The Prints of the Generalized Equations of $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$ in a Free Group

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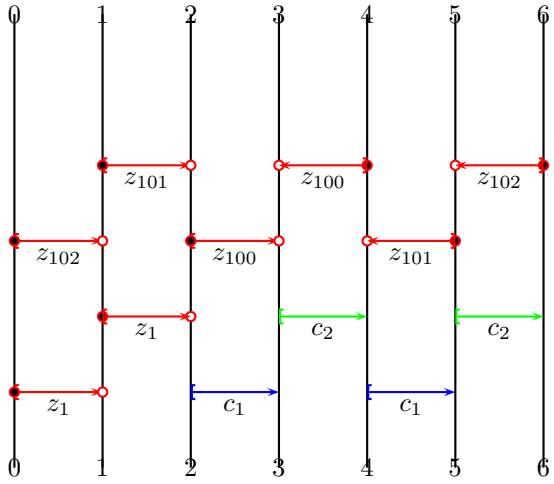
The purpose of this report is to demonstrate the logging facilities of the MKSolver. In particular, here we see that the MKSolver is able to accurately carry out the process of replacig equals by equals, as specified by the Makanin-Razborov machinery. The terminology and process in MKSolver follows the series of papers of Miasnikov and Kharlampovich, concerning the Tarski problem, and moreover has been algorithmically streamlined by extending some of the ideas developed by Gutierrez (2000).

1 Generalized Equation #1

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



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†This report was generated automatically by software developed with support from the National Security Agency Grant H98230-06-1-0042.

GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-2:z1+.] ; Critical Boundary: 1; **Prints**

```
Print 0: =0=1*<1=2*
```

Total number of prints: 1
Next, we consider

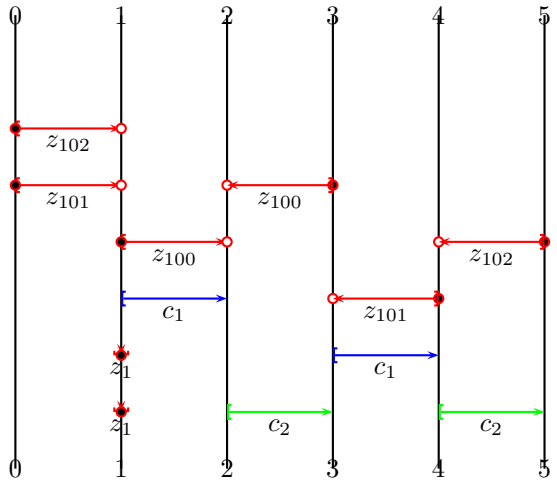
```
Print 1: =0=1*<1=2*
```

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 2.
Step 2: Moved (old) base [0-1:z102+.] to (new) boundaries 1 - 2.
Step 3: Collapsed (new) base [1-2:z1+.] to the empty base (2,2).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=1*<1=2*
```

is shown below:

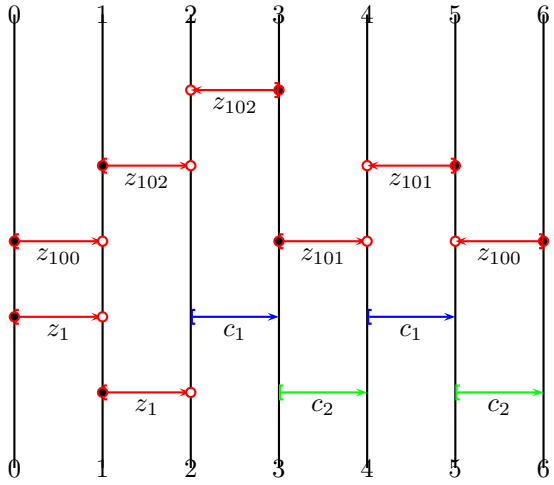


The GE above is non-degenerate.

This completes the consideration of Print 1.

2 Generalized Equation #2

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-2:z1+.] ; Critical Boundary: 1; **Prints**

```
Print 0: =0=1*<1=2*
```

Total number of prints: 1
Next, we consider

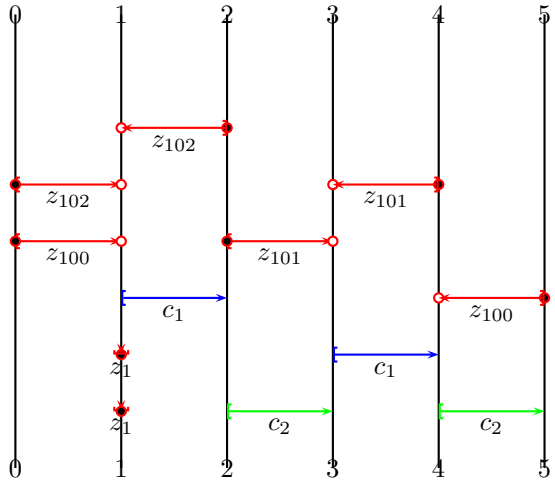
```
Print 1: =0=1*<1=2*
```

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 2.
Step 2: Moved (old) base [0-1:z100+.] to (new) boundaries 1 - 2.
Step 3: Collapsed (new) base [1-2:z1+.] to the empty base (2,2).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=1*<1=2*
```

is shown below:

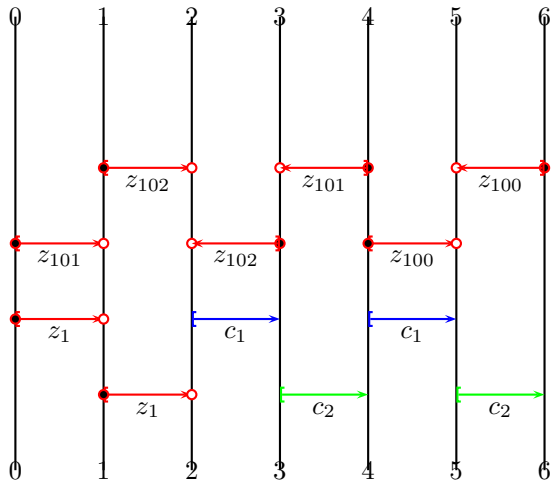


The GE above is non-degenerate.

This completes the consideration of Print 1.

3 Generalized Equation #3

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-2:z1+.] ; Critical Boundary: 1; **Prints**

```
Print 0: =0=1*<1=2*
```

Total number of prints: 1
Next, we consider

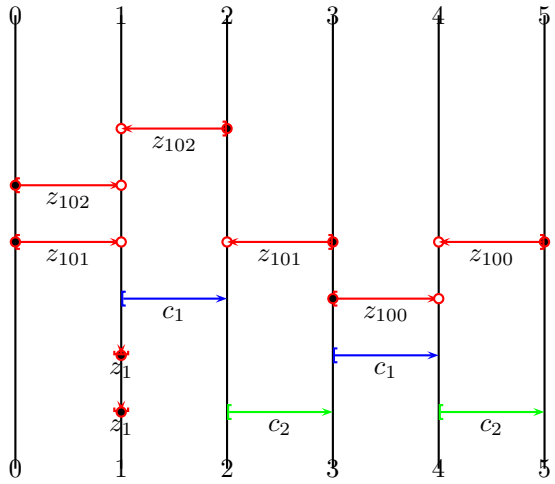
```
Print 1: =0=1*<1=2*
```

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 2.
Step 2: Moved (old) base [0-1:z101+.] to (new) boundaries 1 - 2.
Step 3: Collapsed (new) base [1-2:z1+.] to the empty base (2,2).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=1*<1=2*
```

is shown below:

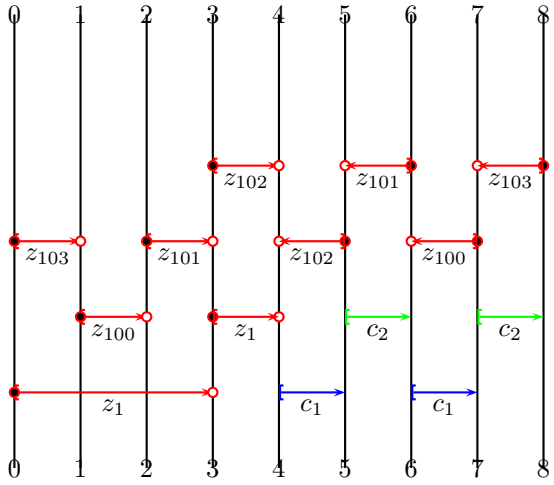


The GE above is non-degenerate.

This completes the consideration of Print 1.

4 Generalized Equation #4

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-3:z1+.] ; Carrier Dual: [3-4:z1+.] ; Critical Boundary: 3; **Prints**

```
Print 0: =0=3*<1<2<3=4*
```

Total number of prints: 1
Next, we consider

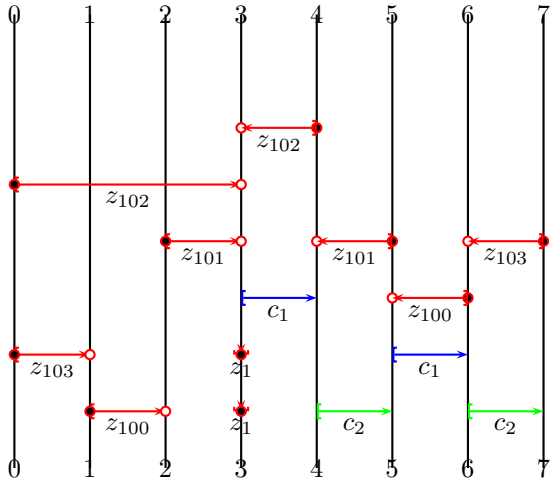
```
Print 1: =0=3*<1<2<3=4*
```

Sequence of Actions in performing the Print 1:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 5.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 6.
Step 4: Moved (old) base [1-2:z100+.] to (new) boundaries 4 - 5.
Step 5: Moved (old) base [2-3:z101+.] to (new) boundaries 5 - 6.
Step 6: Moved (old) base [0-1:z103+.] to (new) boundaries 3 - 4.
Step 7: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=3*<1<2<3=4*
```

is shown below:

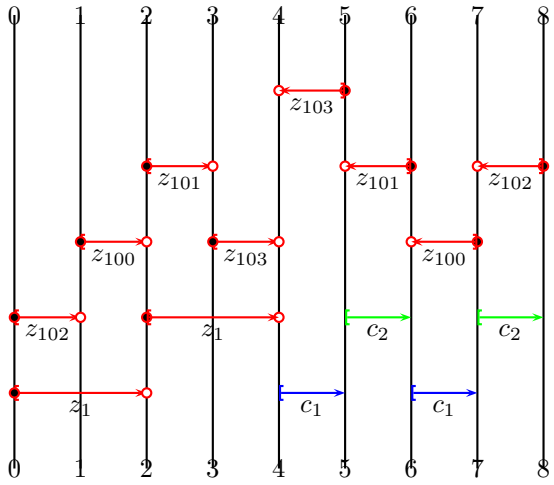


Observe the following facts about this GE: The base [3-4:z102-.] has constraints with its dual that stretch the constant segment 3 - 4 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

5 Generalized Equation #5

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-2:z1+.] ; Carrier Dual: [2-4:z1+.] ; Critical Boundary: 2; **Prints**

```
Print 0: =0=2*<1=3*<2=4*
Print 1: =0=2*<1<3*<2=4*
Print 2: =0=2*<3*<1<2=4*
```

Total number of prints: 3
Next, we consider

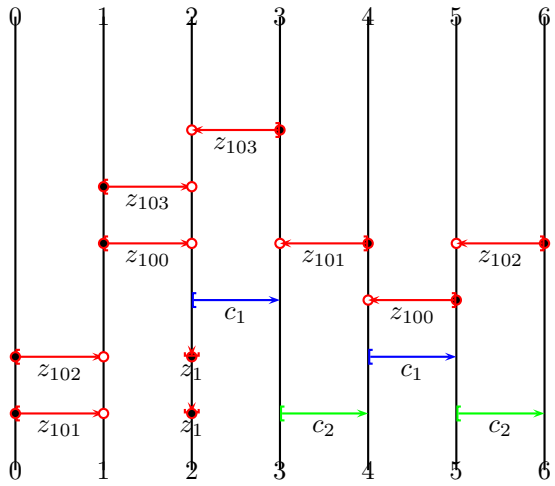
```
Print 1: =0=2*<1=3*<2=4*
```

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 4.
Step 2: Moved (old) base [1-2:z100+.] to (new) boundaries 3 - 4.
Step 3: Moved (old) base [0-1:z102+.] to (new) boundaries 2 - 3.
Step 4: Collapsed (new) base [2-4:z1+.] to the empty base (4,4).
Step 5: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 6: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=2*<1=3*<2=4*
```

is shown below:



The GE above is non-degenerate.

This completes the consideration of Print 1.

Next, we consider

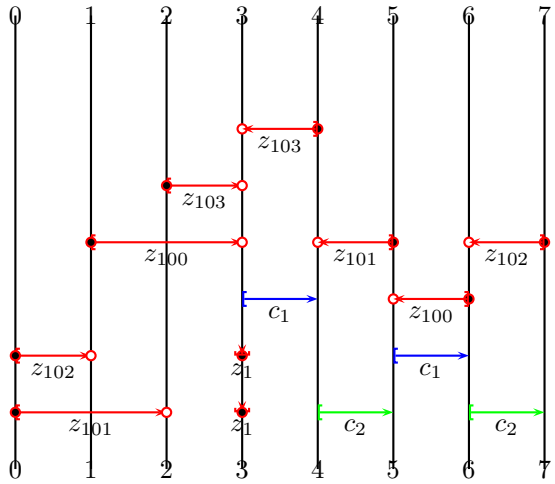
```
Print 2: =0=2*<1<3*<2=4*
```

Sequence of Actions in performing the Print 2:
Step 1: Added (new) boundary 3.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [1-2:z100+.] to (new) boundaries 3 - 5.
Step 4: Moved (old) base [0-1:z102+.] to (new) boundaries 2 - 3.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 2: =0=2*<1<3*<2=4*
```

is shown below:



Observe the following facts about this GE: The base [4-5:z101-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. The base [5-6:z100-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 2.

Next, we consider

```
Print 3: =0=2*<3*<1<2=4*
```

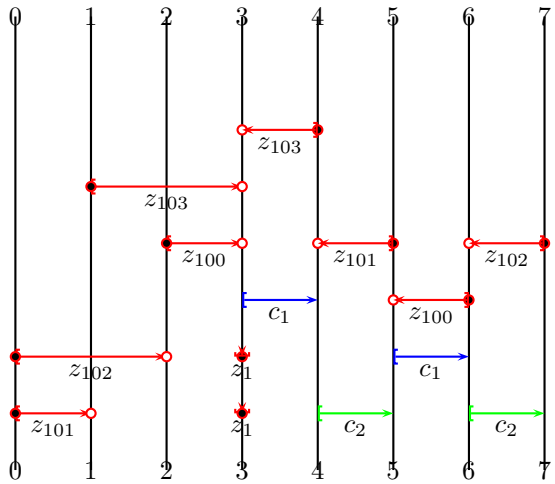
Sequence of Actions in performing the Print 3:
Step 1: Added (new) boundary 4.

Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [1-2:z100+.] to (new) boundaries 4 - 5.
Step 4: Moved (old) base [0-1:z102+.] to (new) boundaries 2 - 4.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 3: =0=2*<3*<1<2=4*
```

is shown below:

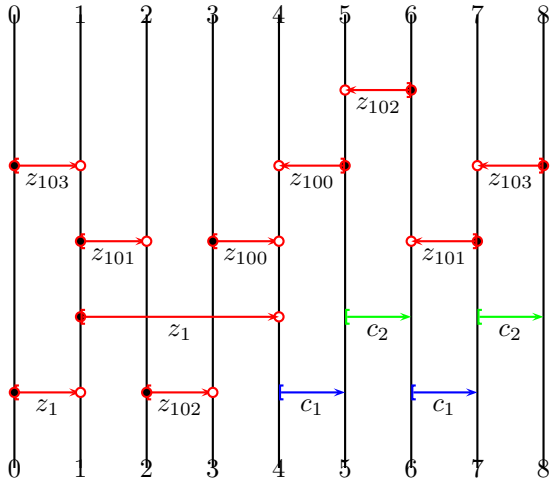


Observe the following facts about this GE: The base [3-4:z103-.] has constraints with its dual that stretch the constant segment 3 - 4 to length different from 1. The base [6-7:z102-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 3.

6 Generalized Equation #6

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-4:z1+.] ; Critical Boundary: 1; **Prints**

```
Print 0: =0=1*<2*<3*<1=4*
```

Total number of prints: 1
Next, we consider

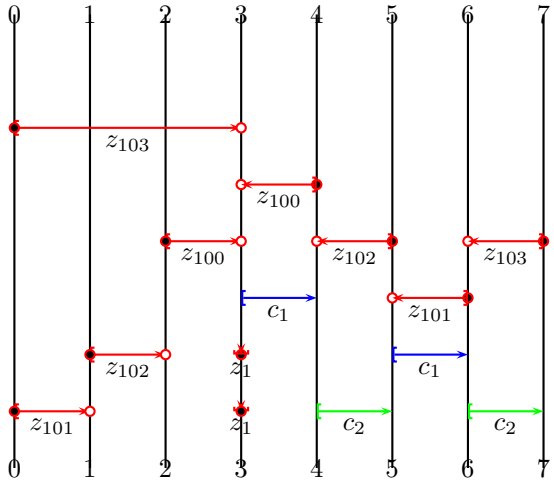
```
Print 1: =0=1*<2*<3*<1=4*
```

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 4.
Step 2: Moved (old) base [0-1:z103+.] to (new) boundaries 1 - 4.
Step 3: Collapsed (new) base [1-4:z1+.] to the empty base (4,4).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=1*<2*<3*<1=4*
```

is shown below:

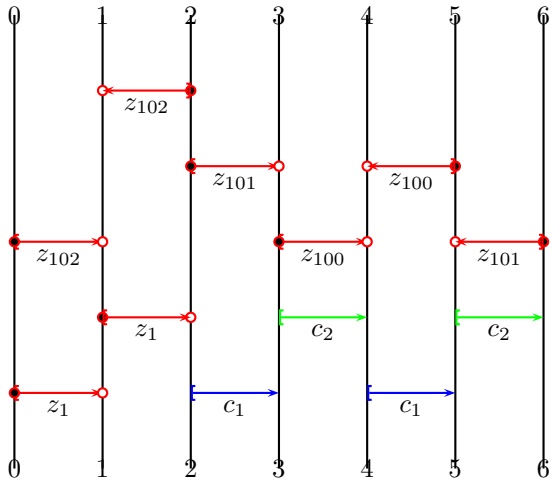


Observe the following facts about this GE: The base [6-7:z103-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

7 Generalized Equation #7

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-2:z1+.] ; Critical Boundary: 1; **Prints**

```
Print 0: =0=1*<1=2*
```

Total number of prints: 1
Next, we consider

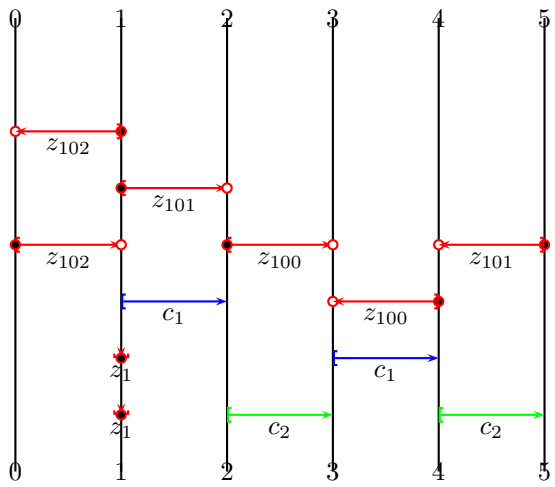
```
Print 1: =0=1*<1=2*
```

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 2.
Step 2: Moved (old) base [0-1:z102+.] to (new) boundaries 1 - 2.
Step 3: Collapsed (new) base [1-2:z1+.] to the empty base (2,2).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=1*<1=2*
```

is shown below:

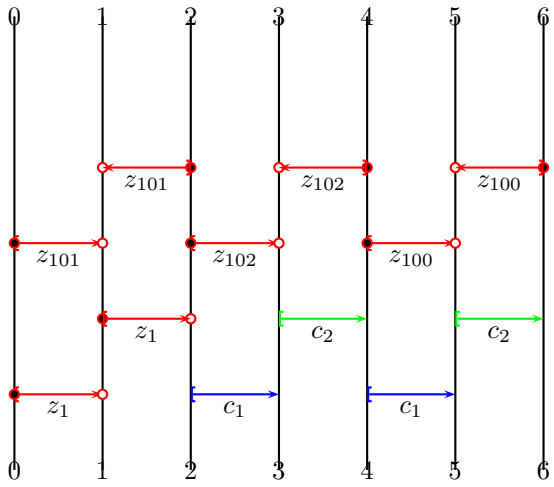


Observe the following facts about this GE: The base [0-1:z102+.] and its dual are of opposite polarity, yet intersect. The base [0-1:z102-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

8 Generalized Equation #8

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-2:z1+.] ; Critical Boundary: 1; **Prints**

```
Print 0: =0=1*<1=2*
```

Total number of prints: 1
Next, we consider

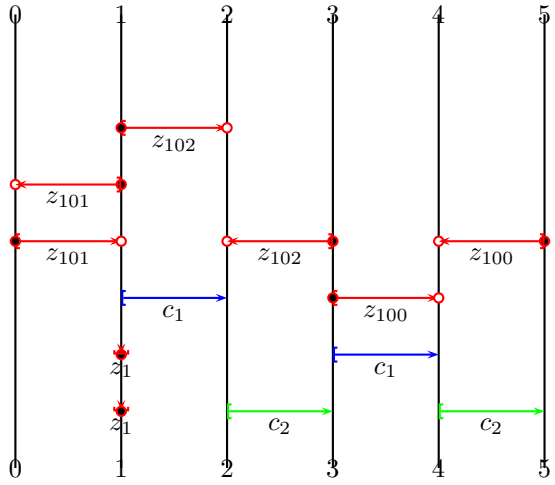
```
Print 1: =0=1*<1=2*
```

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 2.
Step 2: Moved (old) base [0-1:z101+.] to (new) boundaries 1 - 2.
Step 3: Collapsed (new) base [1-2:z1+.] to the empty base (2,2).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=1*<1=2*
```

is shown below:

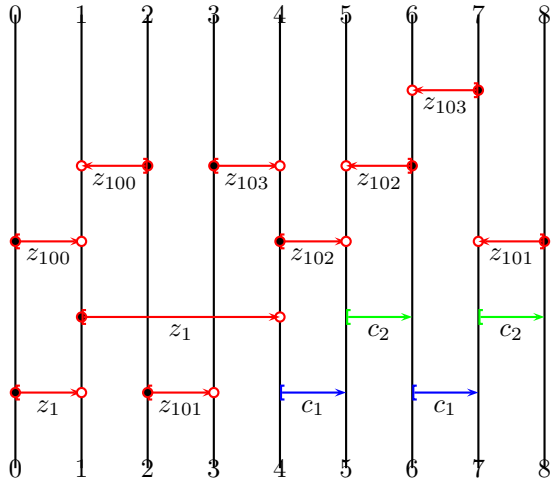


Observe the following facts about this GE: The base [0-1:z101+.] and its dual are of opposite polarity, yet intersect. The base [0-1:z101-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

9 Generalized Equation #9

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-4:z1+.] ; Critical Boundary: 1; **Prints**

```
Print 0: =0=1*<2*<3*<1=4*
```

Total number of prints: 1
Next, we consider

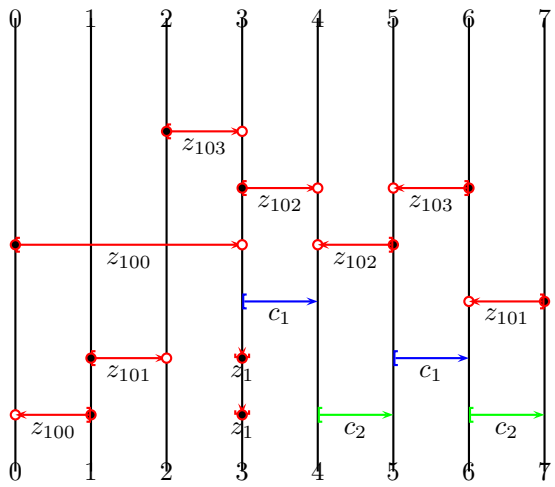
```
Print 1: =0=1*<2*<3*<1=4*
```

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 4.
Step 2: Moved (old) base [0-1:z100+.] to (new) boundaries 1 - 4.
Step 3: Collapsed (new) base [1-4:z1+.] to the empty base (4,4).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=1*<2*<3*<1=4*
```

is shown below:

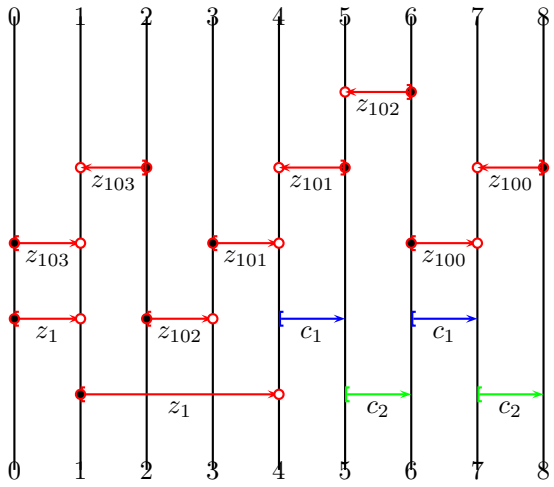


Observe the following facts about this GE: The base [0-3:z100+.] and its dual are of opposite polarity, yet intersect. The base [0-1:z100-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

10 Generalized Equation #10

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-4:z1+.] ; Critical Boundary: 1; **Prints**

```
Print 0: =0=1*<2*<3*<1=4*
```

Total number of prints: 1
Next, we consider

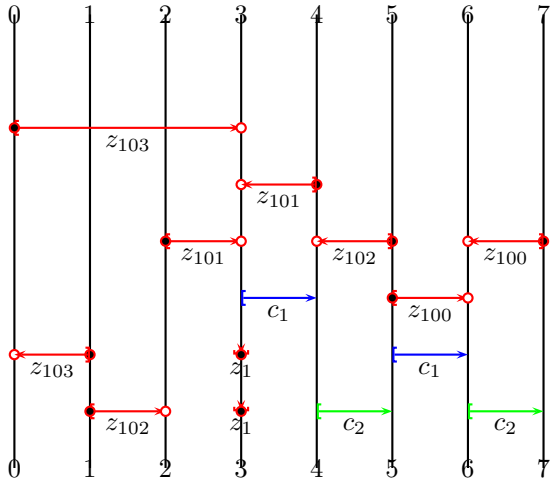
```
Print 1: =0=1*<2*<3*<1=4*
```

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 4.
Step 2: Moved (old) base [0-1:z103+.] to (new) boundaries 1 - 4.
Step 3: Collapsed (new) base [1-4:z1+.] to the empty base (4,4).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=1*<2*<3*<1=4*
```

is shown below:

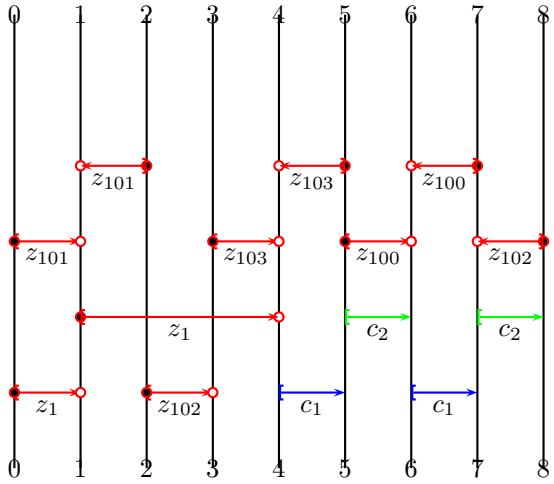


Observe the following facts about this GE: The base [0-3:z103+.] and its dual are of opposite polarity, yet intersect. The base [0-1:z103-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

11 Generalized Equation #11

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-4:z1+.] ; Critical Boundary: 1; **Prints**

```
Print 0: =0=1*<2*<3*<1=4*
```

Total number of prints: 1
Next, we consider

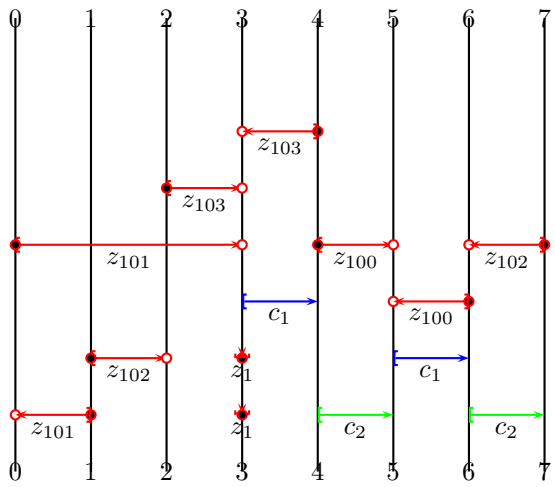
```
Print 1: =0=1*<2*<3*<1=4*
```

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 4.
Step 2: Moved (old) base [0-1:z101+.] to (new) boundaries 1 - 4.
Step 3: Collapsed (new) base [1-4:z1+.] to the empty base (4,4).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=1*<2*<3*<1=4*
```

is shown below:

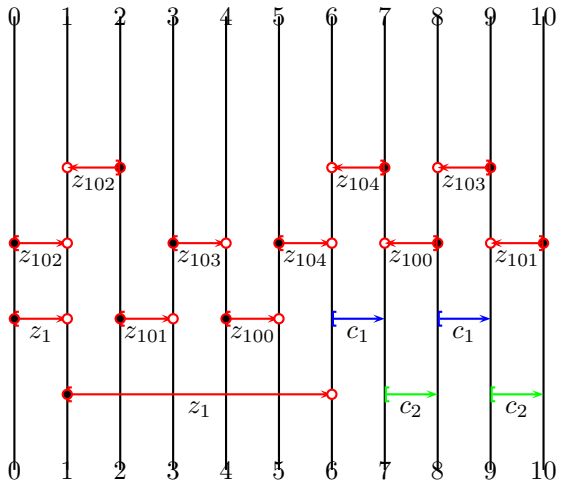


Observe the following facts about this GE: The base [0-3:z101+.] and its dual are of opposite polarity, yet intersect. The base [0-1:z101-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

12 Generalized Equation #12

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-6:z1+.] ; Critical Boundary: 1; **Prints**

```
Print 0: =0=1*<2*<3*<4*<5*<1=6*
```

Total number of prints: 1
Next, we consider

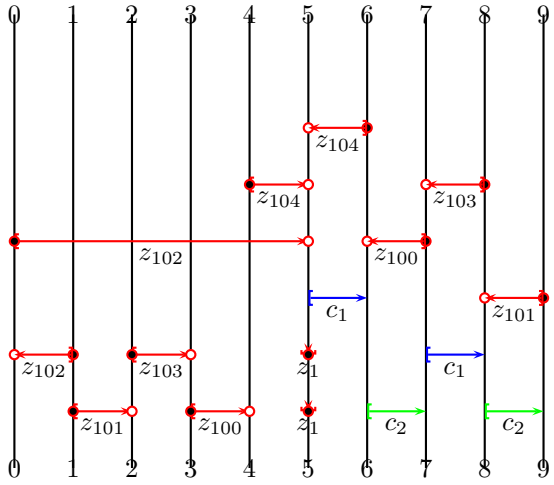
```
Print 1: =0=1*<2*<3*<4*<5*<1=6*
```

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 6.
Step 2: Moved (old) base [0-1:z102+.] to (new) boundaries 1 - 6.
Step 3: Collapsed (new) base [1-6:z1+.] to the empty base (6,6).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=1*<2*<3*<4*<5*<1=6*
```

is shown below:

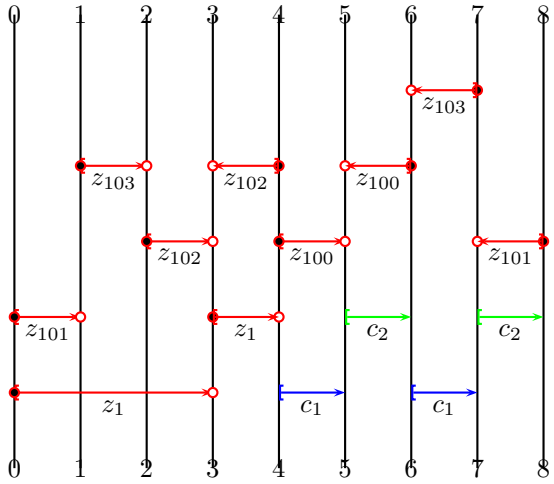


Observe the following facts about this GE: The base [0-5:z102+.] and its dual are of opposite polarity, yet intersect. The base [0-1:z102-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

13 Generalized Equation #13

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-3:z1+.] ; Carrier Dual: [3-4:z1+.] ; Critical Boundary: 3; **Prints**

```
Print 0: =0=3*<1<2<3=4*
```

Total number of prints: 1
Next, we consider

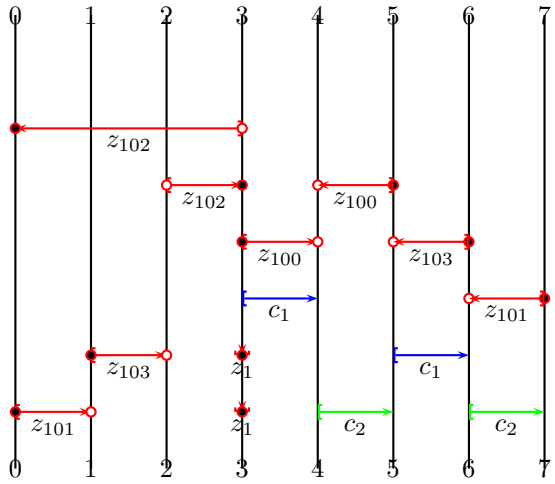
```
Print 1: =0=3*<1<2<3=4*
```

Sequence of Actions in performing the Print 1:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 5.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 6.
Step 4: Moved (old) base [0-1:z101+.] to (new) boundaries 3 - 4.
Step 5: Moved (old) base [2-3:z102+.] to (new) boundaries 5 - 6.
Step 6: Moved (old) base [1-2:z103+.] to (new) boundaries 4 - 5.
Step 7: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=3*<1<2<3=4*
```

is shown below:

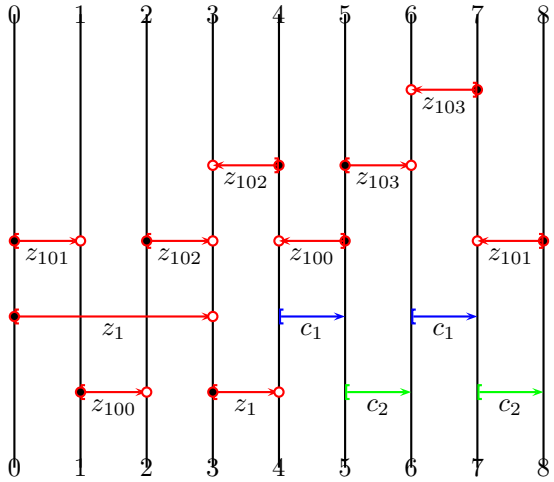


Observe the following facts about this GE: The base [2-3:z102+.] and its dual are of opposite polarity, yet intersect. The base [0-3:z102-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

14 Generalized Equation #14

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-3:z1+.] ; Carrier Dual: [3-4:z1+.] ; Critical Boundary: 3; **Prints**

```
Print 0: =0=3*<1<2<3=4*
```

Total number of prints: 1
Next, we consider

```
Print 1: =0=3*<1<2<3=4*
```

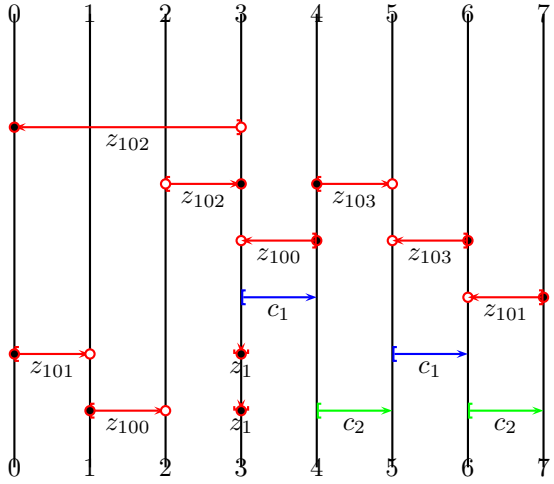
Sequence of Actions in performing the Print 1:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 5.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 6.
Step 4: Moved (old) base [1-2:z100+.] to (new) boundaries 4 - 5.
Step 5: Moved (old) base [0-1:z101+.] to (new) boundaries 3 - 4.
Step 6: Moved (old) base [2-3:z102+.] to (new) boundaries 5 - 6.

Step 7: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=3*<1<2<3=4*
```

is shown below:

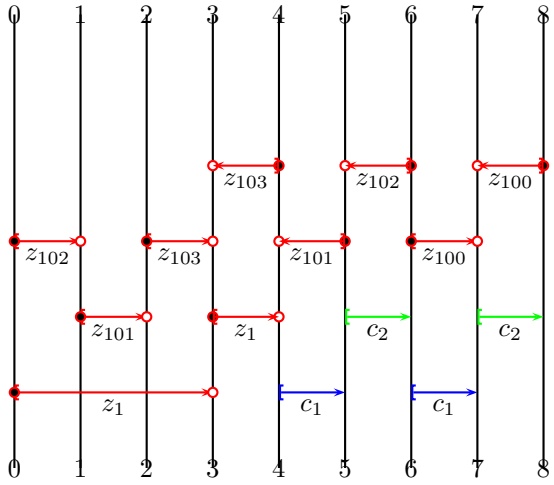


Observe the following facts about this GE: The base [2-3:z102+.] and its dual are of opposite polarity, yet intersect. The base [0-3:z102-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

15 Generalized Equation #15

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-3:z1+.] ; Carrier Dual: [3-4:z1+.] ; Critical Boundary: 3; **Prints**

```
Print 0: =0=3*<1<2<3=4*
```

Total number of prints: 1
Next, we consider

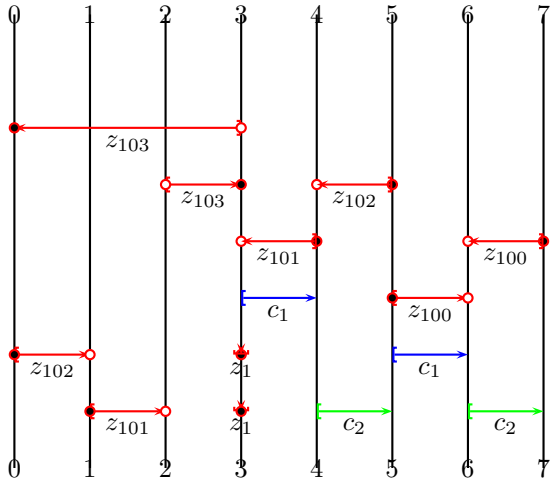
```
Print 1: =0=3*<1<2<3=4*
```

Sequence of Actions in performing the Print 1:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 5.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 6.
Step 4: Moved (old) base [1-2:z101+.] to (new) boundaries 4 - 5.
Step 5: Moved (old) base [0-1:z102+.] to (new) boundaries 3 - 4.
Step 6: Moved (old) base [2-3:z103+.] to (new) boundaries 5 - 6.
Step 7: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=3*<1<2<3=4*
```

is shown below:

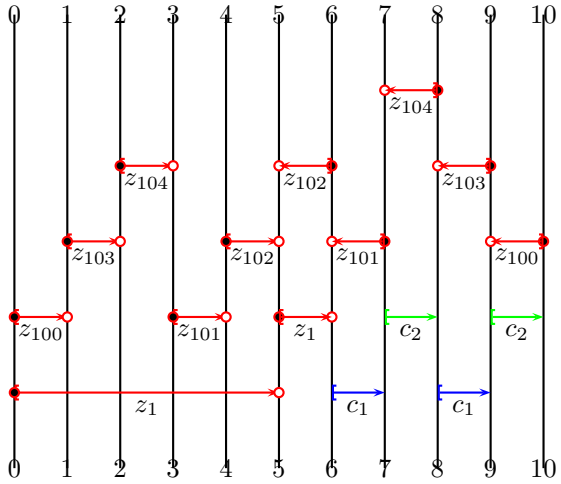


Observe the following facts about this GE: The base [2-3:z103+.] and its dual are of opposite polarity, yet intersect. The base [0-3:z103-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

16 Generalized Equation #16

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-5:z1+.] ; Carrier Dual: [5-6:z1+.] ; Critical Boundary: 5; **Prints**

```
Print 0: =0=5*<1<2<3<4<5=6*
```

Total number of prints: 1
Next, we consider

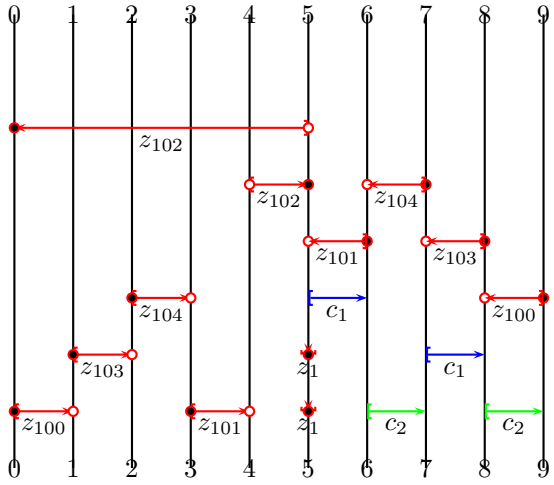
```
Print 1: =0=5*<1<2<3<4<5=6*
```

Sequence of Actions in performing the Print 1:
Step 1: Added (new) boundary 6.
Step 2: Added (new) boundary 7.
Step 3: Added (new) boundary 8.
Step 4: Added (new) boundary 9.
Step 5: Moved (old) base [0-5:z1+.] to (new) boundaries 5 - 10.
Step 6: Moved (old) base [0-1:z100+.] to (new) boundaries 5 - 6.
Step 7: Moved (old) base [3-4:z101+.] to (new) boundaries 8 - 9.
Step 8: Moved (old) base [4-5:z102+.] to (new) boundaries 9 - 10.
Step 9: Moved (old) base [1-2:z103+.] to (new) boundaries 6 - 7.
Step 10: Moved (old) base [2-3:z104+.] to (new) boundaries 7 - 8.
Step 11: Collapsed (new) base [5-10:z1+.] to the empty base (10,10).
Step 12: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 13: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 14: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 15: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 16: Deleted (new) boundary 4 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=5*<1<2<3<4<5=6*
```

is shown below:

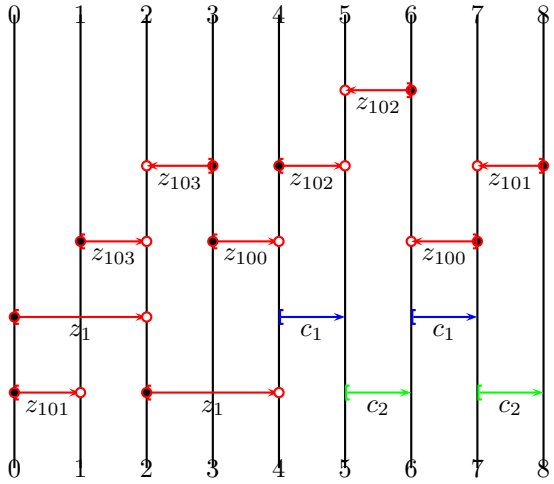


Observe the following facts about this GE: The base [4-5:z102+.] and its dual are of opposite polarity, yet intersect. The base [0-5:z102-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

17 Generalized Equation #17

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-2:z1+.] ; Carrier Dual: [2-4:z1+.] ; Critical Boundary: 2; **Prints**

```
Print 0: =0=2*<1<3*<2=4*
Print 1: =0=2*<3*<1<2=4*
```

Total number of prints: 2
Next, we consider

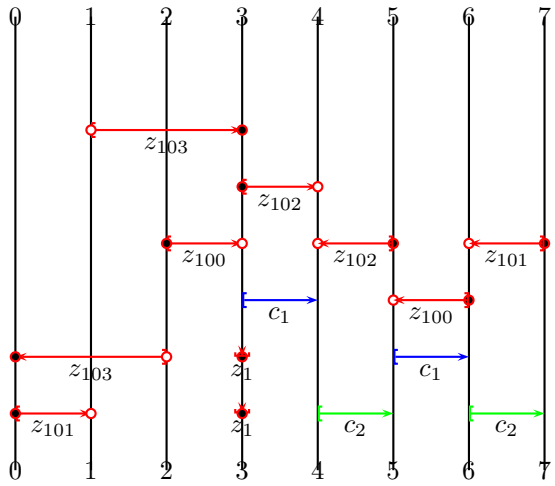
```
Print 1: =0=2*<1<3*<2=4*
```

Sequence of Actions in performing the Print 1:
Step 1: Added (new) boundary 3.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [0-1:z101+.] to (new) boundaries 2 - 3.
Step 4: Moved (old) base [1-2:z103+.] to (new) boundaries 3 - 5.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=2*<1<3*<2=4*
```

is shown below:



Observe the following facts about this GE: The base [1-3:z103+.] and its dual are of opposite polarity, yet intersect. The base [0-2:z103-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is

degenerate.

This completes the consideration of Print 1.

Next, we consider

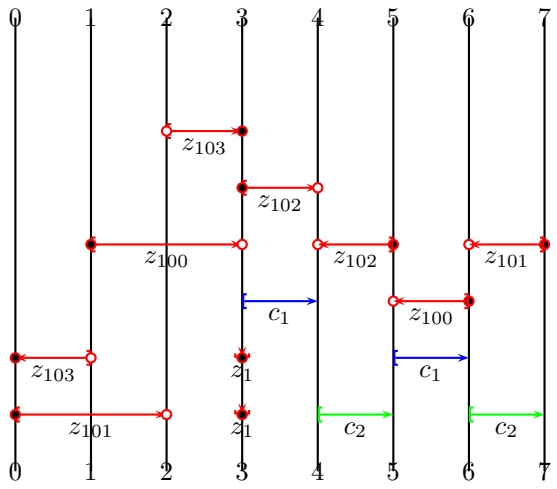
```
Print 2: =0=2*<3*<1<2=4*
```

Sequence of Actions in performing the Print 2:
Step 1: Added (new) boundary 4.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [0-1:z101+.] to (new) boundaries 2 - 4.
Step 4: Moved (old) base [1-2:z103+.] to (new) boundaries 4 - 5.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 2: =0=2*<3*<1<2=4*
```

is shown below:

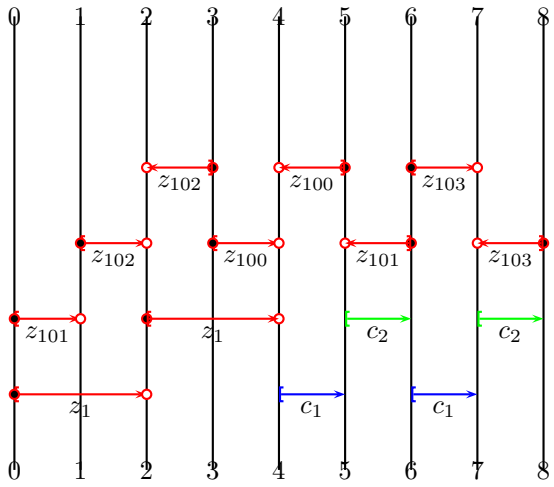


Observe the following facts about this GE: The base [5-6:z100-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [6-7:z101-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 2.

18 Generalized Equation #18

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-2:z1+.] ; Carrier Dual: [2-4:z1+.] ; Critical Boundary: 2; **Prints**

```
Print 0: =0=2*<1<3*<2=4*
Print 1: =0=2*<3*<1<2=4*
```

Total number of prints: 2
Next, we consider

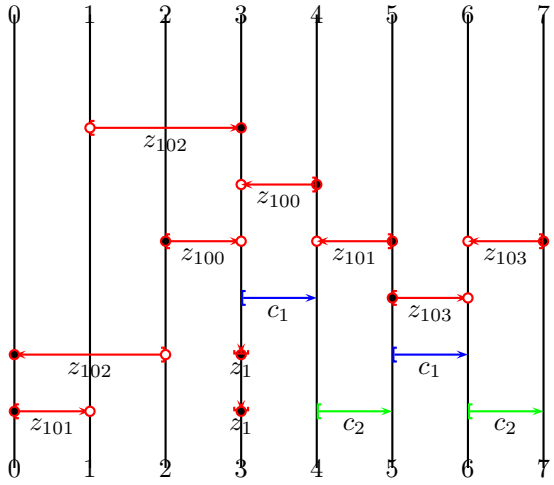
```
Print 1: =0=2*<1<3*<2=4*
```

Sequence of Actions in performing the Print 1:
Step 1: Added (new) boundary 3.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [0-1:z101+.] to (new) boundaries 2 - 3.
Step 4: Moved (old) base [1-2:z102+.] to (new) boundaries 3 - 5.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=2*<1<3*<2=4*
```

is shown below:



Observe the following facts about this GE: The base [1-3:z102+.] and its dual are of opposite polarity, yet intersect. The base [0-2:z102-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

Next, we consider

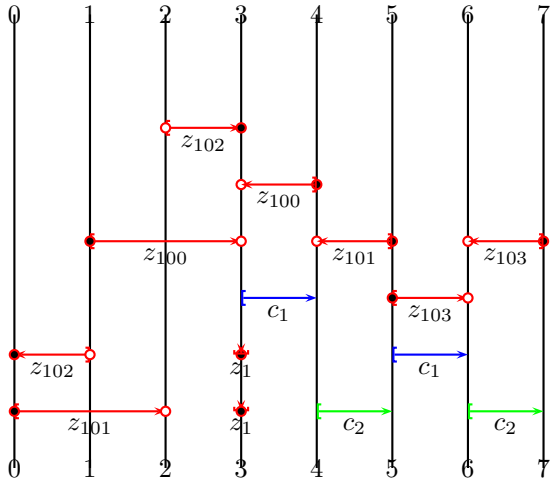
```
Print 2: =0=2*<3*<1<2=4*
```

Sequence of Actions in performing the Print 2:
Step 1: Added (new) boundary 4.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [0-1:z101+.] to (new) boundaries 2 - 4.
Step 4: Moved (old) base [1-2:z102+.] to (new) boundaries 4 - 5.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 2: =0=2*<3*<1<2=4*
```

is shown below:

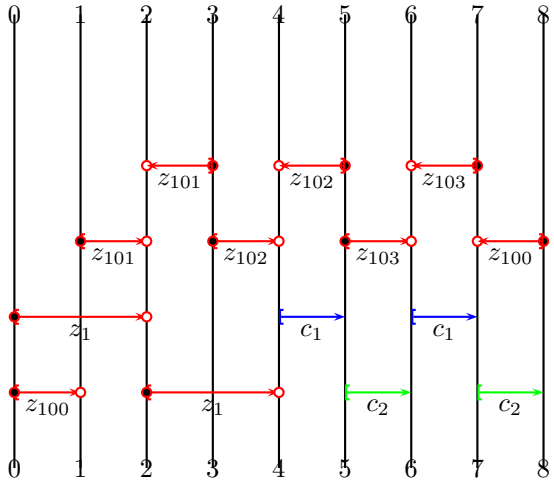


Observe the following facts about this GE: The base [3-4:z100-.] has constraints with its dual that stretch the constant segment 3 - 4 to length different from 1. The base [4-5:z101-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 2.

19 Generalized Equation #19

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-2:z1+.] ; Carrier Dual: [2-4:z1+.] ; Critical Boundary: 2; **Prints**

```
Print 0: =0=2*<1<3*<2=4*
Print 1: =0=2*<3*<1<2=4*
```

Total number of prints: 2
Next, we consider

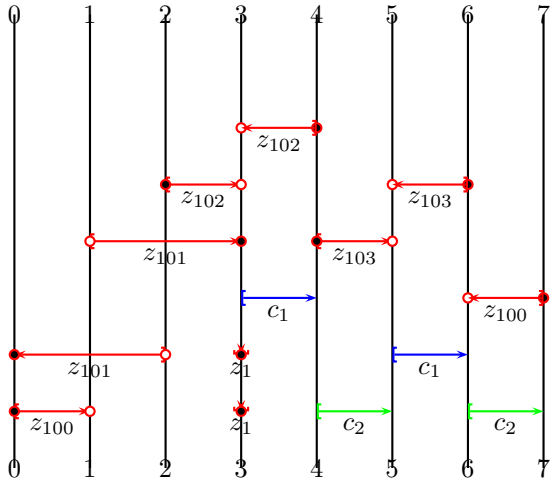
```
Print 1: =0=2*<1<3*<2=4*
```

Sequence of Actions in performing the Print 1:
Step 1: Added (new) boundary 3.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [0-1:z100+.] to (new) boundaries 2 - 3.
Step 4: Moved (old) base [1-2:z101+.] to (new) boundaries 3 - 5.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=2*<1<3*<2=4*
```

is shown below:



Observe the following facts about this GE: The base [1-3:z101+.] and its dual are of opposite polarity, yet intersect. The base [0-2:z101-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

Next, we consider

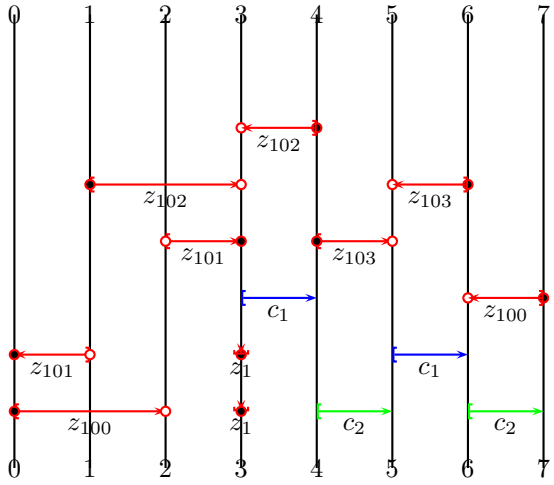
```
Print 2: =0=2*<3*<1<2=4*
```

Sequence of Actions in performing the Print 2:
Step 1: Added (new) boundary 4.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [0-1:z100+.] to (new) boundaries 2 - 4.
Step 4: Moved (old) base [1-2:z101+.] to (new) boundaries 4 - 5.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 2: =0=2*<3*<1<2=4*
```

is shown below:

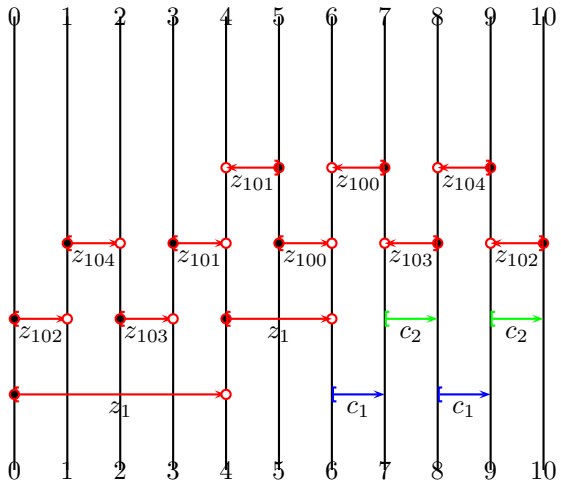


Observe the following facts about this GE: The base [3-4:z102-.] has constraints with its dual that stretch the constant segment 3 - 4 to length different from 1. The base [6-7:z100-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 2.

20 Generalized Equation #20

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-4:z1+.] ; Carrier Dual: [4-6:z1+.] ; Critical Boundary: 4; **Prints**

```
Print 0: =0=4*<1<2<3<5*<4=6*
Print 1: =0=4*<1<2=5*<3<4=6*
Print 2: =0=4*<1<2<5*<3<4=6*
Print 3: =0=4*<1=5*<2<3<4=6*
Print 4: =0=4*<1<5*<2<3<4=6*
Print 5: =0=4*<5*<1<2<3<4=6*
```

Total number of prints: 6
Next, we consider

```
Print 1: =0=4*<1<2<3<5*<4=6*
```

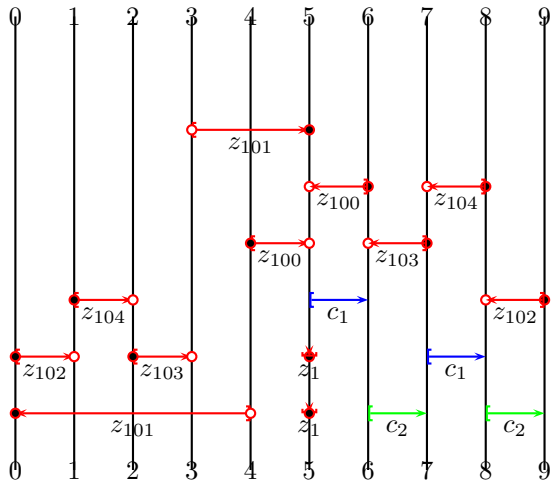
Sequence of Actions in performing the Print 1:
Step 1: Added (new) boundary 5.
Step 2: Added (new) boundary 6.
Step 3: Added (new) boundary 7.
Step 4: Moved (old) base [0-4:z1+.] to (new) boundaries 4 - 9.
Step 5: Moved (old) base [3-4:z101+.] to (new) boundaries 7 - 9.
Step 6: Moved (old) base [0-1:z102+.] to (new) boundaries 4 - 5.
Step 7: Moved (old) base [2-3:z103+.] to (new) boundaries 6 - 7.
Step 8: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 6.
Step 9: Collapsed (new) base [4-9:z1+.] to the empty base (9,9).
Step 10: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 11: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 12: Deleted (new) boundary 2 because it is not used inside any base. This

will cause renumbering of higher numbered boundaries.
Step 13: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=4*<1<2<3<5*<4=6*
```

is shown below:



Observe the following facts about this GE: The base [3-5:z101+.] and its dual are of opposite polarity, yet intersect. The base [0-4:z101-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

Next, we consider

```
Print 2: =0=4*<1<2=5*<3<4=6*
```

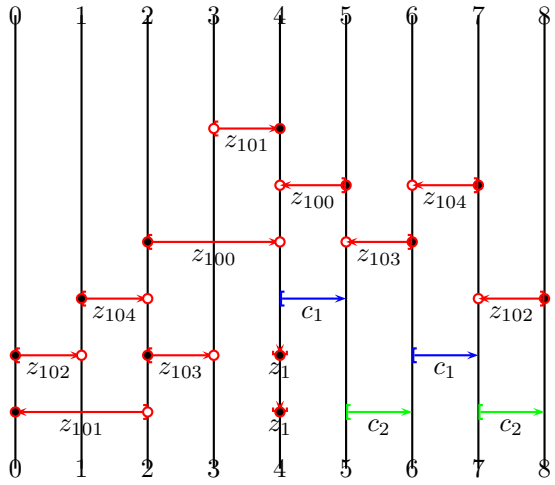
Sequence of Actions in performing the Print 2:
Step 1: Added (new) boundary 5.
Step 2: Added (new) boundary 7.
Step 3: Moved (old) base [0-4:z1+.] to (new) boundaries 4 - 8.
Step 4: Moved (old) base [3-4:z101+.] to (new) boundaries 7 - 8.
Step 5: Moved (old) base [0-1:z102+.] to (new) boundaries 4 - 5.
Step 6: Moved (old) base [2-3:z103+.] to (new) boundaries 6 - 7.
Step 7: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 6.
Step 8: Collapsed (new) base [4-8:z1+.] to the empty base (8,8).
Step 9: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Step 10: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 11: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 12: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 2: =0=4*<1<2=5*<3<4=6*
```

is shown below:



Observe the following facts about this GE: The base [4-5:z100-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 2.

Next, we consider

```
Print 3: =0=4*<1<2<5*<3<4=6*
```

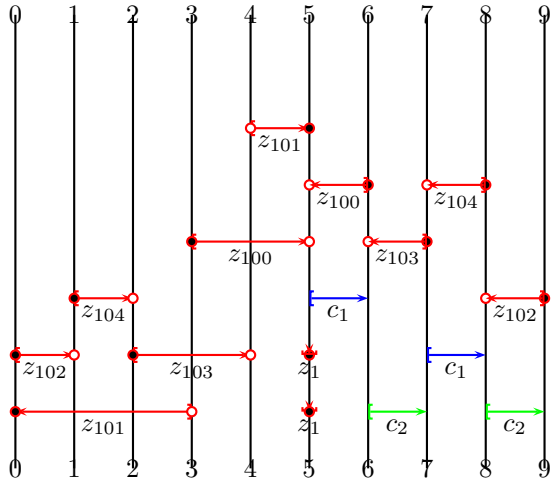
Sequence of Actions in performing the Print 3:
Step 1: Added (new) boundary 5.
Step 2: Added (new) boundary 6.
Step 3: Added (new) boundary 8.
Step 4: Moved (old) base [0-4:z1+.] to (new) boundaries 4 - 9.
Step 5: Moved (old) base [3-4:z101+.] to (new) boundaries 8 - 9.
Step 6: Moved (old) base [0-1:z102+.] to (new) boundaries 4 - 5.
Step 7: Moved (old) base [2-3:z103+.] to (new) boundaries 6 - 8.
Step 8: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 6.

Step 9: Collapsed (new) base [4-9:z1+.] to the empty base (9,9).
Step 10: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 11: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 12: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 13: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 3: =0=4*<1<2<5*<3<4=6*
```

is shown below:



Observe the following facts about this GE: The base [5-6:z100-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [6-7:z103-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 3.

Next, we consider

```
Print 4: =0=4*<1=5*<2<3<4=6*
```

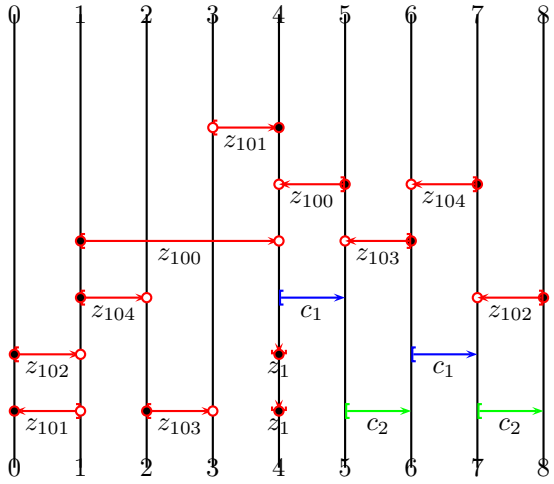
Sequence of Actions in performing the Print 4:
Step 1: Added (new) boundary 6.
Step 2: Added (new) boundary 7.
Step 3: Moved (old) base [0-4:z1+.] to (new) boundaries 4 - 8.

Step 4: Moved (old) base [3-4:z101+.] to (new) boundaries 7 - 8.
Step 5: Moved (old) base [0-1:z102+.] to (new) boundaries 4 - 5.
Step 6: Moved (old) base [2-3:z103+.] to (new) boundaries 6 - 7.
Step 7: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 6.
Step 8: Collapsed (new) base [4-8:z1+.] to the empty base (8,8).
Step 9: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 11: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 12: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 4: =0=4*<1=5*<2<3<4=6*
```

is shown below:



Observe the following facts about this GE: The base [4-5:z100-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 4.

Next, we consider

```
Print 5: =0=4*<1<5*<2<3<4=6*
```

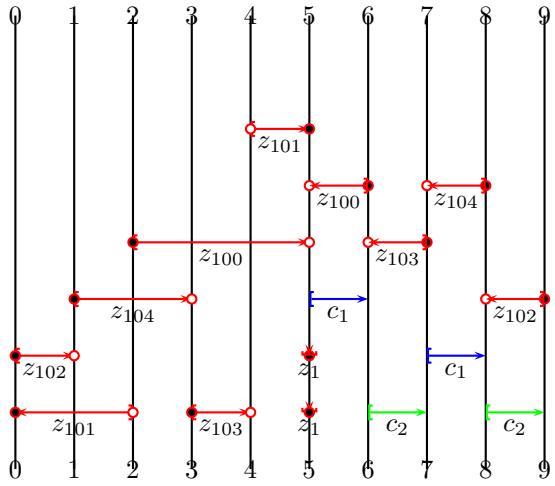
Sequence of Actions in performing the Print 5:
Step 1: Added (new) boundary 5.

Step 2: Added (new) boundary 7.
Step 3: Added (new) boundary 8.
Step 4: Moved (old) base [0-4:z1+.] to (new) boundaries 4 - 9.
Step 5: Moved (old) base [3-4:z101+.] to (new) boundaries 8 - 9.
Step 6: Moved (old) base [0-1:z102+.] to (new) boundaries 4 - 5.
Step 7: Moved (old) base [2-3:z103+.] to (new) boundaries 7 - 8.
Step 8: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 7.
Step 9: Collapsed (new) base [4-9:z1+.] to the empty base (9,9).
Step 10: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 11: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 12: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 13: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 5: =0=4*<1<5*<2<3<4=6*
```

is shown below:



Observe the following facts about this GE: The base [5-6:z100-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [7-8:z104-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 5.

Next, we consider

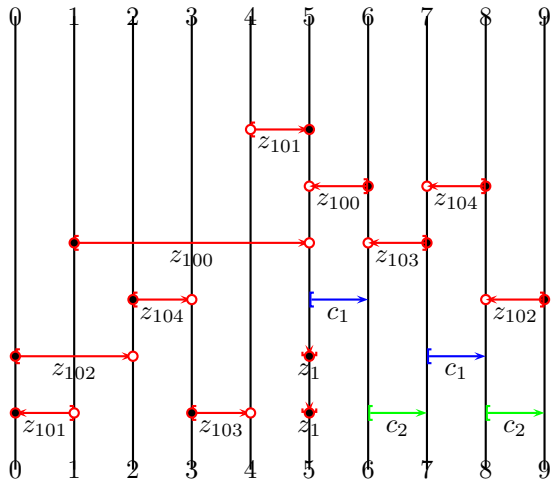
```
Print 6: =0=4*<5*<1<2<3<4=6*
```

Sequence of Actions in performing the Print 6:
Step 1: Added (new) boundary 6.
Step 2: Added (new) boundary 7.
Step 3: Added (new) boundary 8.
Step 4: Moved (old) base [0-4:z1+.] to (new) boundaries 4 - 9.
Step 5: Moved (old) base [3-4:z101+.] to (new) boundaries 8 - 9.
Step 6: Moved (old) base [0-1:z102+.] to (new) boundaries 4 - 6.
Step 7: Moved (old) base [2-3:z103+.] to (new) boundaries 7 - 8.
Step 8: Moved (old) base [1-2:z104+.] to (new) boundaries 6 - 7.
Step 9: Collapsed (new) base [4-9:z1+.] to the empty base (9,9).
Step 10: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 11: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 12: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 13: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 6: =0=4*<5*<1<2<3<4=6*
```

is shown below:



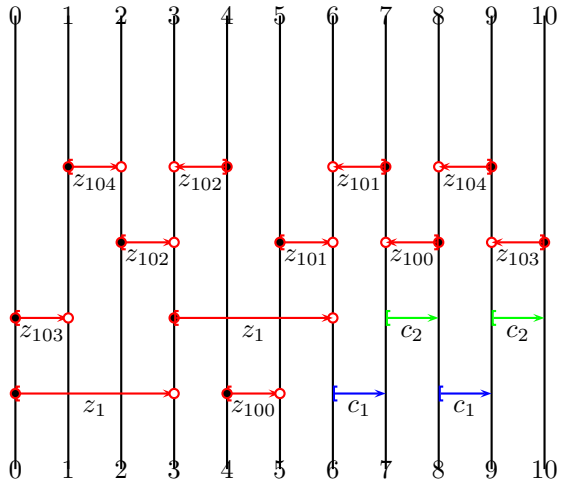
Observe the following facts about this GE: The base [5-6:z100-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from

1. The base [8-9:z102-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 6.

21 Generalized Equation #21

Quadratic System: $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.



GE Information: Carrier: [0-3:z1+.] ; Carrier Dual: [3-6:z1+.] ; Critical Boundary: 3; **Prints**

```
Print 0: =0=3*<1<2<4*<5*<3=6*
Print 1: =0=3*<1=4*<2=5*<3=6*
Print 2: =0=3*<1=4*<2<5*<3=6*
Print 3: =0=3*<1=4*<5*<2<3=6*
Print 4: =0=3*<1<4*<2=5*<3=6*
Print 5: =0=3*<1<4*<2<5*<3=6*
Print 6: =0=3*<1<4*<5*<2<3=6*
Print 7: =0=3*<4*<1<2=5*<3=6*
Print 8: =0=3*<4*<1<2<5*<3=6*
Print 9: =0=3*<4*<1=5*<2<3=6*
Print 10: =0=3*<4*<1<5*<2<3=6*
Print 11: =0=3*<4*<5*<1<2<3=6*
```

Total number of prints: 12
Next, we consider

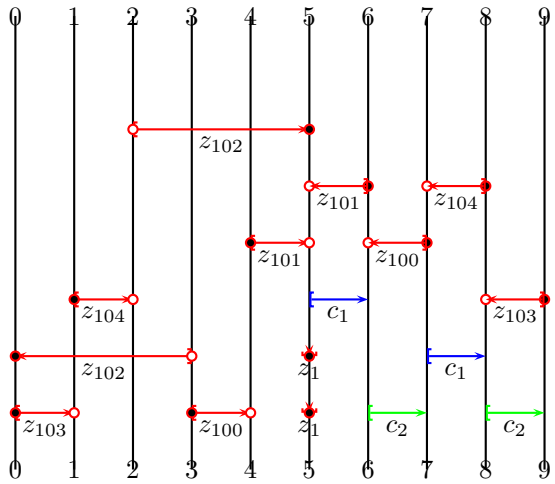
```
Print 1: =0=3*<1<2<4*<5*<3=6*
```

Sequence of Actions in performing the Print 1:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 5.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 8.
Step 4: Moved (old) base [2-3:z102+.] to (new) boundaries 5 - 8.
Step 5: Moved (old) base [0-1:z103+.] to (new) boundaries 3 - 4.
Step 6: Moved (old) base [1-2:z104+.] to (new) boundaries 4 - 5.
Step 7: Collapsed (new) base [3-8:z1+.] to the empty base (8,8).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=3*<1<2<4*<5*<3=6*
```

is shown below:



Observe the following facts about this GE: The base [2-5:z102+.] and its dual are of opposite polarity, yet intersect. The base [0-3:z102-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

Next, we consider

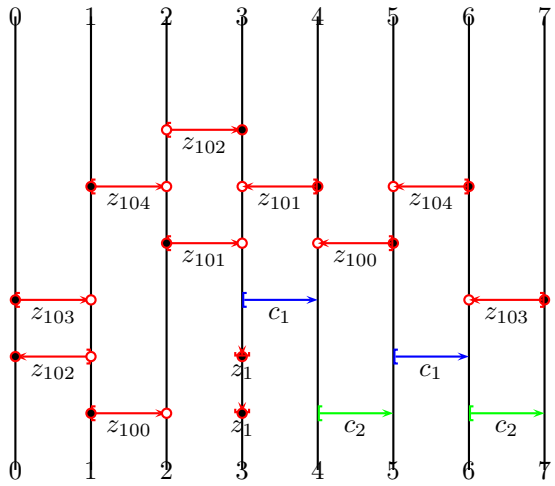
```
Print 2: =0=3*<1=4*<2=5*<3=6*
```

Sequence of Actions in performing the Print 2:
Step 1: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 6.
Step 2: Moved (old) base [2-3:z102+.] to (new) boundaries 5 - 6.
Step 3: Moved (old) base [0-1:z103+.] to (new) boundaries 3 - 4.
Step 4: Moved (old) base [1-2:z104+.] to (new) boundaries 4 - 5.
Step 5: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 8: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 2: =0=3*<1=4*<2=5*<3=6*
```

is shown below:



The GE above is non-degenerate.

This completes the consideration of Print 2.

Next, we consider

```
Print 3: =0=3*<1=4*<2<5*<3=6*
```

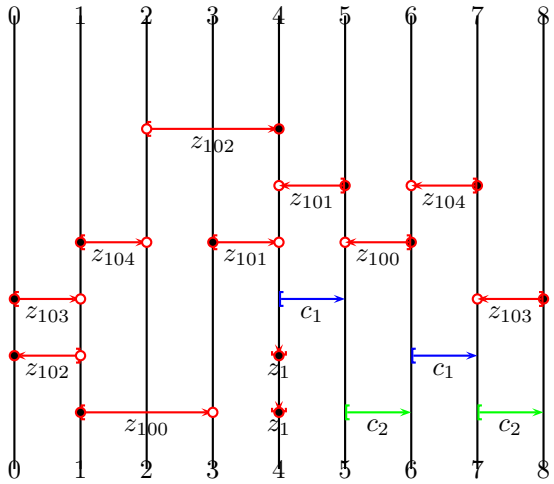
Sequence of Actions in performing the Print 3:
Step 1: Added (new) boundary 5.
Step 2: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 7.

Step 3: Moved (old) base [2-3:z102+.] to (new) boundaries 5 - 7.
Step 4: Moved (old) base [0-1:z103+.] to (new) boundaries 3 - 4.
Step 5: Moved (old) base [1-2:z104+.] to (new) boundaries 4 - 5.
Step 6: Collapsed (new) base [3-7:z1+.] to the empty base (7,7).
Step 7: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 8: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 3: =0=3*<1=4*<2<5*<3=6*
```

is shown below:



Observe the following facts about this GE: The base [5-6:z100-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 3.

Next, we consider

```
Print 4: =0=3*<1=4*<5*<2<3=6*
```

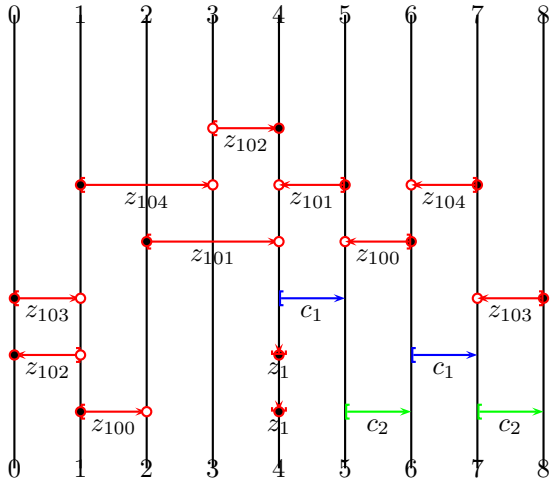
Sequence of Actions in performing the Print 4:
Step 1: Added (new) boundary 6.
Step 2: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 7.
Step 3: Moved (old) base [2-3:z102+.] to (new) boundaries 6 - 7.
Step 4: Moved (old) base [0-1:z103+.] to (new) boundaries 3 - 4.

Step 5: Moved (old) base [1-2:z104+.] to (new) boundaries 4 - 6.
Step 6: Collapsed (new) base [3-7:z1+.] to the empty base (7,7).
Step 7: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 8: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 4: =0=3*<1=4*<5*<2<3=6*
```

is shown below:



Observe the following facts about this GE: The base [4-5:z101-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. The base [6-7:z104-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 4.

Next, we consider

```
Print 5: =0=3*<1<4*<2=5*<3=6*
```

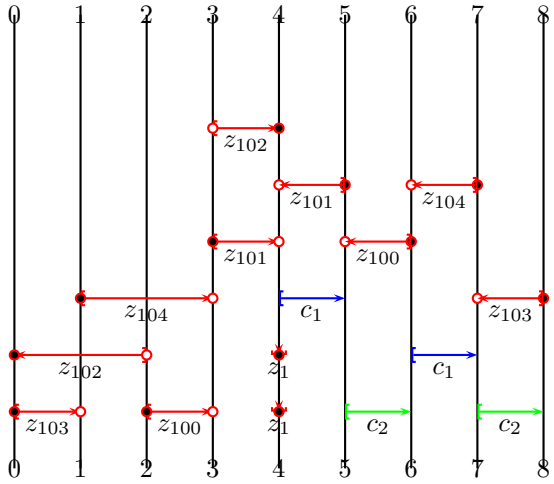
Sequence of Actions in performing the Print 5:
Step 1: Added (new) boundary 4.
Step 2: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 7.
Step 3: Moved (old) base [2-3:z102+.] to (new) boundaries 6 - 7.
Step 4: Moved (old) base [0-1:z103+.] to (new) boundaries 3 - 4.

Step 5: Moved (old) base [1-2:z104+.] to (new) boundaries 4 - 6.
Step 6: Collapsed (new) base [3-7:z1+.] to the empty base (7,7).
Step 7: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 8: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 5: =0=3*<1<4*<2=5*<3=6*
```

is shown below:



Observe the following facts about this GE: The base [6-7:z104-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 5.

Next, we consider

```
Print 6: =0=3*<1<4*<2<5*<3=6*
```

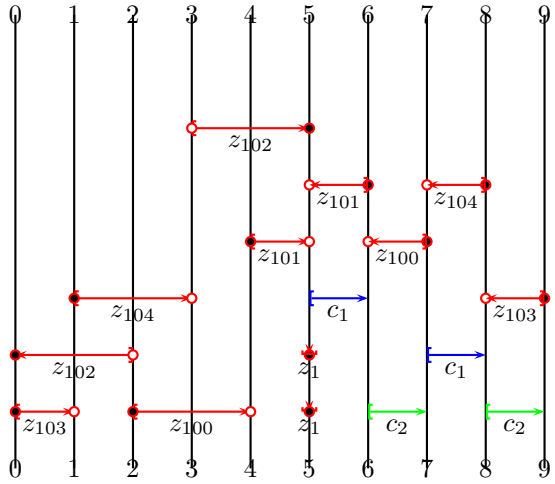
Sequence of Actions in performing the Print 6:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 6.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 8.
Step 4: Moved (old) base [2-3:z102+.] to (new) boundaries 6 - 8.
Step 5: Moved (old) base [0-1:z103+.] to (new) boundaries 3 - 4.
Step 6: Moved (old) base [1-2:z104+.] to (new) boundaries 4 - 6.

Step 7: Collapsed (new) base [3-8:z1+.] to the empty base (8,8).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 6: =0=3*<1<4*<2<5*<3=6*
```

is shown below:



Observe the following facts about this GE: The base [6-7:z100-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. The base [7-8:z104-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 6.

Next, we consider

```
Print 7: =0=3*<1<4*<5*<2<3=6*
```

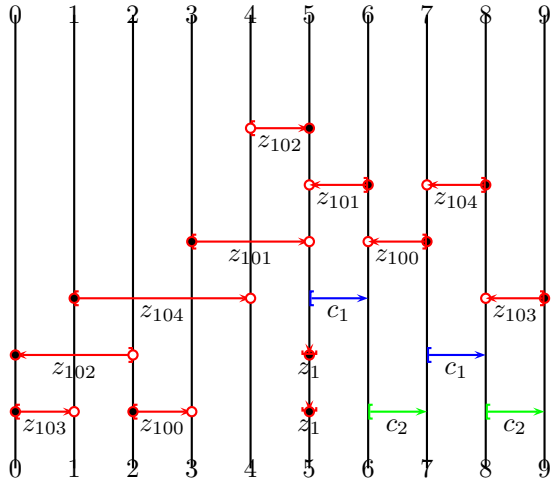
Sequence of Actions in performing the Print 7:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 7.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 8.
Step 4: Moved (old) base [2-3:z102+.] to (new) boundaries 7 - 8.
Step 5: Moved (old) base [0-1:z103+.] to (new) boundaries 3 - 4.

Step 6: Moved (old) base [1-2:z104+.] to (new) boundaries 4 - 7.
Step 7: Collapsed (new) base [3-8:z1+.] to the empty base (8,8).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 7: =0=3*<1<4*<5*<2<3=6*
```

is shown below:



Observe the following facts about this GE: The base [5-6:z101-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [7-8:z104-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 7.

Next, we consider

```
Print 8: =0=3*<4*<1<2=5*<3=6*
```

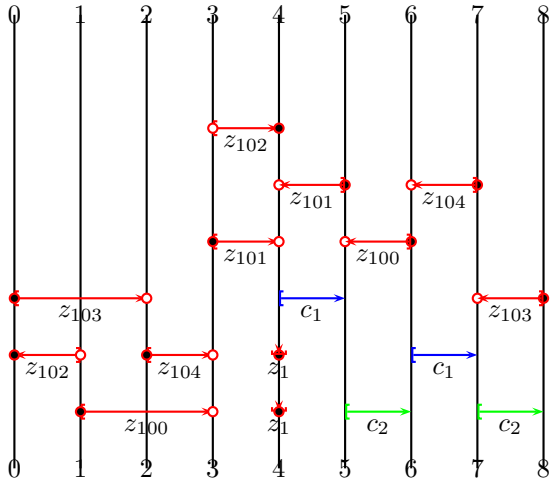
Sequence of Actions in performing the Print 8:
Step 1: Added (new) boundary 5.
Step 2: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 7.
Step 3: Moved (old) base [2-3:z102+.] to (new) boundaries 6 - 7.
Step 4: Moved (old) base [0-1:z103+.] to (new) boundaries 3 - 5.

Step 5: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 6.
Step 6: Collapsed (new) base [3-7:z1+.] to the empty base (7,7).
Step 7: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 8: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 8: =0=3*<4*<1<2=5*<3=6*
```

is shown below:



Observe the following facts about this GE: The base [5-6:z100-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [7-8:z103-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 8.

Next, we consider

```
Print 9: =0=3*<4*<1<2<5*<3=6*
```

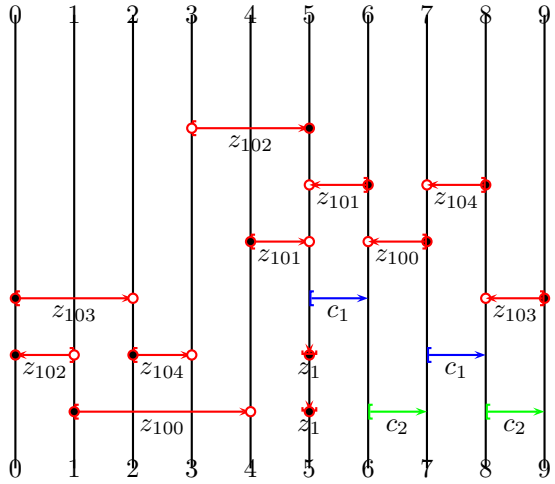
Sequence of Actions in performing the Print 9:
Step 1: Added (new) boundary 5.
Step 2: Added (new) boundary 6.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 8.
Step 4: Moved (old) base [2-3:z102+.] to (new) boundaries 6 - 8.

Step 5: Moved (old) base [0-1:z103+.] to (new) boundaries 3 - 5.
Step 6: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 6.
Step 7: Collapsed (new) base [3-8:z1+.] to the empty base (8,8).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 9: =0=3*<4*<1<2<5*<3=6*
```

is shown below:



Observe the following facts about this GE: The base [6-7:z100-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. The base [8-9:z103-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 9.

Next, we consider

```
Print 10: =0=3*<4*<1=5*<2<3=6*
```

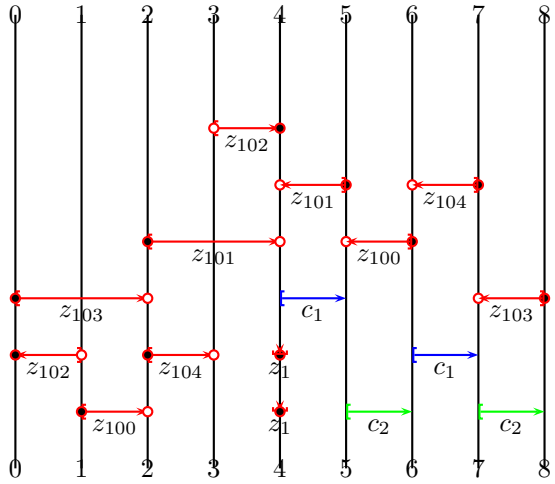
Sequence of Actions in performing the Print 10:
Step 1: Added (new) boundary 6.
Step 2: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 7.
Step 3: Moved (old) base [2-3:z102+.] to (new) boundaries 6 - 7.

Step 4: Moved (old) base [0-1:z103+.] to (new) boundaries 3 - 5.
Step 5: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 6.
Step 6: Collapsed (new) base [3-7:z1+.] to the empty base (7,7).
Step 7: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 8: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 10: =0=3*<4*<1=5*<2<3=6*
```

is shown below:



Observe the following facts about this GE: The base [4-5:z101-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. The base [7-8:z103-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 10.

Next, we consider

```
Print 11: =0=3*<4*<1<5*<2<3=6*
```

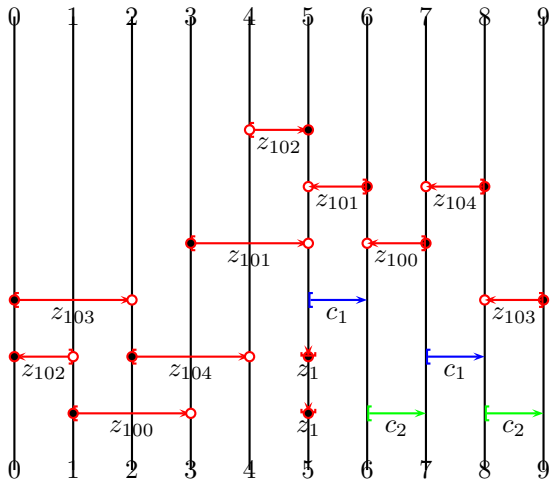
Sequence of Actions in performing the Print 11:
Step 1: Added (new) boundary 5.
Step 2: Added (new) boundary 7.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 8.

Step 4: Moved (old) base [2-3:z102+.] to (new) boundaries 7 - 8.
Step 5: Moved (old) base [0-1:z103+.] to (new) boundaries 3 - 5.
Step 6: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 7.
Step 7: Collapsed (new) base [3-8:z1+.] to the empty base (8,8).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 11: =0=3*<4*<1<5*<2<3=6*
```

is shown below:



Observe the following facts about this GE: The base [5-6:z101-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [6-7:z100-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. The base [7-8:z104-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. The base [8-9:z103-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 11.

Next, we consider

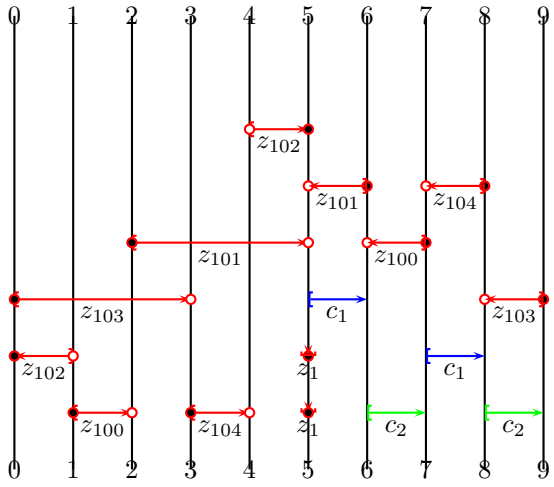
```
Print 12: =0=3*<4*<5*<1<2<3=6*
```

Sequence of Actions in performing the Print 12:
Step 1: Added (new) boundary 6.
Step 2: Added (new) boundary 7.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 8.
Step 4: Moved (old) base [2-3:z102+.] to (new) boundaries 7 - 8.
Step 5: Moved (old) base [0-1:z103+.] to (new) boundaries 3 - 6.
Step 6: Moved (old) base [1-2:z104+.] to (new) boundaries 6 - 7.
Step 7: Collapsed (new) base [3-8:z1+.] to the empty base (8,8).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 12: =0=3*<4*<5*<1<2<3=6*
```

is shown below:

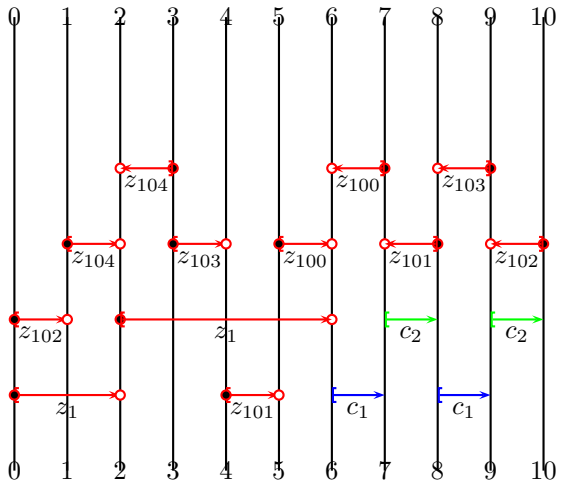


Observe the following facts about this GE: The base [5-6:z101-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [8-9:z103-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 12.

22 Generalized Equation #22

Quadratic System: $z_1z_1c_1c_2c_1c_2 =_F 1$.



GE Information: Carrier: [0-2:z1+.] ; Carrier Dual: [2-6:z1+.] ; Critical Boundary: 2; **Prints**

```
Print 0: =0=2*<1<3*<4*<5*<2=6*
Print 1: =0=2*<3*<1=4*<5*<2=6*
Print 2: =0=2*<3*<1<4*<5*<2=6*
Print 3: =0=2*<3*<4*<1=5*<2=6*
Print 4: =0=2*<3*<4*<1<5*<2=6*
Print 5: =0=2*<3*<4*<5*<1<2=6*
```

Total number of prints: 6
Next, we consider

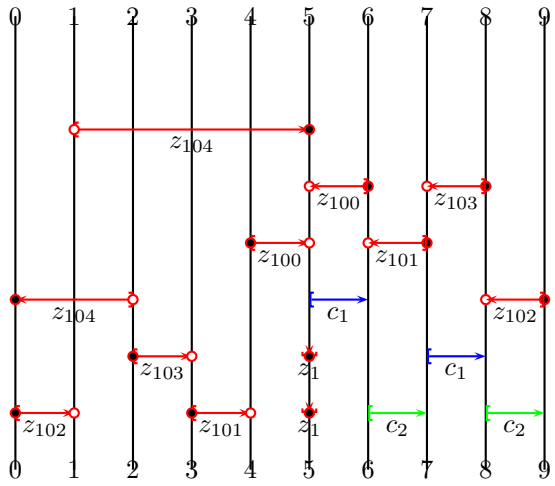
```
Print 1: =0=2*<1<3*<4*<5*<2=6*
```

Sequence of Actions in performing the Print 1:
Step 1: Added (new) boundary 3.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 7.
Step 3: Moved (old) base [0-1:z102+.] to (new) boundaries 2 - 3.
Step 4: Moved (old) base [1-2:z104+.] to (new) boundaries 3 - 7.
Step 5: Collapsed (new) base [2-7:z1+.] to the empty base (7,7).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=2*<1<3*<4*<5*<2=6*
```

is shown below:



Observe the following facts about this GE: The base [1-5:z104+.] and its dual are of opposite polarity, yet intersect. The base [0-2:z104-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

Next, we consider

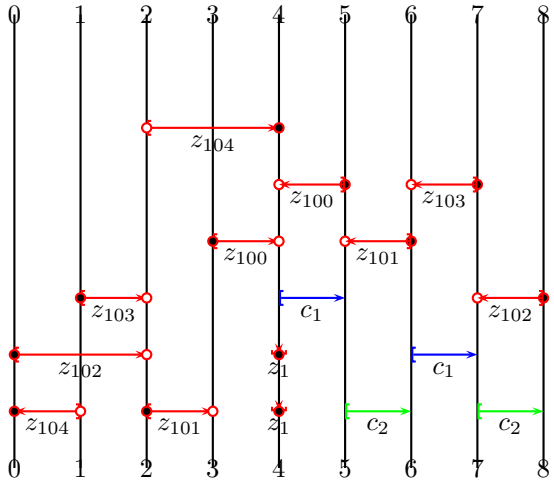
```
Print 2: =0=2*<3*<1=4*<5*<2=6*
```

Sequence of Actions in performing the Print 2:
Step 1: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 6.
Step 2: Moved (old) base [0-1:z102+.] to (new) boundaries 2 - 4.
Step 3: Moved (old) base [1-2:z104+.] to (new) boundaries 4 - 6.
Step 4: Collapsed (new) base [2-6:z1+.] to the empty base (6,6).
Step 5: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 6: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 2: =0=2*<3*<1=4*<5*<2=6*
```

is shown below:



Observe the following facts about this GE: The base [7-8:z102-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 2.

Next, we consider

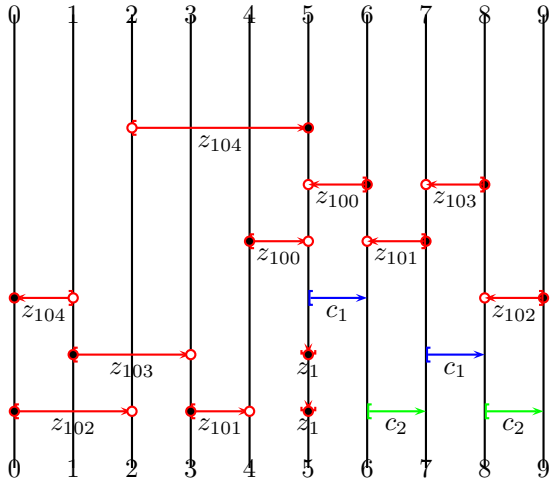
```
Print 3: =0=2*<3*<1<4*<5*<2=6*
```

Sequence of Actions in performing the Print 3:
Step 1: Added (new) boundary 4.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 7.
Step 3: Moved (old) base [0-1:z102+.] to (new) boundaries 2 - 4.
Step 4: Moved (old) base [1-2:z104+.] to (new) boundaries 4 - 7.
Step 5: Collapsed (new) base [2-7:z1+.] to the empty base (7,7).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 3: =0=2*<3*<1<4*<5*<2=6*
```

is shown below:



Observe the following facts about this GE: The base [7-8:z103-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. The base [8-9:z102-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 3.

Next, we consider

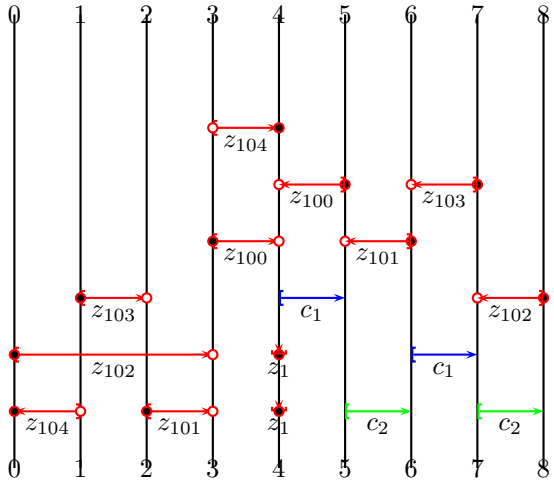
```
Print 4: =0=2*<3*<4*<1=5*<2=6*
```

Sequence of Actions in performing the Print 4:
Step 1: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 6.
Step 2: Moved (old) base [0-1:z102+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 6.
Step 4: Collapsed (new) base [2-6:z1+.] to the empty base (6,6).
Step 5: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 6: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 4: =0=2*<3*<4*<1=5*<2=6*
```

is shown below:



Observe the following facts about this GE: The base [7-8:z102-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 4.

Next, we consider

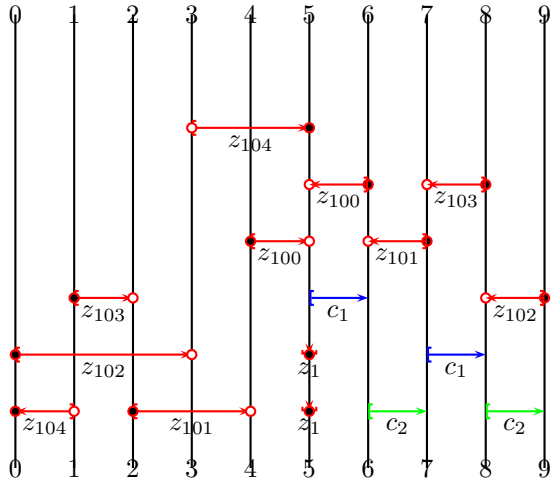
```
Print 5: =0=2*<3*<4*<1<5*<2=6*
```

Sequence of Actions in performing the Print 5:
Step 1: Added (new) boundary 5.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 7.
Step 3: Moved (old) base [0-1:z102+.] to (new) boundaries 2 - 5.
Step 4: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 7.
Step 5: Collapsed (new) base [2-7:z1+.] to the empty base (7,7).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 5: =0=2*<3*<4*<1<5*<2=6*
```

is shown below:



Observe the following facts about this GE: The base [6-7:z101-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. The base [8-9:z102-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 5.

Next, we consider

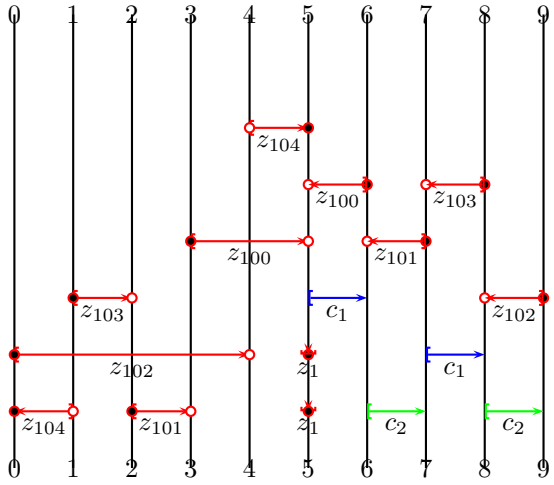
```
Print 6: =0=2*<3*<4*<5*<1<2=6*
```

Sequence of Actions in performing the Print 6:
Step 1: Added (new) boundary 6.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 7.
Step 3: Moved (old) base [0-1:z102+.] to (new) boundaries 2 - 6.
Step 4: Moved (old) base [1-2:z104+.] to (new) boundaries 6 - 7.
Step 5: Collapsed (new) base [2-7:z1+.] to the empty base (7,7).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 6: =0=2*<3*<4*<5*<1<2=6*
```

is shown below:



Observe the following facts about this GE: The base [5-6:z100-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [8-9:z102-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 6.

23 Acknowledgements

The authors acknowledge that this report was generated by software developed as part of a funded project supported by a research grant (H98230-06-1-0042) from the National Security Agency. We also give special thanks to Alexei Miasnikov and Olga Kharlampovich for many helpful discussions along the way.