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Issue I: Internalizing Martin-Löf Type Theory

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This article demonstrates formal Martin-Löf Type Theory (MLTT) embedding into the host type system with constructive proofs of the complete set of inference rules. This was recently made possible by cubical type theory and cubical type checker **cubicaltt**[1] in 2017. The long road from pure type systems of AUTOMATH by de Bruijn to type checkers with homotopical core was made. This article touches only the formal MLTT core type system with Π and Σ types (that correspond to $\forall$ and $\exists$ quantifiers for mathematical reasoning) and identity type.

Each language implementation needs to be checked. One of the possible test cases for type checkers is the direct embedding of type theory model into the language of the type checker. As types are formulated using 5 types of rules (formation, intro, elimination, computation, uniqueness), we construct aliases for the host language primitives and use type checker to prove that it has the realization of MLTT. This could be seen as an ultimate test sample for type checker as intro-elimination fusion resides in beta-eta rules, so by proving them, we prove properties of the host type checker.

More formally cubical MLTT internalization proofs the J eliminator and its beta rule, something that was impossible without geometrical realization of cubical type theory. Also, this issue opens a series of articles dedicated to the formalization of the foundations of mathematics in cubical type theory, MLTT modeling and cubical proof verification. As many may not be familiar with types, this issue presents different interpretations of core types from other areas of mathematics.

We should note that this is an entrance to the internalization technique, and after formal MLTT embedding, we could go through inductive types up to embedding of CW-complexes as the indexed gluing of the higher inductive types.

[1] http://github.com/mortberg/cubicaltt