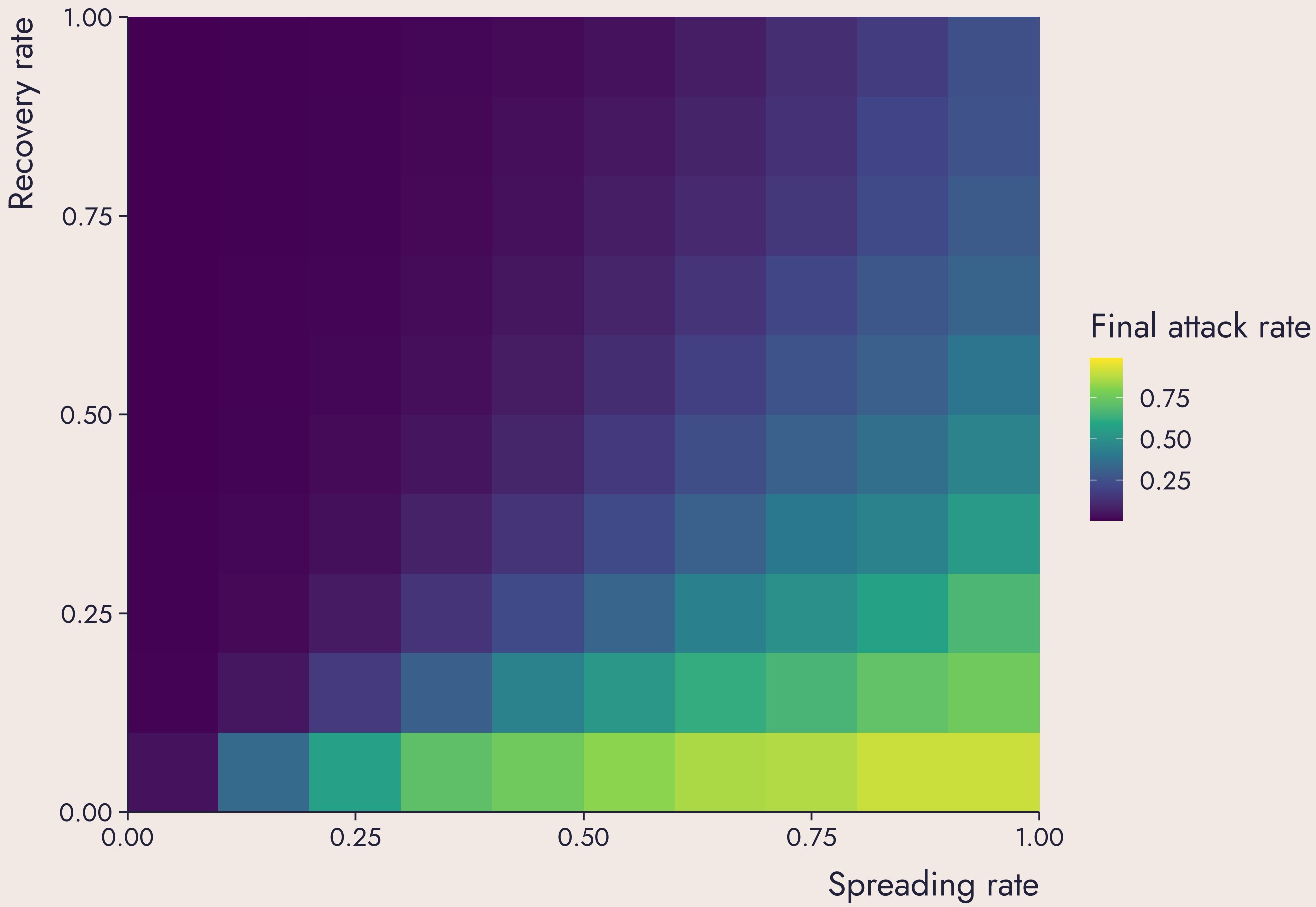




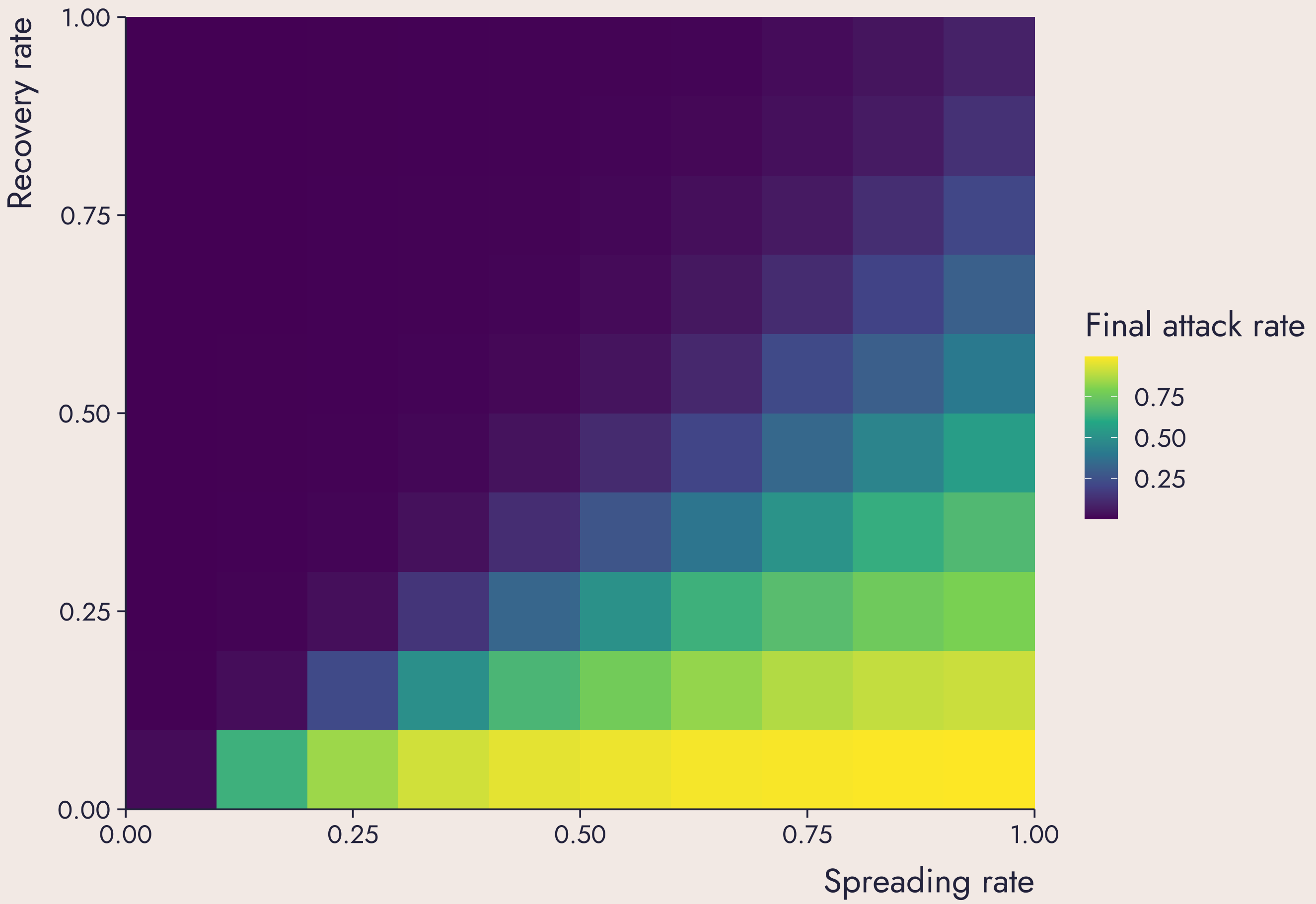
The Results

Heatmaps - Final attack rate

Simple contagion

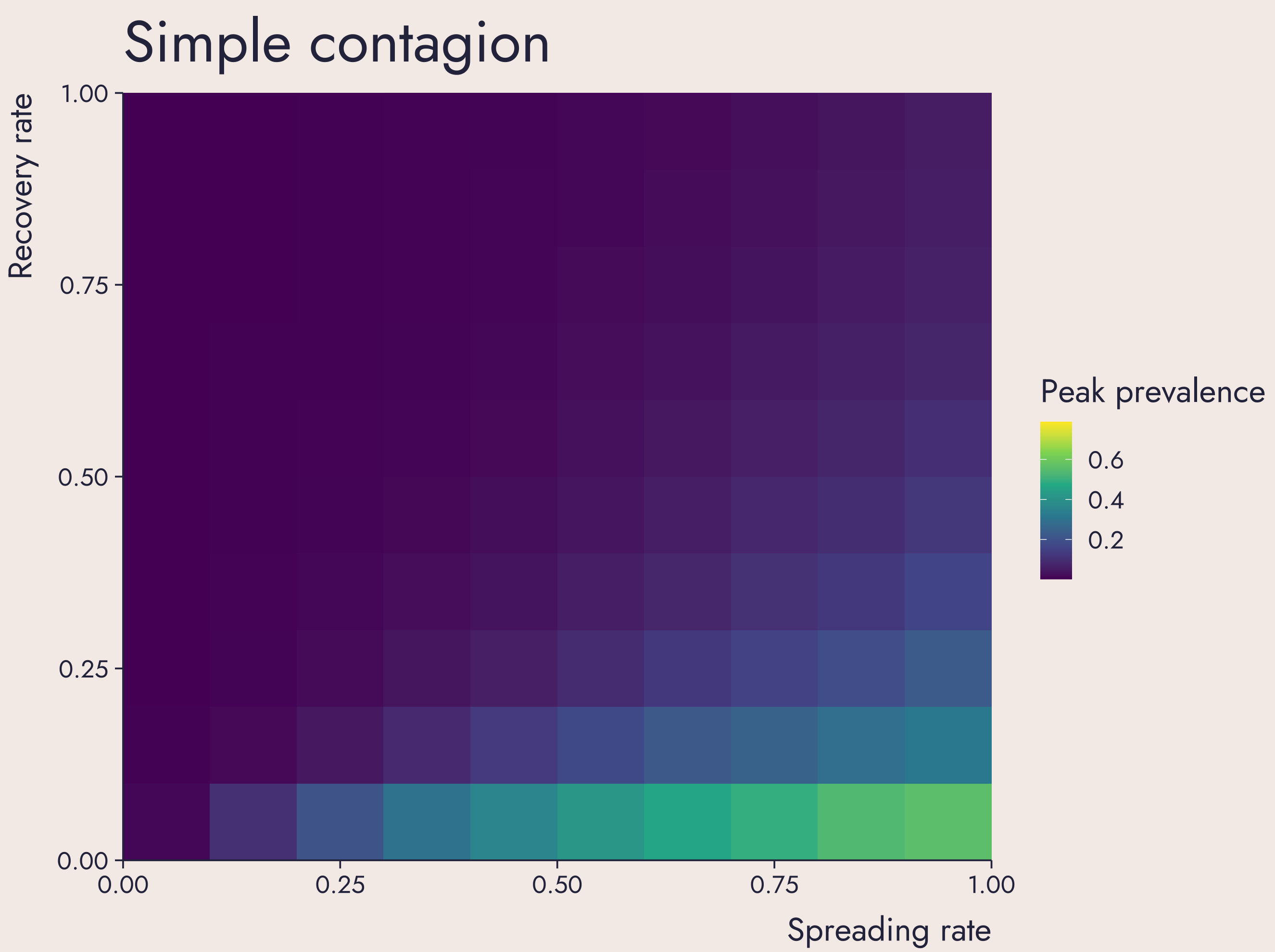


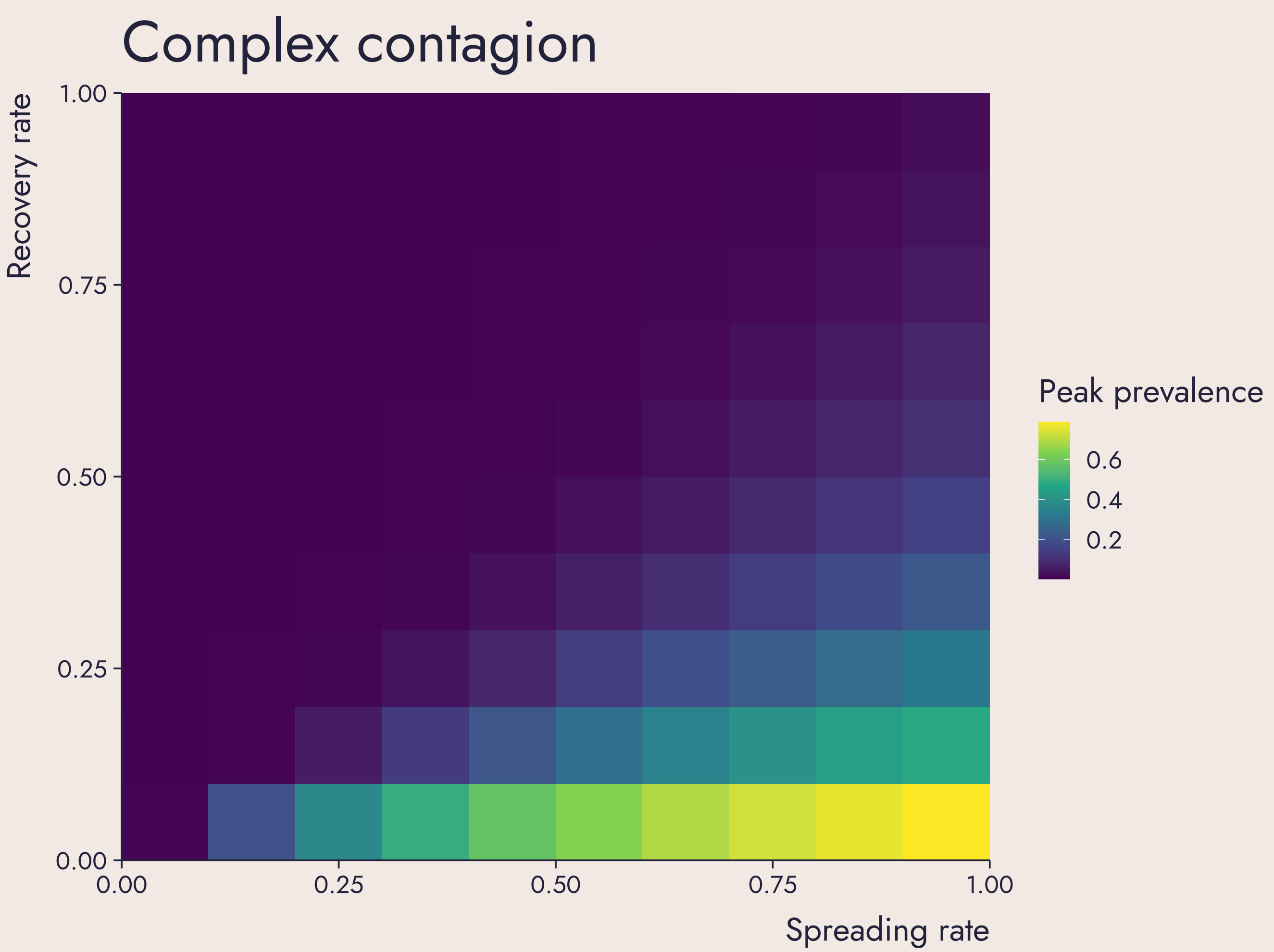
Complex contagion



The Results

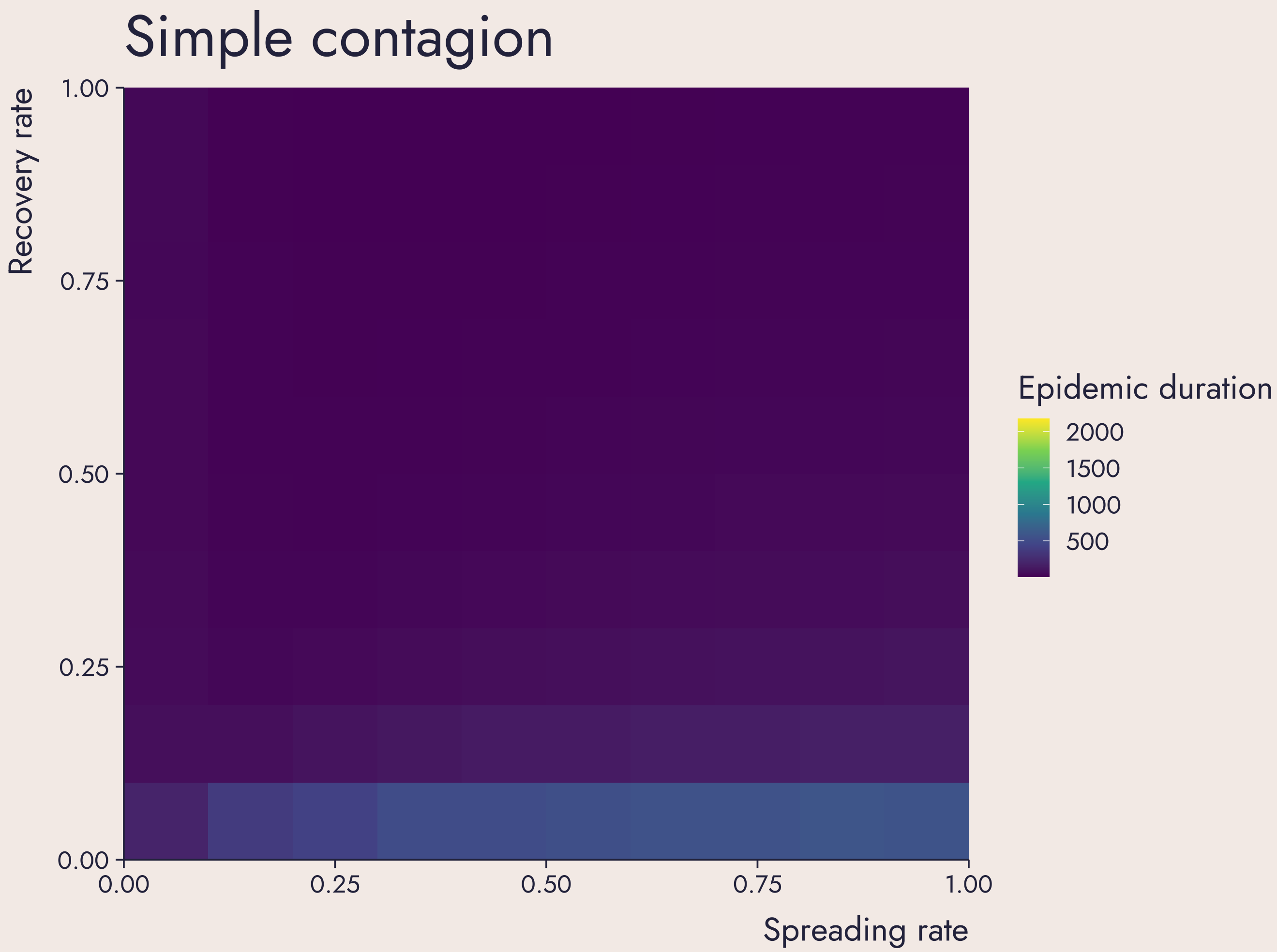
Heatmaps - Peak prevalence

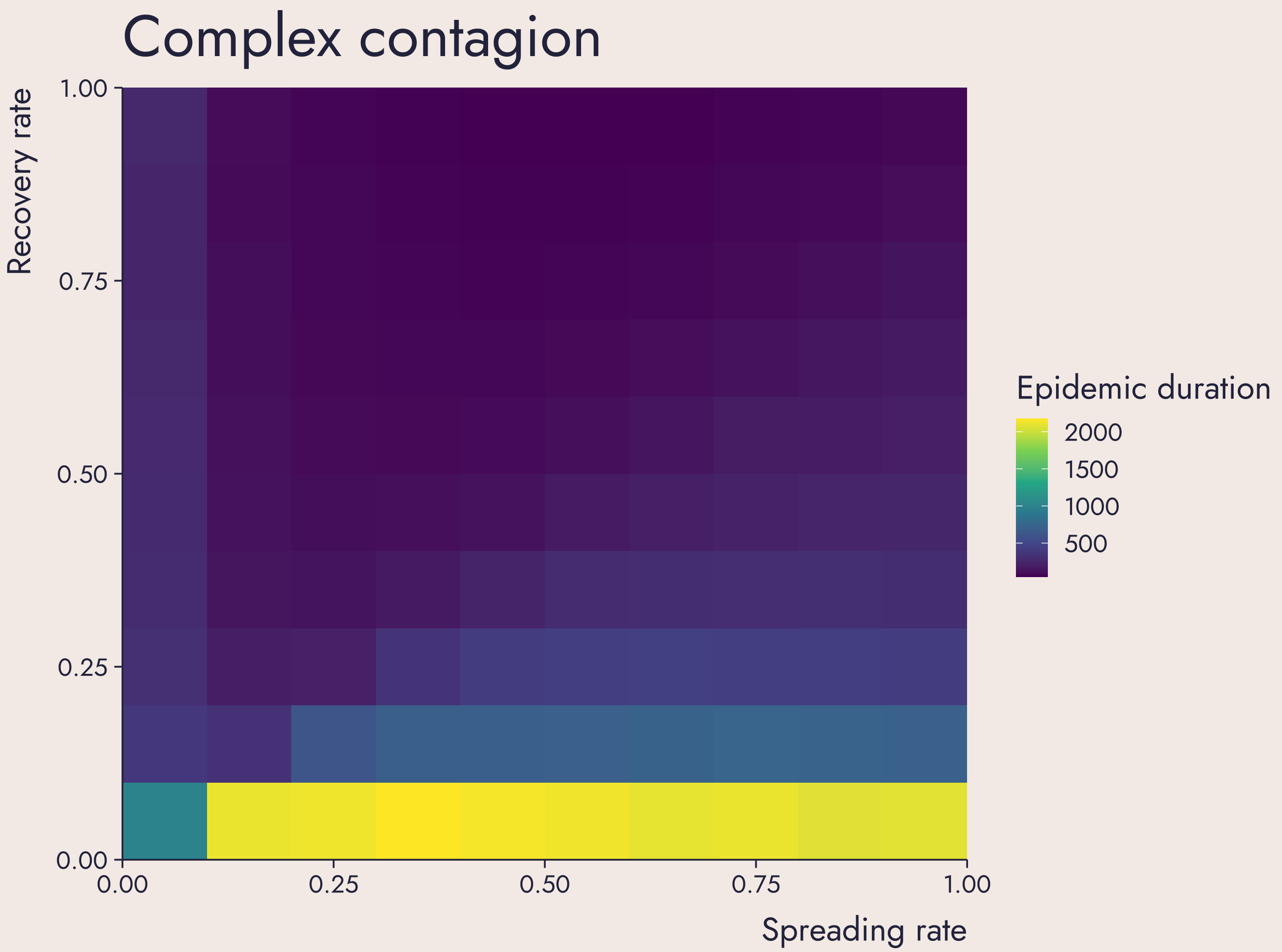




The Results

Heatmaps - Epidemic duration



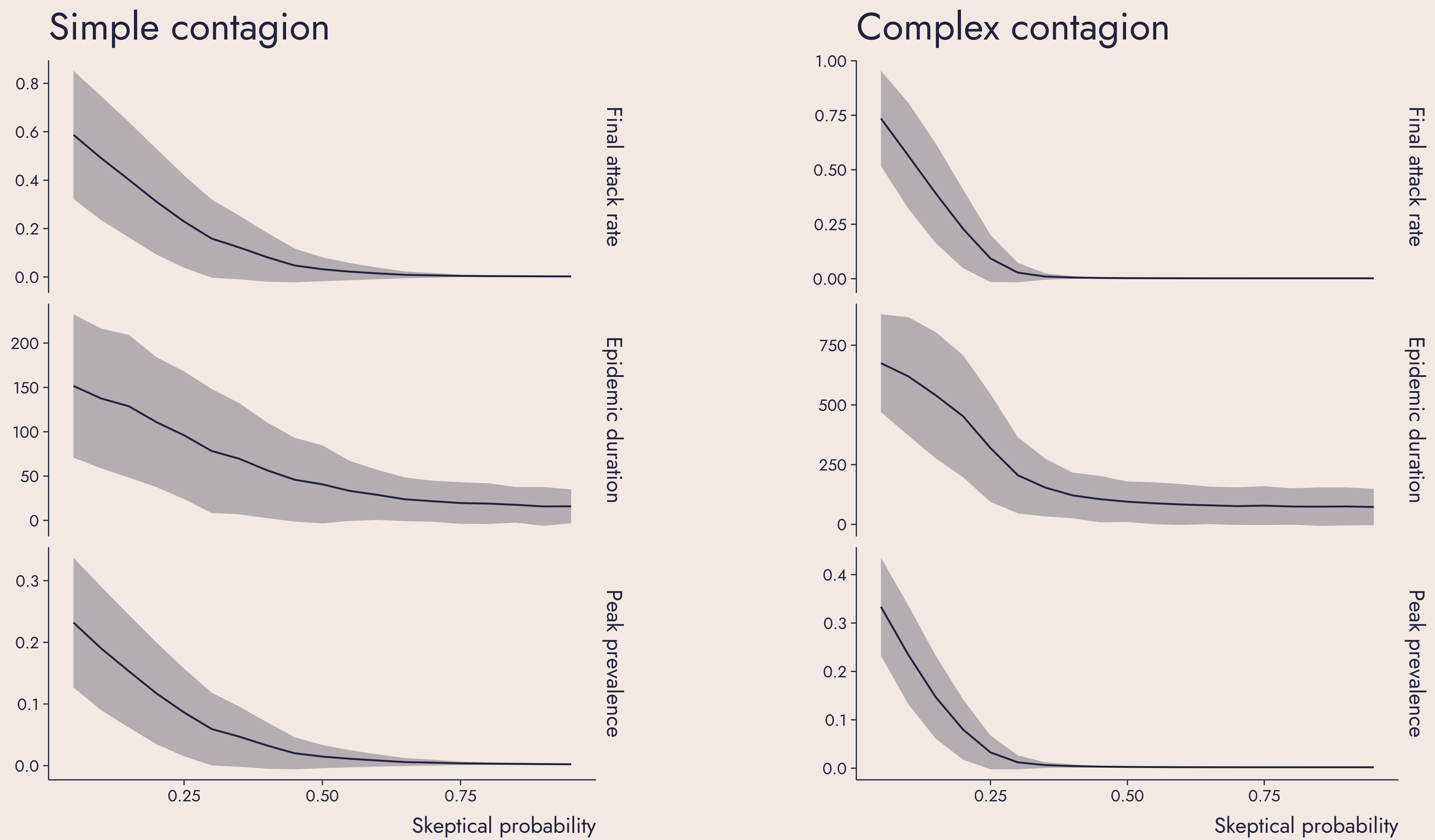


The Results

Effect of skepticism

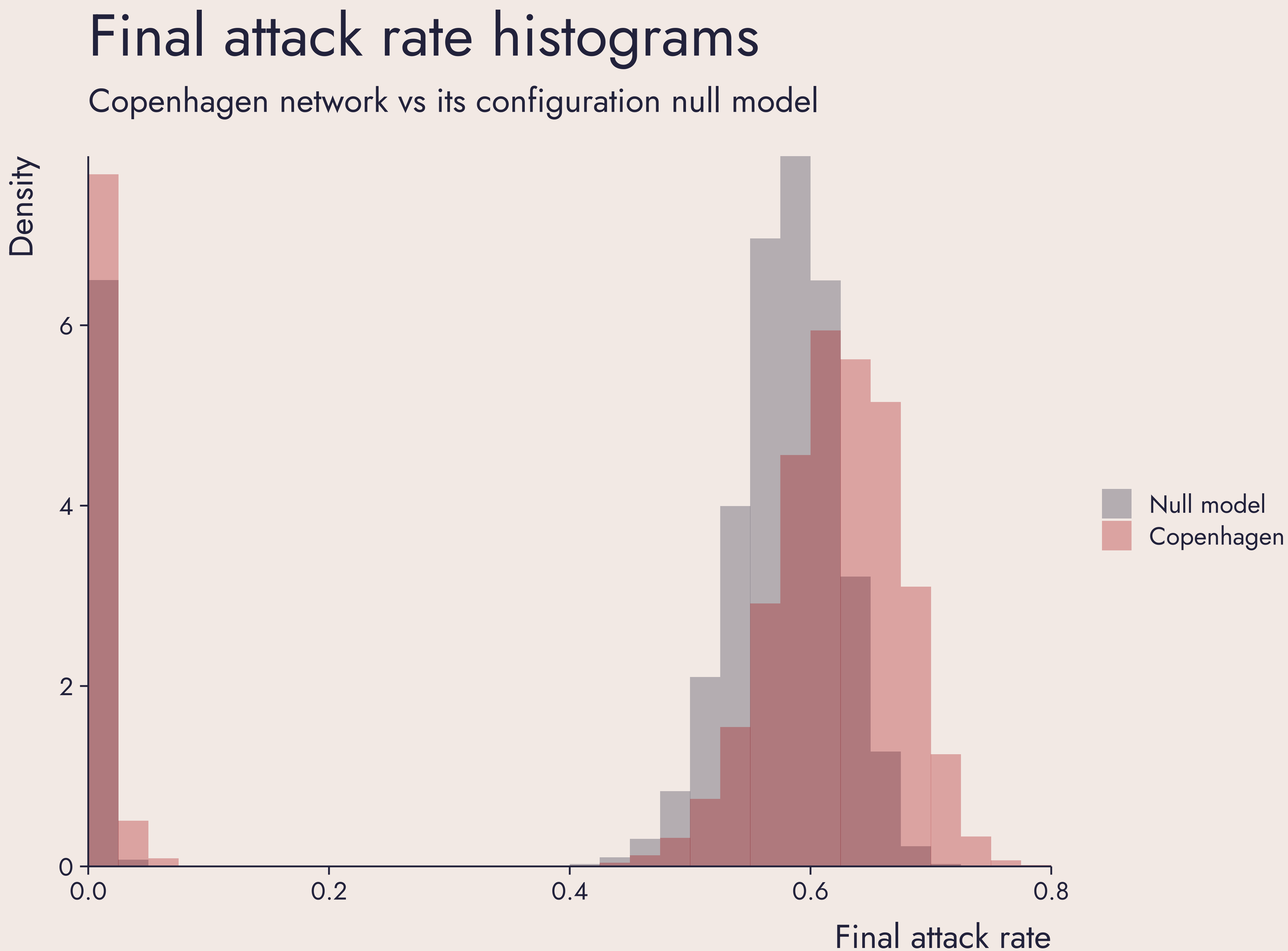
$\lambda = 0.85$

$\alpha = 0.15$



The Results

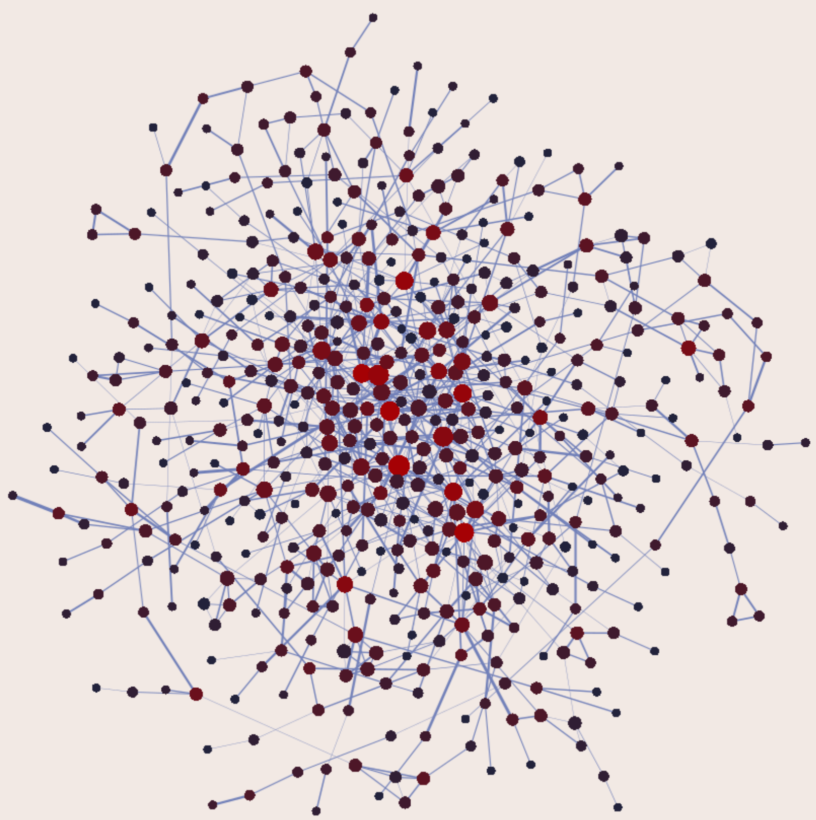
Null model



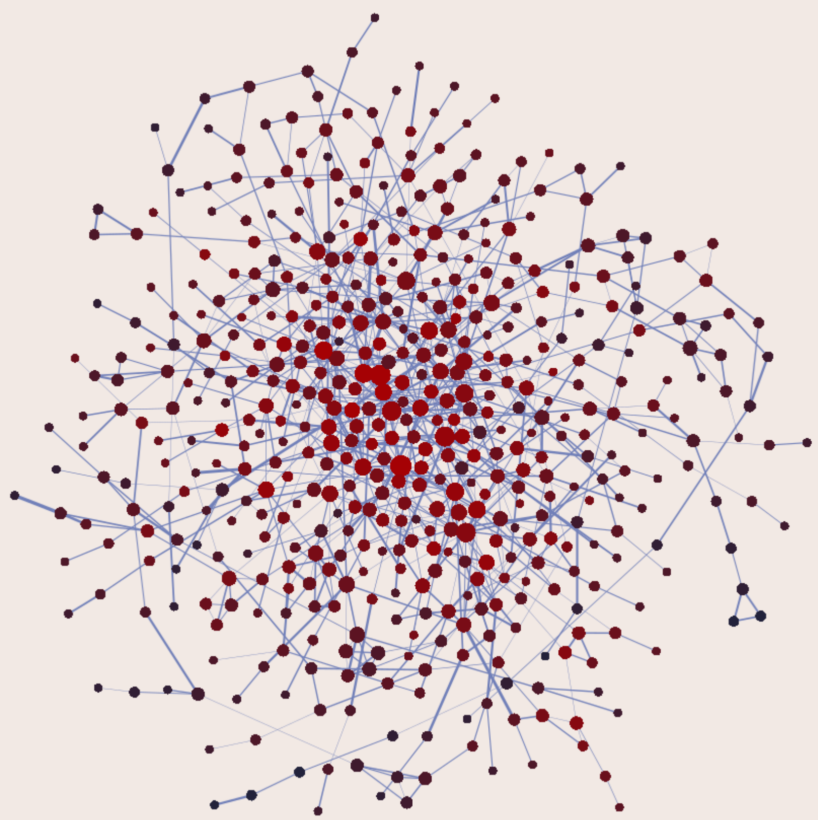
The Results - Node by node

Centrality

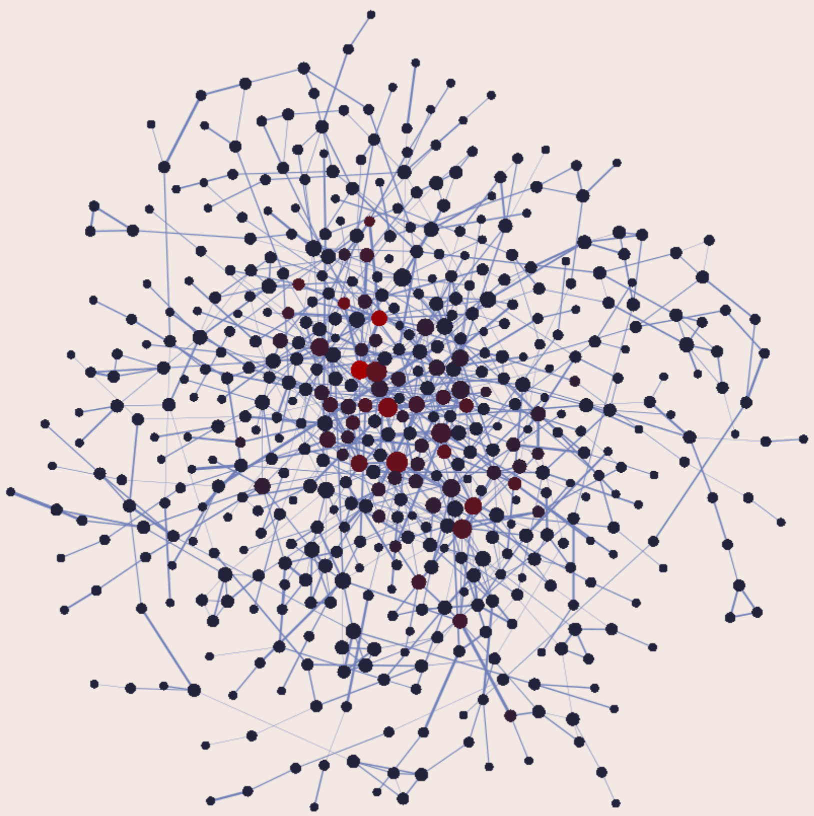
PageRank centrality



Closeness centrality

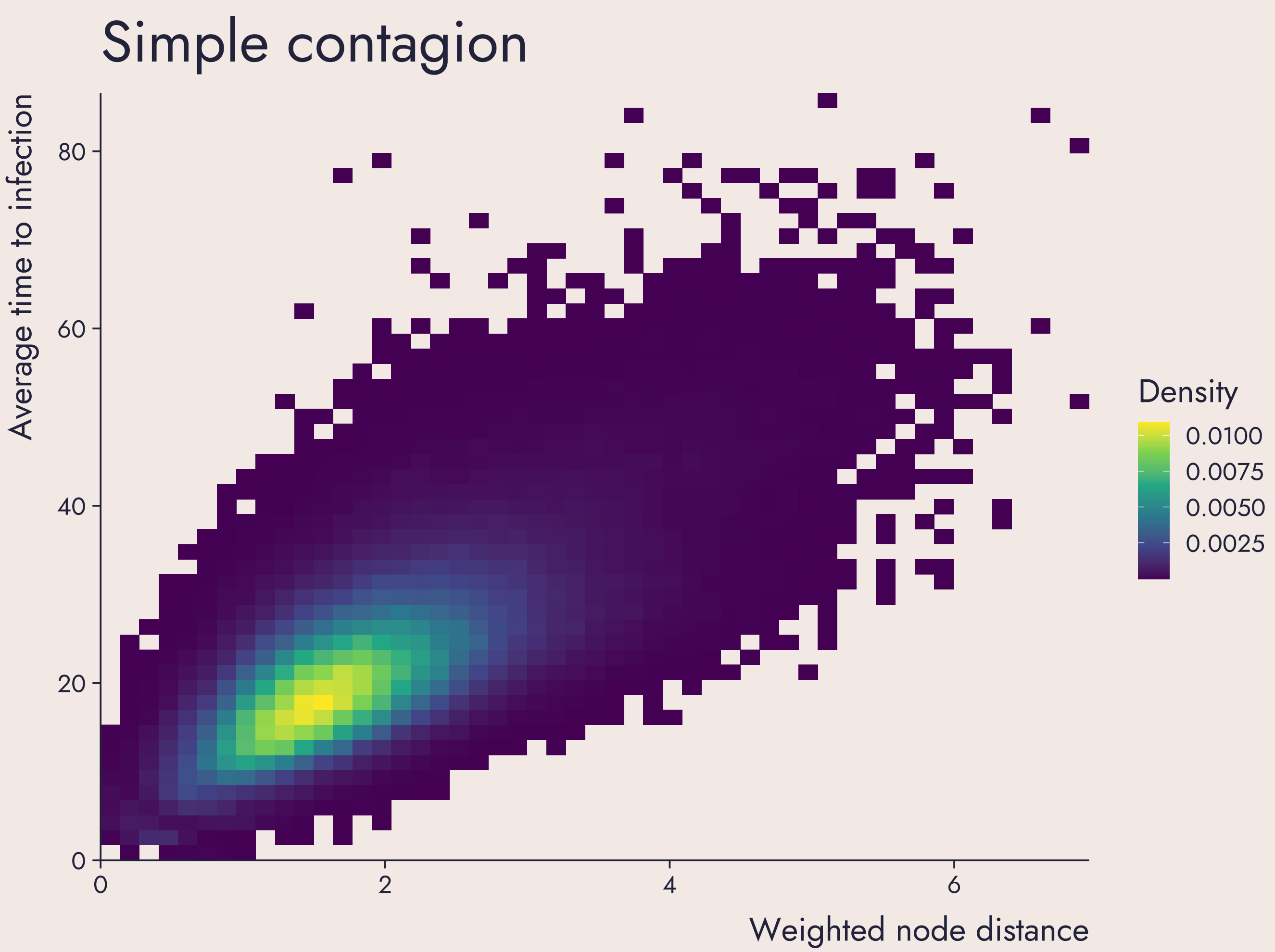


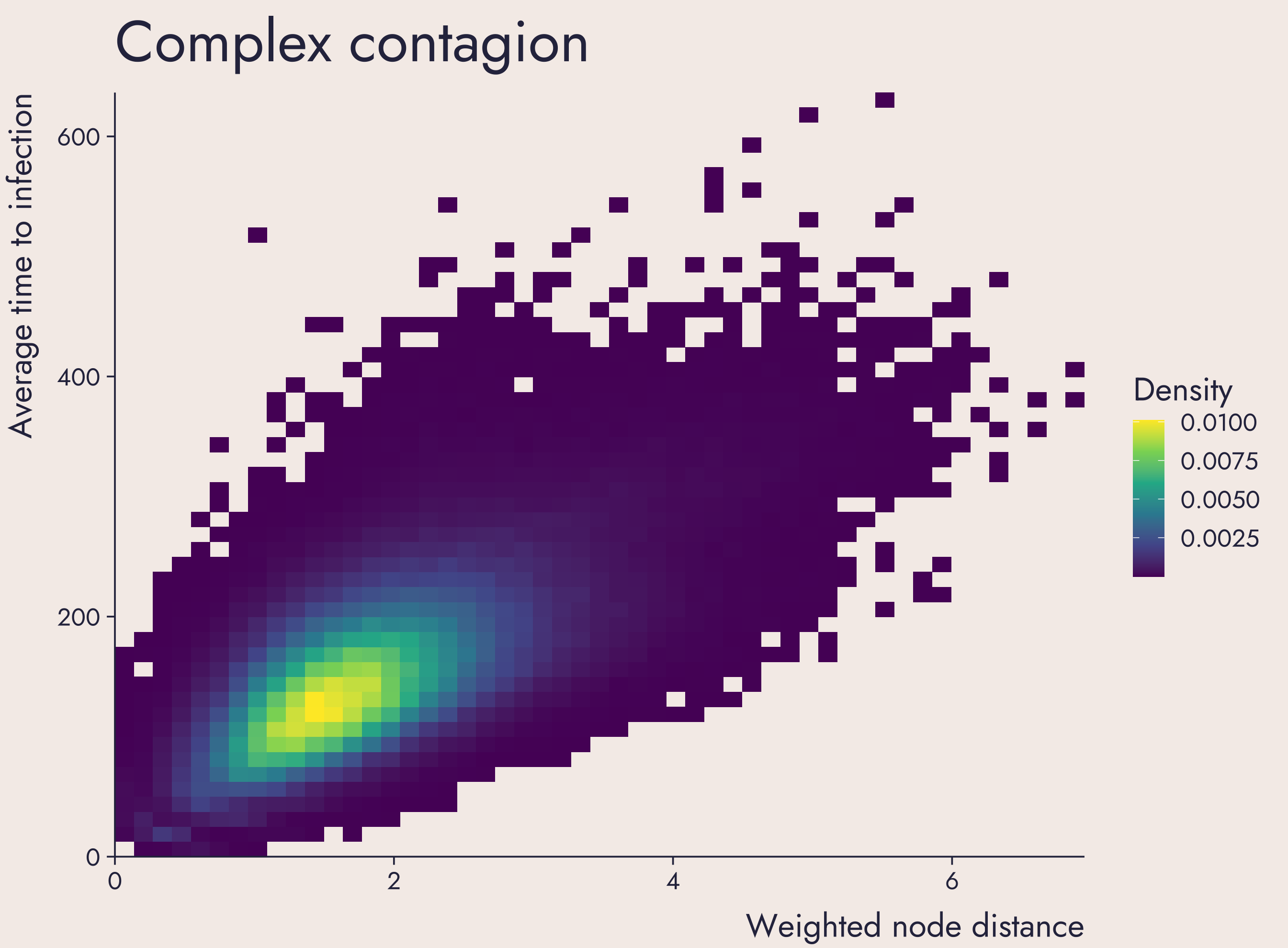
Eigenvector centrality



The Results - Node by node

Time to infection by nodes distance

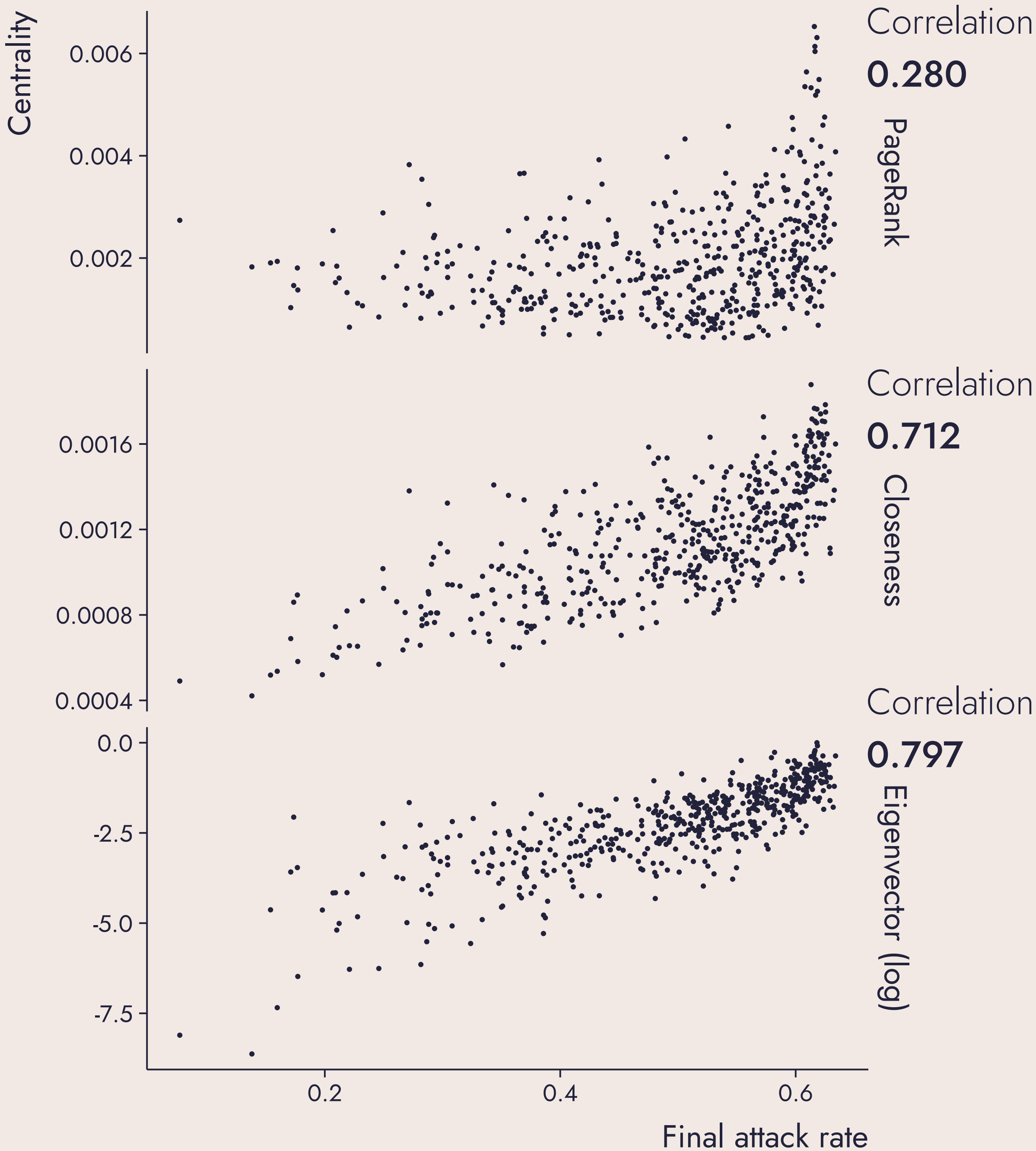




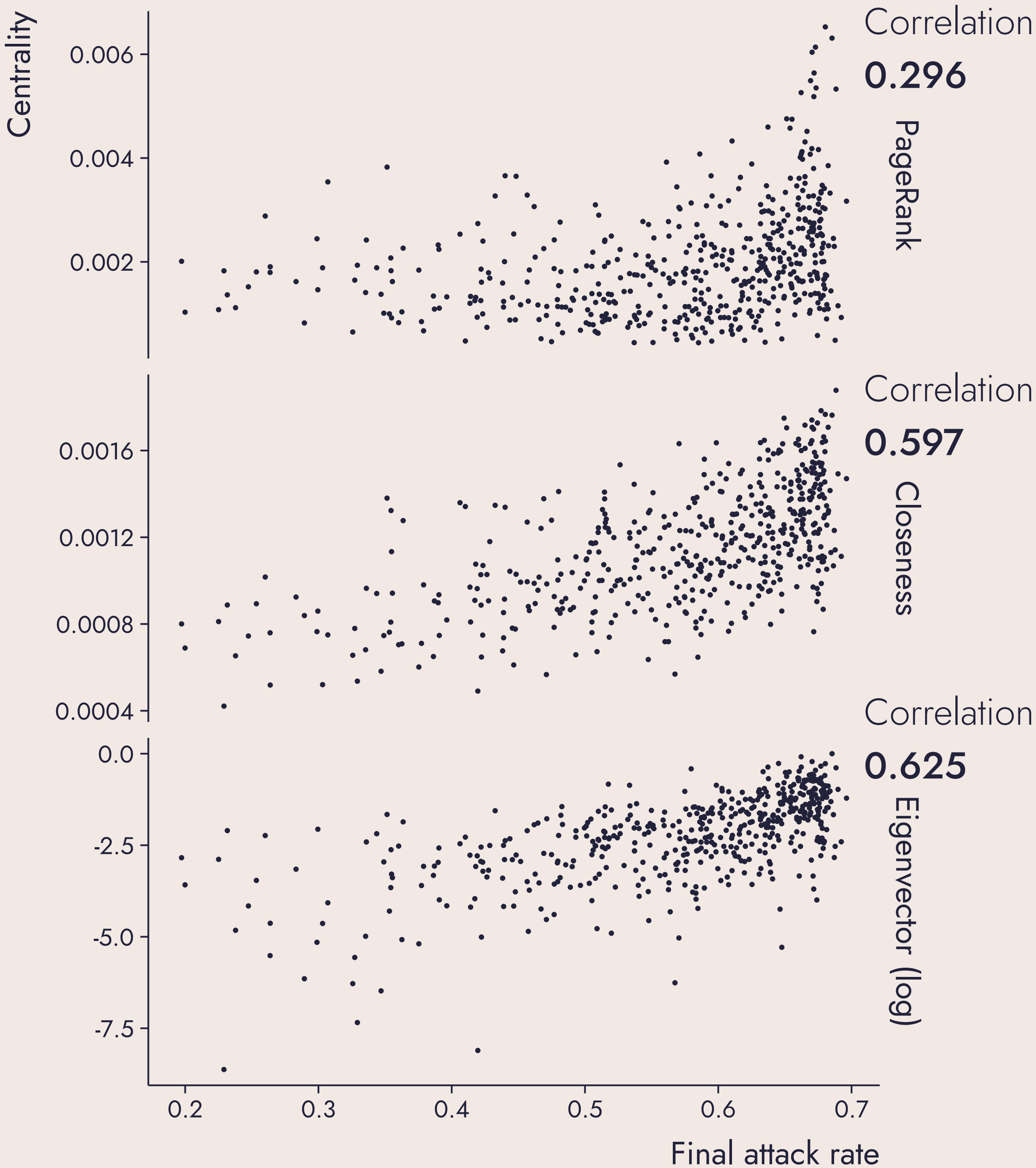
The Results - Node by node

Centralities correlations - Attack rate

Simple contagion



Complex contagion

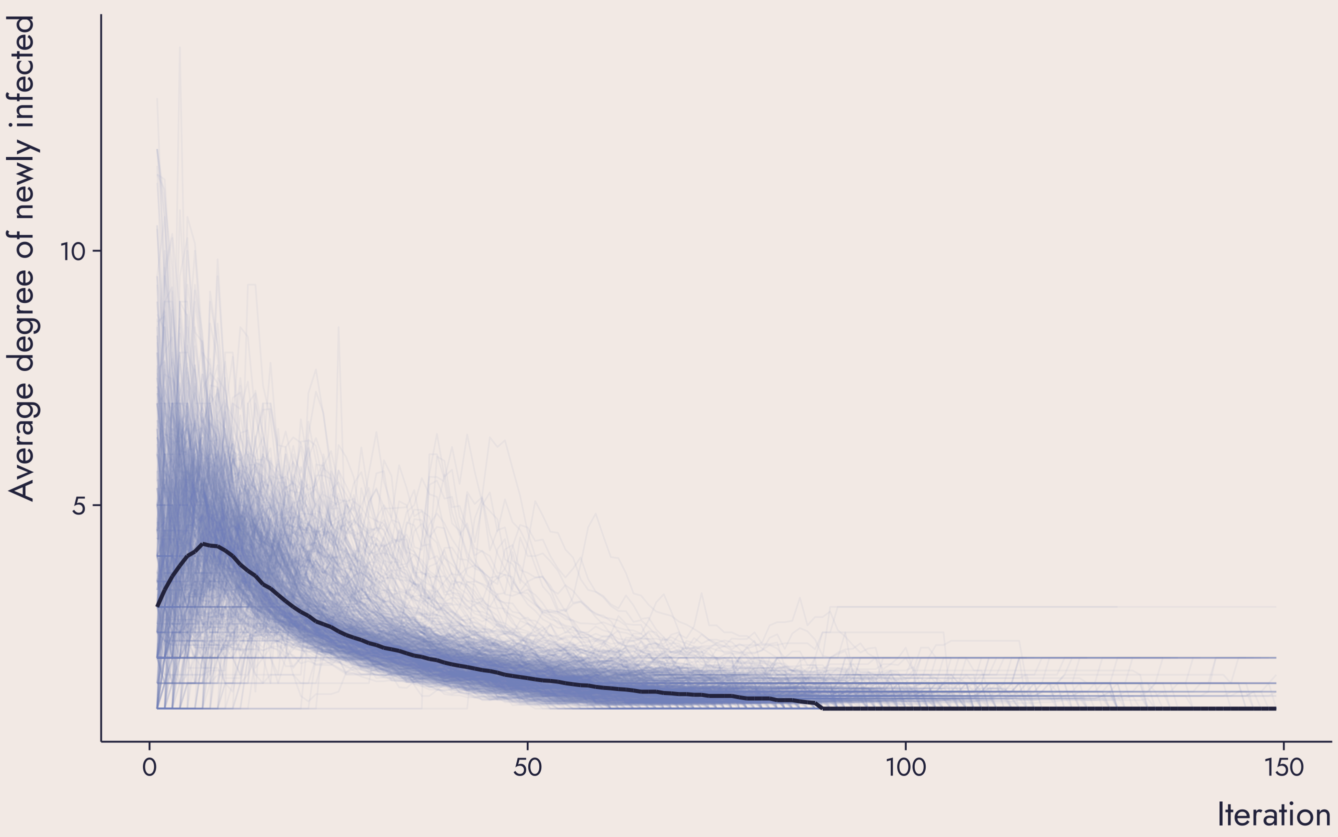


The Results - Node by node

Degree of newly infected nodes

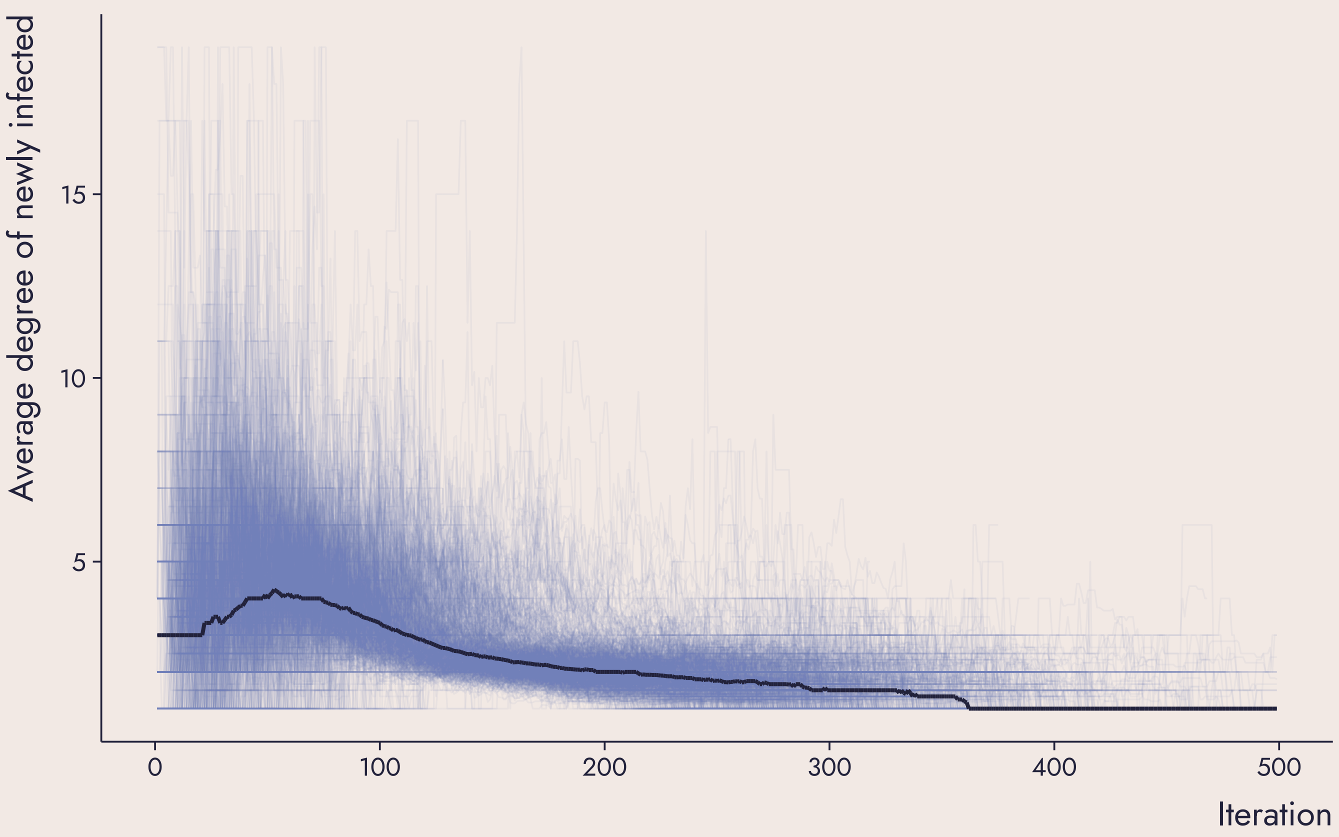
Simple contagion

750 stochastic trajectories



Complex contagion

750 stochastic trajectories

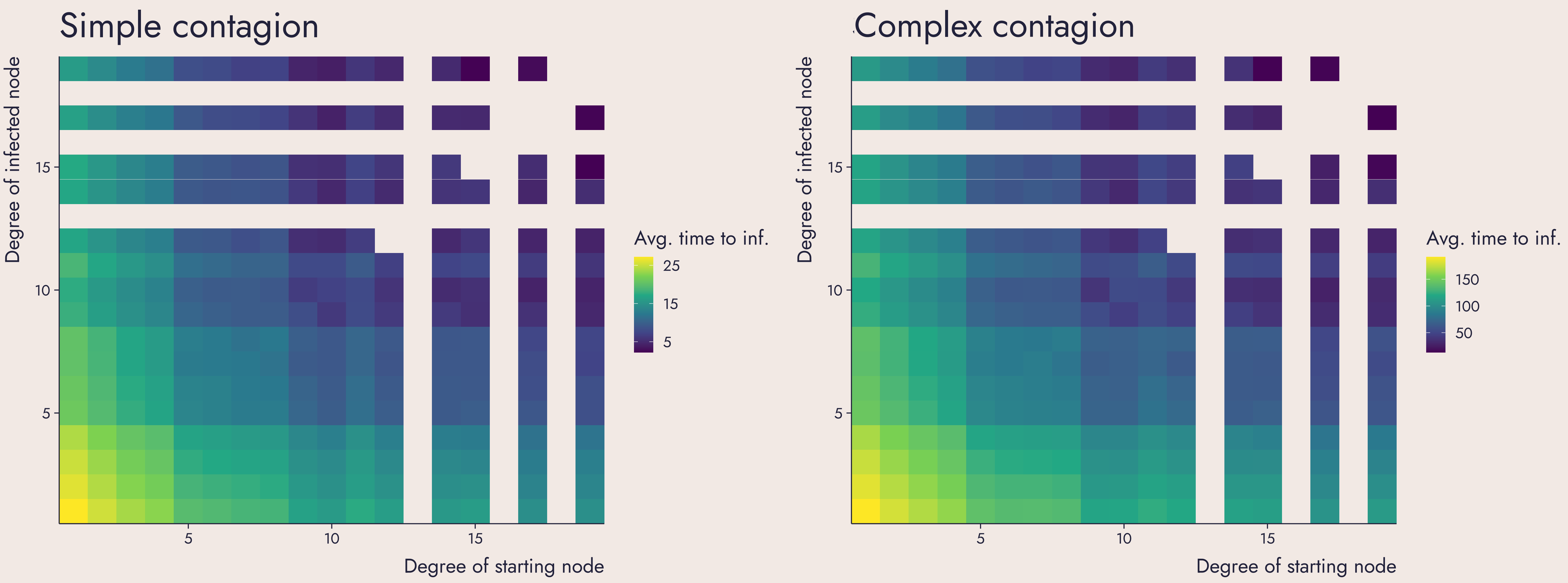


Higher degree nodes are, on average, reached earlier in the epidemic.

Notice the increased stochasticity for the complex contagion

The Results - Node by node

Time to infection by degree



Behavior is quite symmetrical, it takes pretty much the same time to start from high degree nodes and reach the low degree ones as the other way around

Conclusion

Conclusions

We carefully analyzed an established model in the literature and applied it to a real world, small to medium scale network

We proposed an alternative model that takes into account threshold dynamics

The results showed increased stochasticity but overall similar qualitative and quantitative behavior

In both cases the spread of the epidemic is reasonably linked to the structure of the network