The Cancellation Diagrams of $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$ in a Free Group

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The purpose of this report is to demonstrate the logging facilities of the MKSolver. In particular, here we see that the MKSolver is able to accurately enumerate distinct cancellation diagrams corresponding to an equation over a free group.In this manner, the problem of solving equations in free groups, reduces to finding a solution in the free *semigroup* of one of finitely many systems of constraints. This idea dates back to Makanin (1977) in his discussion of "partition tables".We report on the cancellation diagrams of $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$, which can be re-expressed as a quadratic system $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$.. Below, we list the possible cancellation diagrams and then represent each as a generalized equation (GE).

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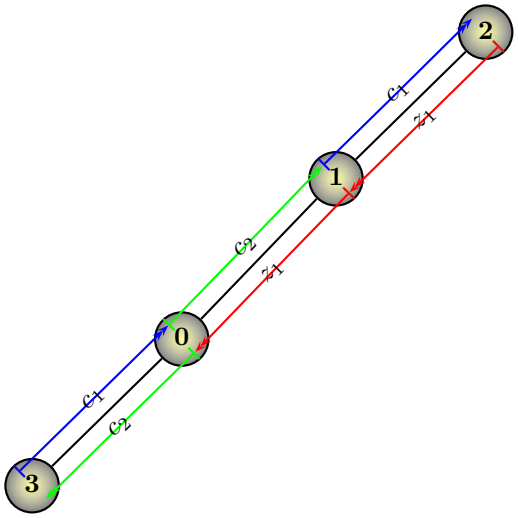
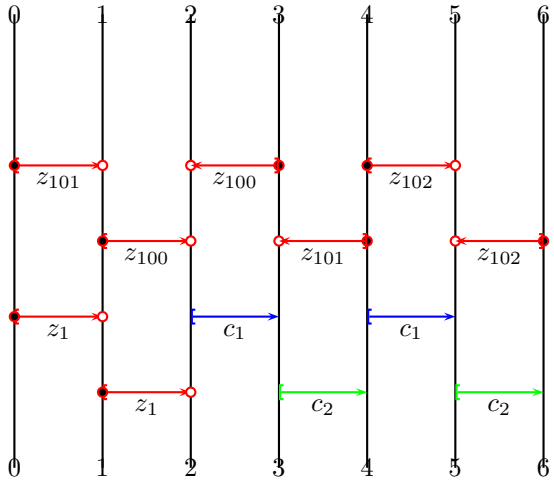
[†]This report was generated automatically by software developed with support from the National Security Agency Grant H98230-06-1-0042.

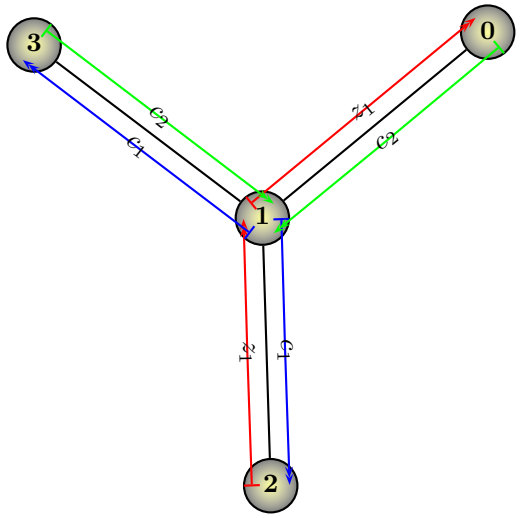
Cancellation diagram root.1.1.1.1.1:



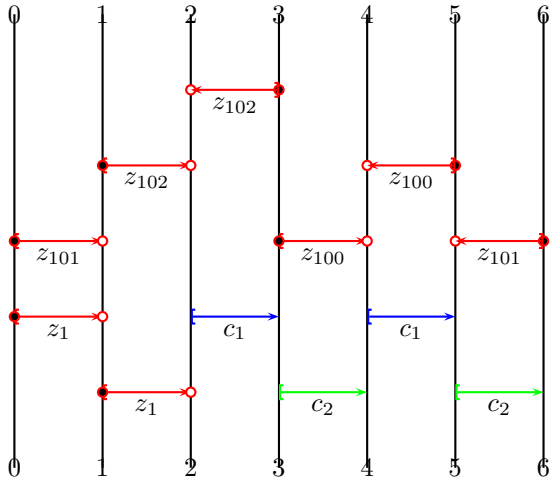
z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$1 \leftarrow 2$
c_2	$0 \leftarrow 1$
c_1	$3 \leftarrow 0$
c_2	$0 \leftarrow 3$



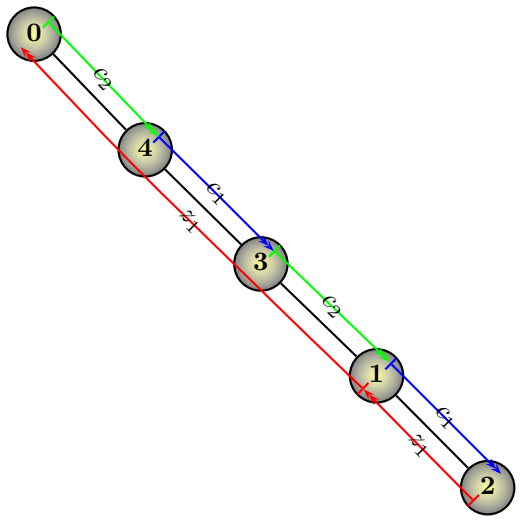
Cancellation diagram root.1.1.1.2.2:



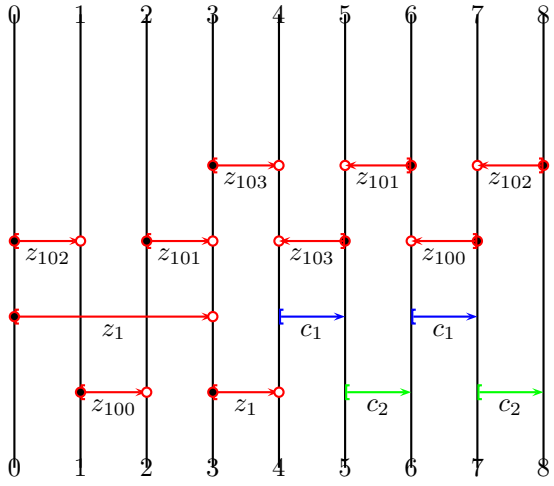
z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$1 \leftarrow 2$
c_2	$3 \leftarrow 1$
c_1	$1 \leftarrow 3$
c_2	$0 \leftarrow 1$



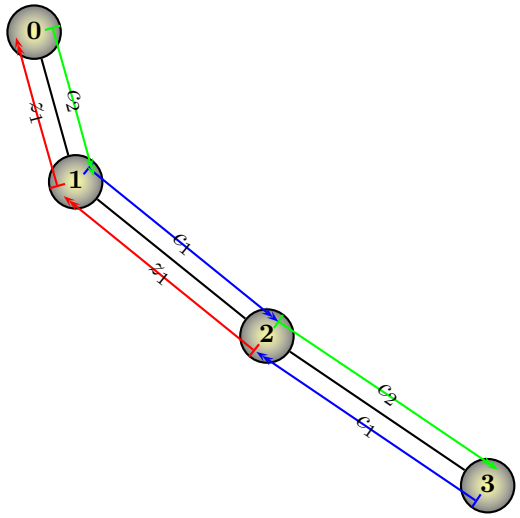
Cancellation diagram root.1.1.1.3.3:



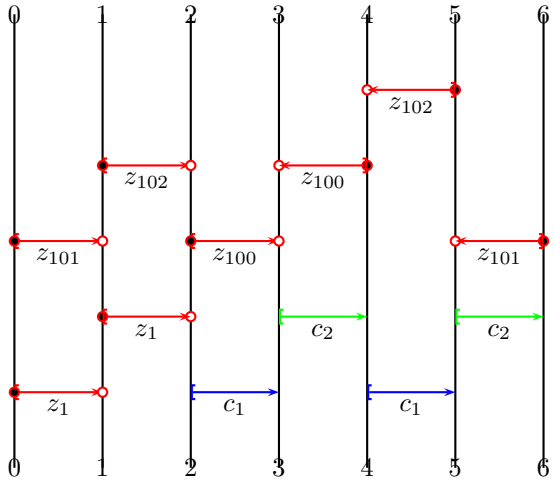
z_1	$1 \leftarrow 3 \leftarrow 4 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$1 \leftarrow 2$
c_2	$3 \leftarrow 1$
c_1	$4 \leftarrow 3$
c_2	$0 \leftarrow 4$



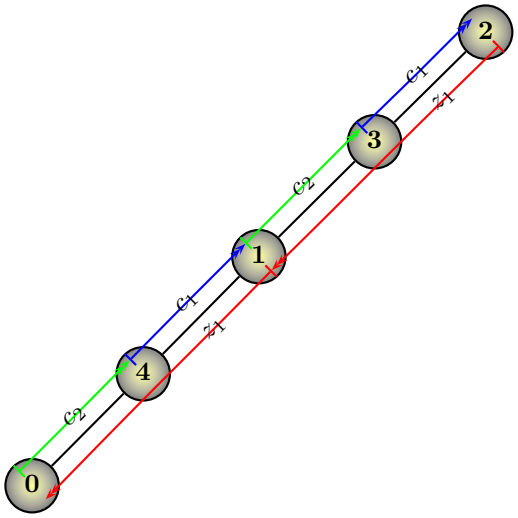
Cancellation diagram root.1.1.2.1.1:



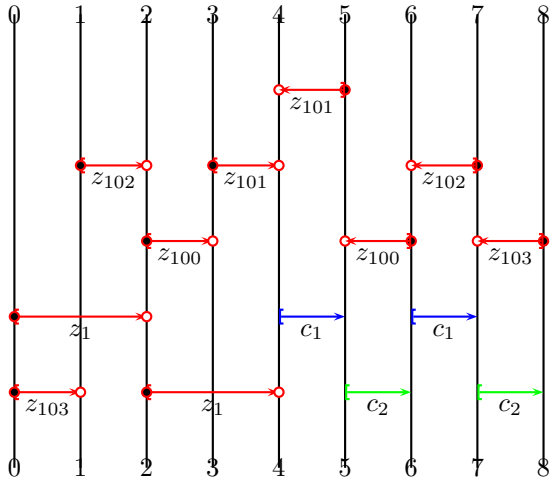
z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$2 \leftarrow 3$
c_1	$1 \leftarrow 2$
c_2	$0 \leftarrow 1$



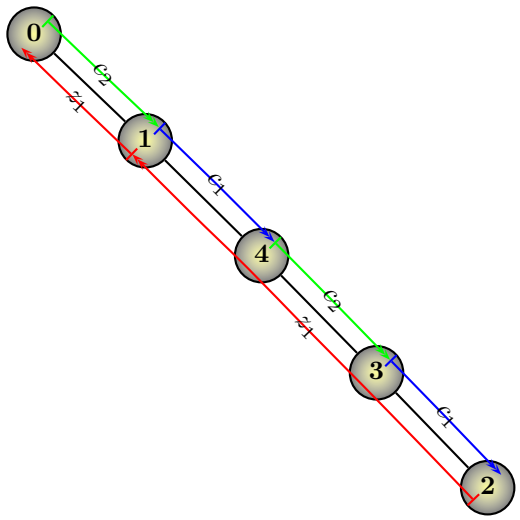
Cancellation diagram root.1.1.3.1.3:



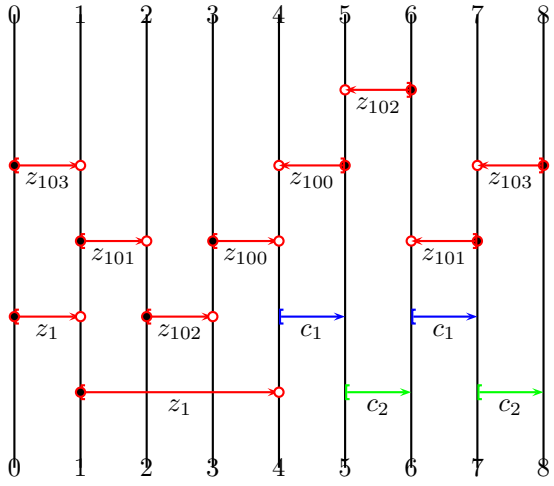
z_1	$1 \leftarrow 4 \leftarrow 0$
z_1	$2 \leftarrow 3 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$1 \leftarrow 3$
c_1	$4 \leftarrow 1$
c_2	$0 \leftarrow 4$



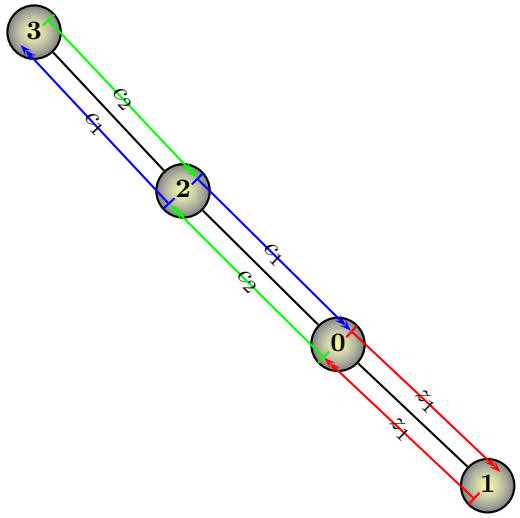
Cancellation diagram root.1.1.3.3.1:



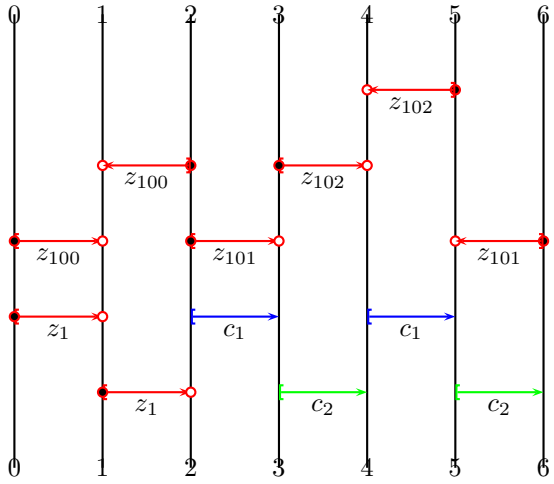
z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 3 \leftarrow 4 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$4 \leftarrow 3$
c_1	$1 \leftarrow 4$
c_2	$0 \leftarrow 1$



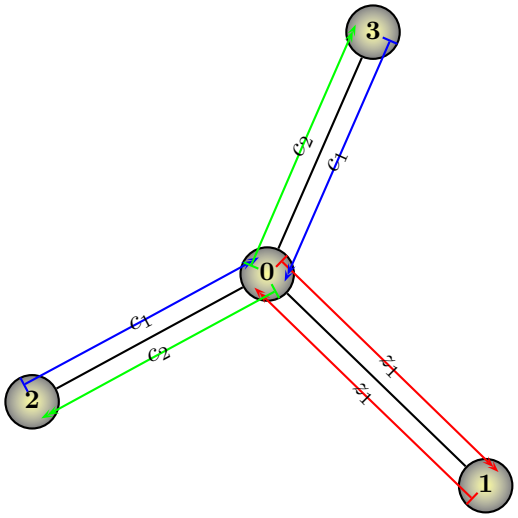
Cancellation diagram root.1.2.1.1.1:



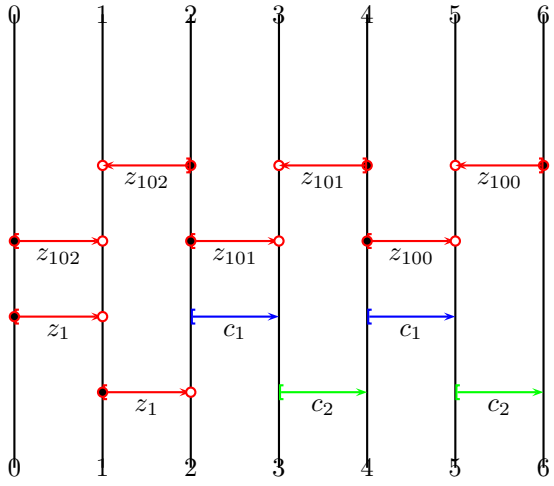
z_1	$1 \leftarrow 0$
z_1	$0 \leftarrow 1$
c_1	$2 \leftarrow 0$
c_2	$3 \leftarrow 2$
c_1	$2 \leftarrow 3$
c_2	$0 \leftarrow 2$



Cancellation diagram root.1.2.1.2.1:



z_1	$1 \leftarrow 0$
z_1	$0 \leftarrow 1$
c_1	$2 \leftarrow 0$
c_2	$0 \leftarrow 2$
c_1	$3 \leftarrow 0$
c_2	$0 \leftarrow 3$



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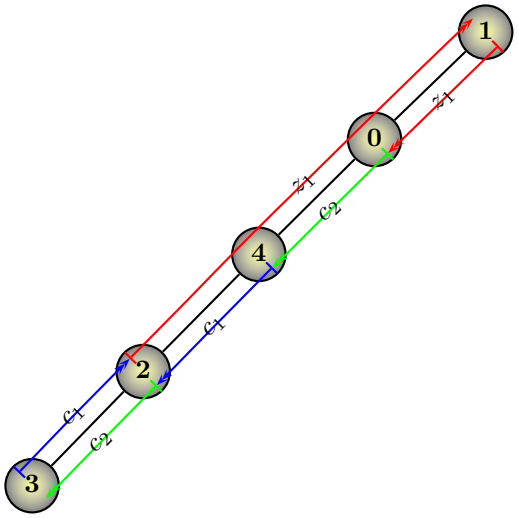
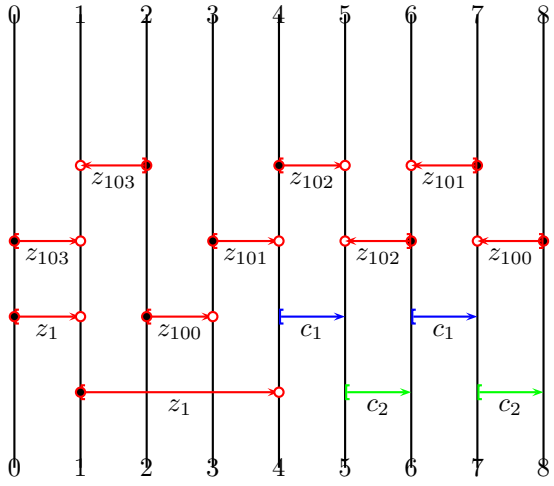
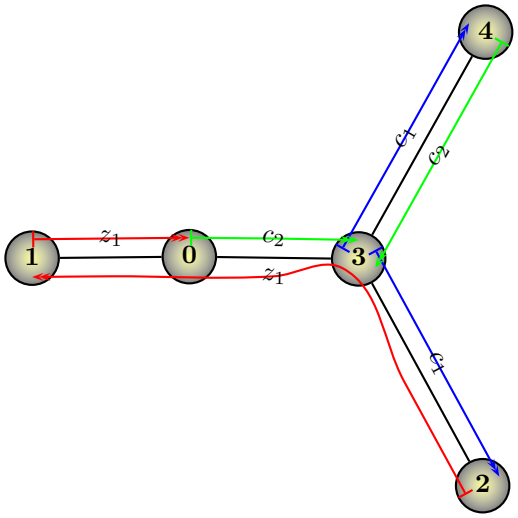


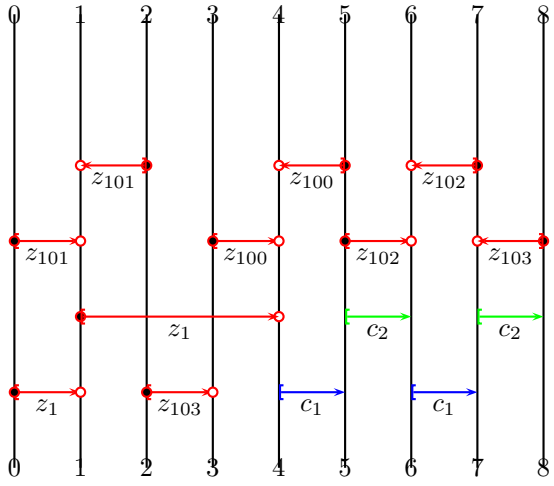
z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 4 \leftarrow 0 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$2 \leftarrow 3$
c_1	$4 \leftarrow 2$
c_2	$0 \leftarrow 4$



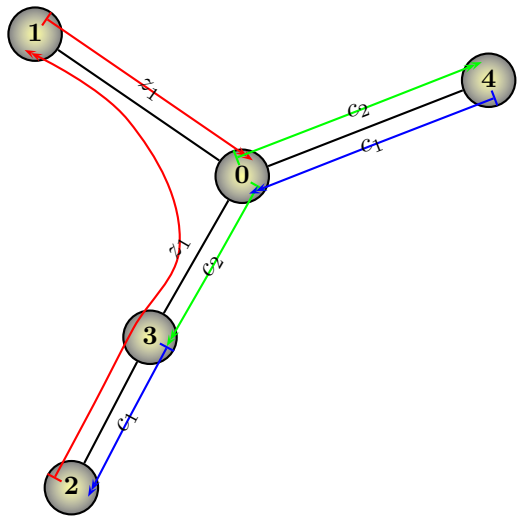
Cancellation diagram root.1.3.3.1.1:



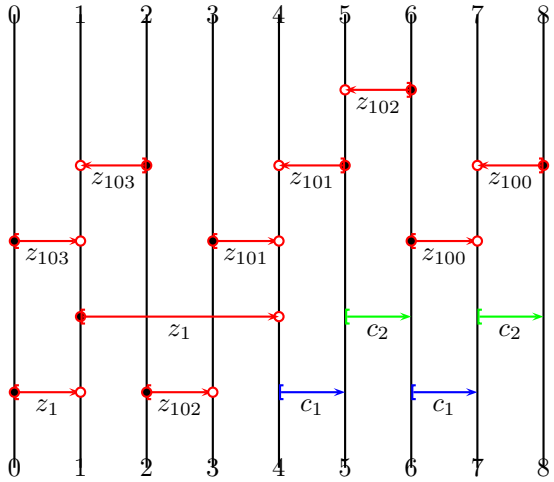
z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 3 \leftarrow 0 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$4 \leftarrow 3$
c_1	$3 \leftarrow 4$
c_2	$0 \leftarrow 3$



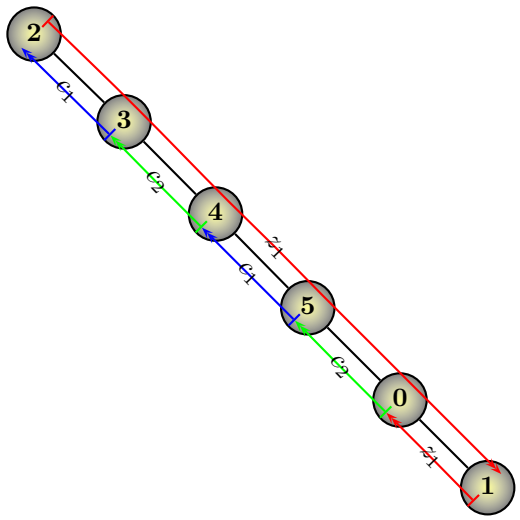
Cancellation diagram root.1.3.3.2.1:



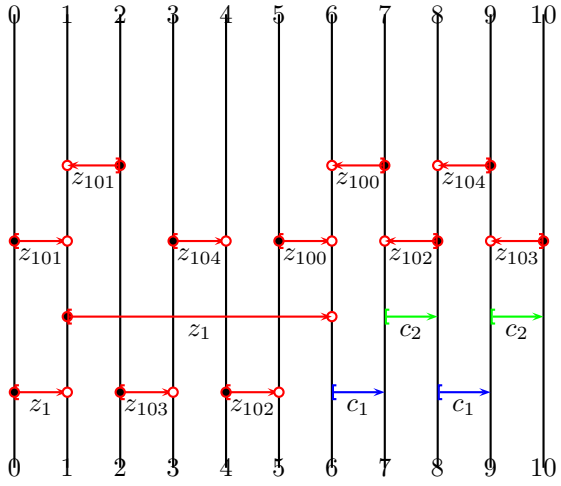
z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 3 \leftarrow 0 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$0 \leftarrow 3$
c_1	$4 \leftarrow 0$
c_2	$0 \leftarrow 4$



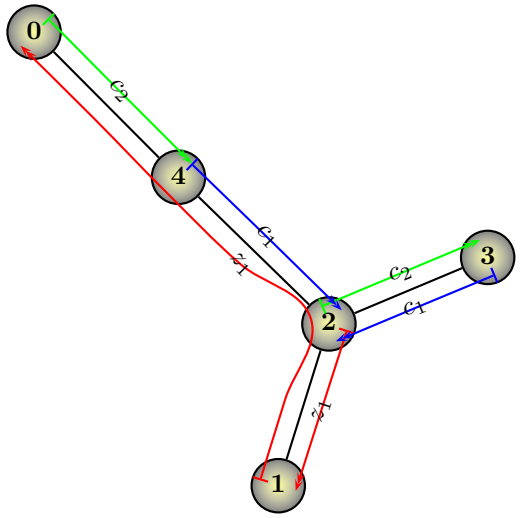
Cancellation diagram root.1.3.3.3.3:



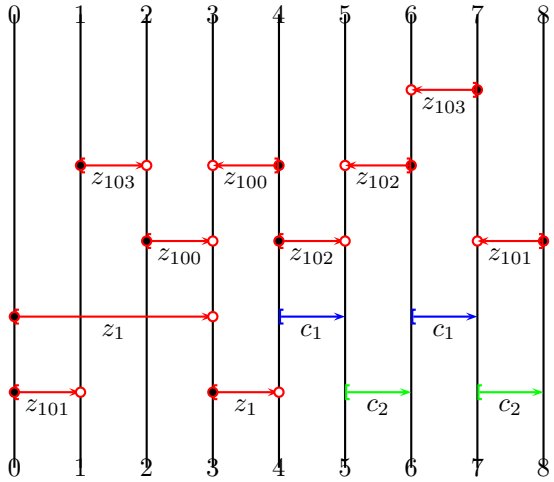
z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 3 \leftarrow 4 \leftarrow 5 \leftarrow 0 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$4 \leftarrow 3$
c_1	$5 \leftarrow 4$
c_2	$0 \leftarrow 5$



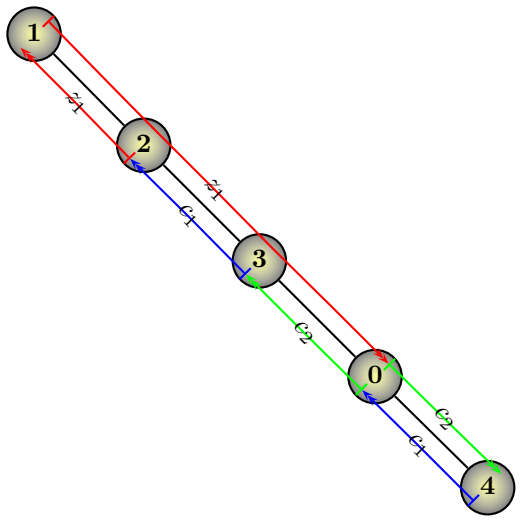
Cancellation diagram root.1.4.1.2.3:



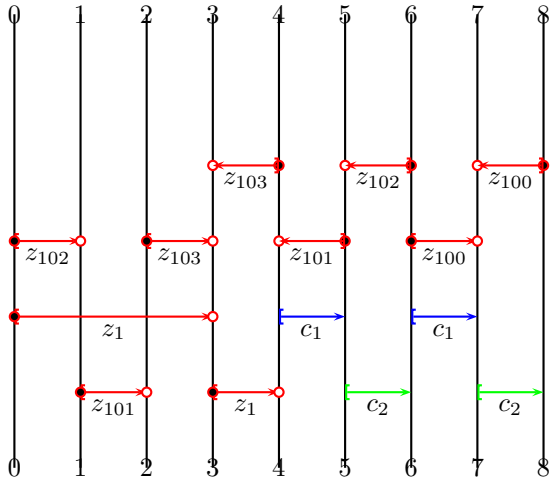
z_1	$1 \leftarrow 2 \leftarrow 4 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$2 \leftarrow 3$
c_1	$4 \leftarrow 2$
c_2	$0 \leftarrow 4$



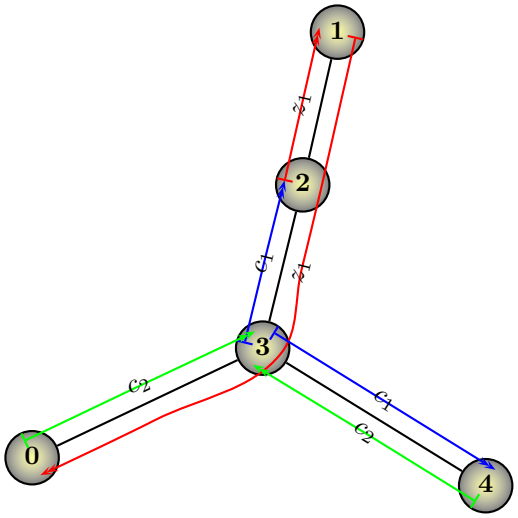
Cancellation diagram root.1.4.3.1.1:



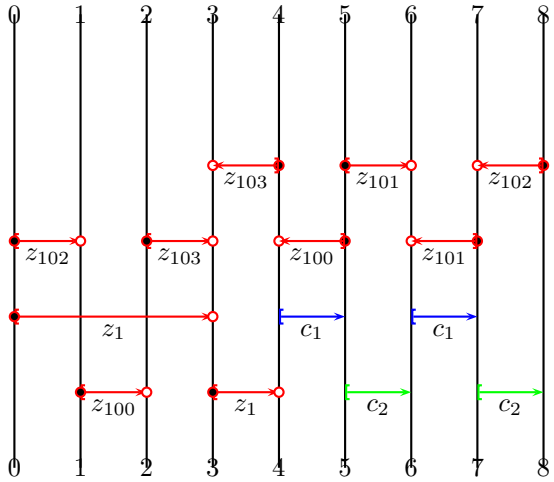
z_1	$1 \leftarrow 2 \leftarrow 3 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$0 \leftarrow 3$
c_1	$4 \leftarrow 0$
c_2	$0 \leftarrow 4$



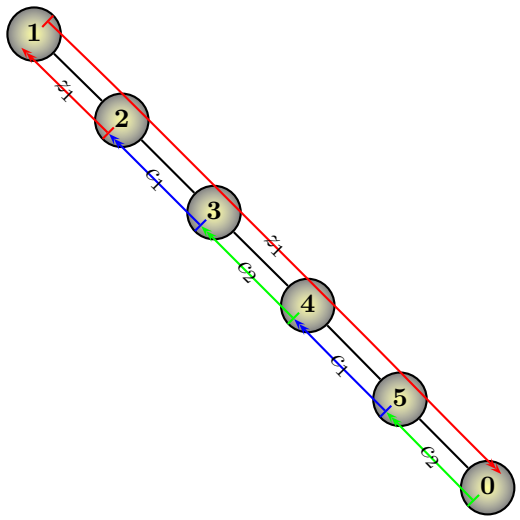
Cancellation diagram root.1.4.3.2.1:



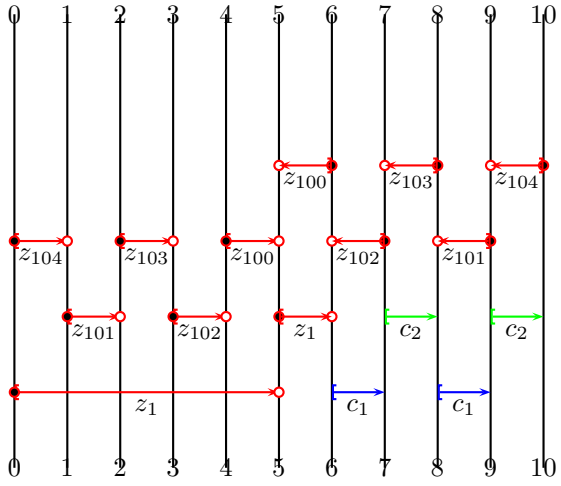
z_1	$1 \leftarrow 2 \leftarrow 3 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$4 \leftarrow 3$
c_1	$3 \leftarrow 4$
c_2	$0 \leftarrow 3$



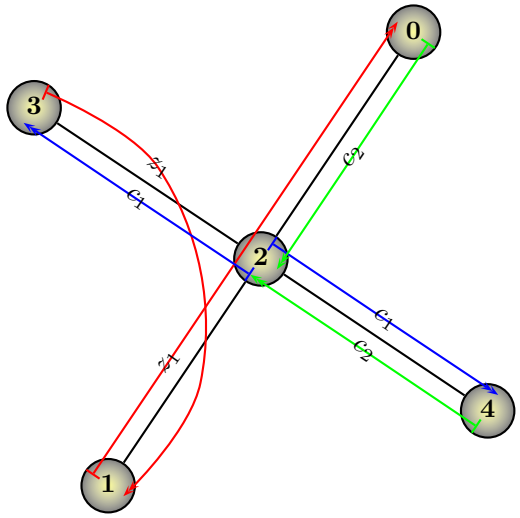
Cancellation diagram root.1.4.3.3.3:



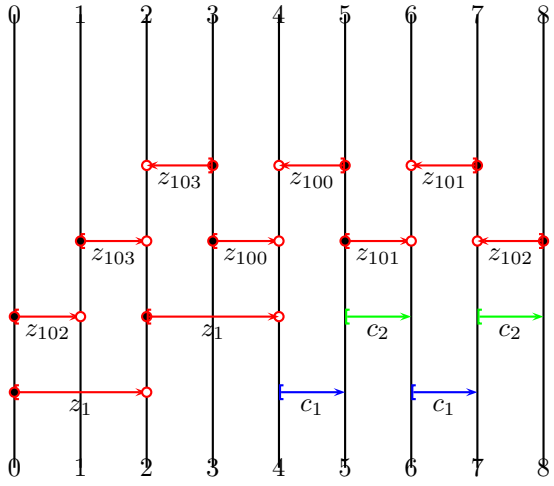
z_1	$1 \leftarrow 2 \leftarrow 3 \leftarrow 4 \leftarrow 5 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$4 \leftarrow 3$
c_1	$5 \leftarrow 4$
c_2	$0 \leftarrow 5$



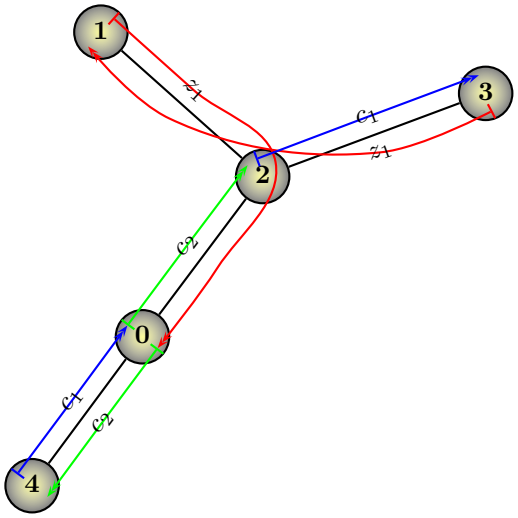
Cancellation diagram root.1.5.1.1.2:



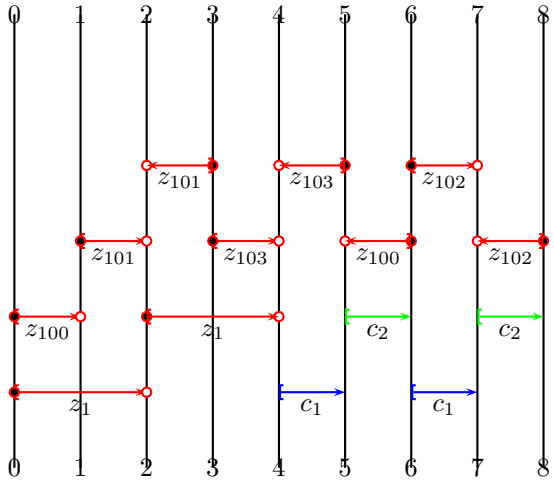
z_1	$1 \leftarrow 2 \leftarrow 0$
z_1	$3 \leftarrow 2 \leftarrow 1$
c_1	$2 \leftarrow 3$
c_2	$4 \leftarrow 2$
c_1	$2 \leftarrow 4$
c_2	$0 \leftarrow 2$



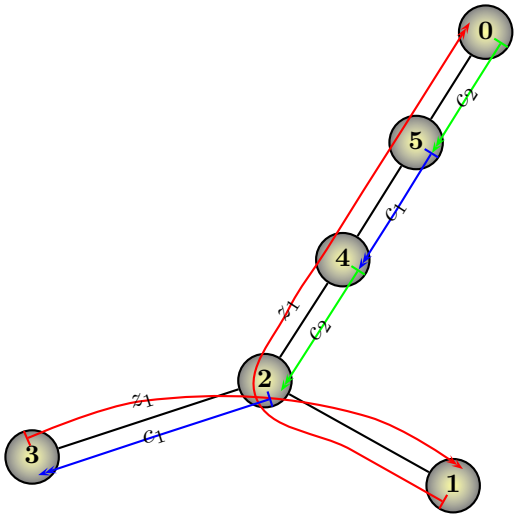
Cancellation diagram root.1.5.1.2.1:



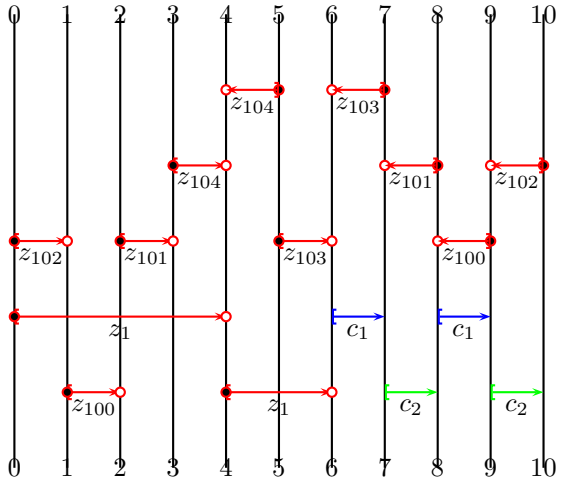
z_1	$1 \leftarrow 2 \leftarrow 0$
z_1	$3 \leftarrow 2 \leftarrow 1$
c_1	$2 \leftarrow 3$
c_2	$0 \leftarrow 2$
c_1	$4 \leftarrow 0$
c_2	$0 \leftarrow 4$



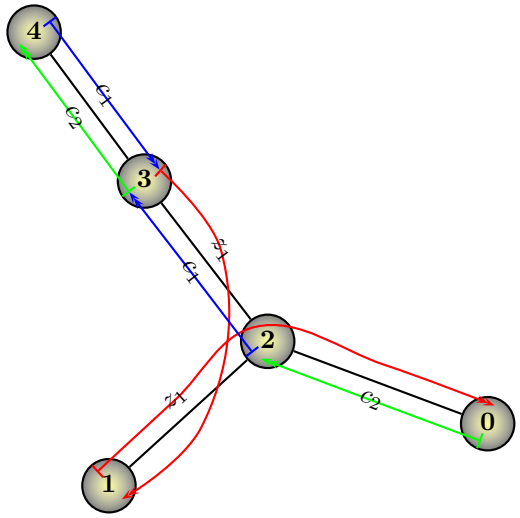
Cancellation diagram root.1.5.1.3.3:



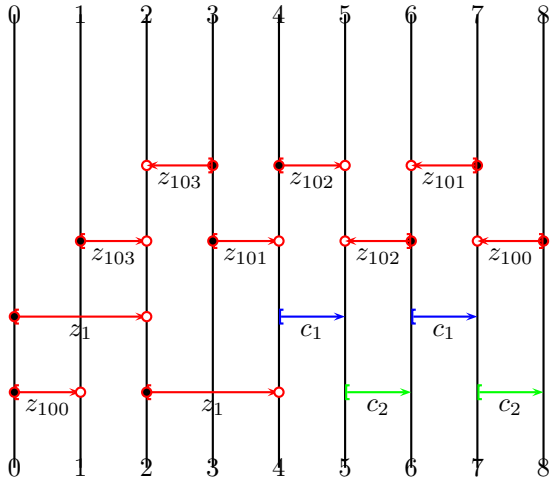
z_1	$1 \leftarrow 2 \leftarrow 4 \leftarrow 5 \leftarrow 0$
z_1	$3 \leftarrow 2 \leftarrow 1$
c_1	$2 \leftarrow 3$
c_2	$4 \leftarrow 2$
c_1	$5 \leftarrow 4$
c_2	$0 \leftarrow 5$



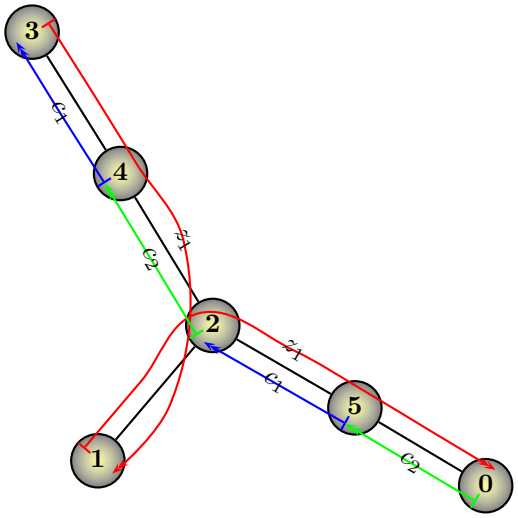
Cancellation diagram root.1.5.2.1.1:



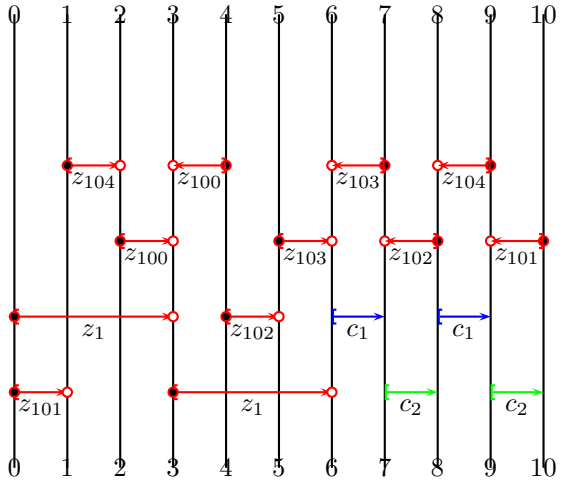
z_1	$1 \leftarrow 2 \leftarrow 0$
z_1	$3 \leftarrow 2 \leftarrow 1$
c_1	$4 \leftarrow 3$
c_2	$3 \leftarrow 4$
c_1	$2 \leftarrow 3$
c_2	$0 \leftarrow 2$



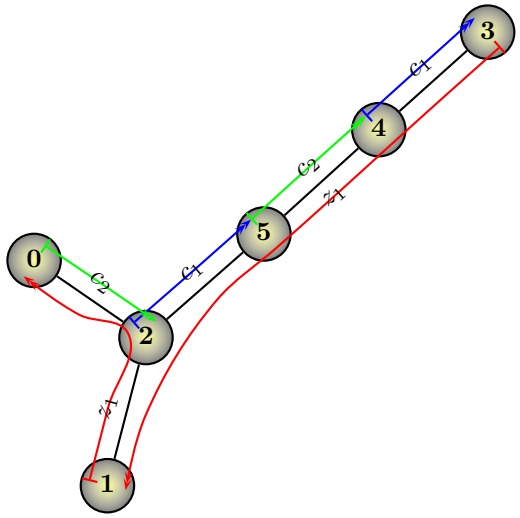
Cancellation diagram root.1.5.3.2.3:



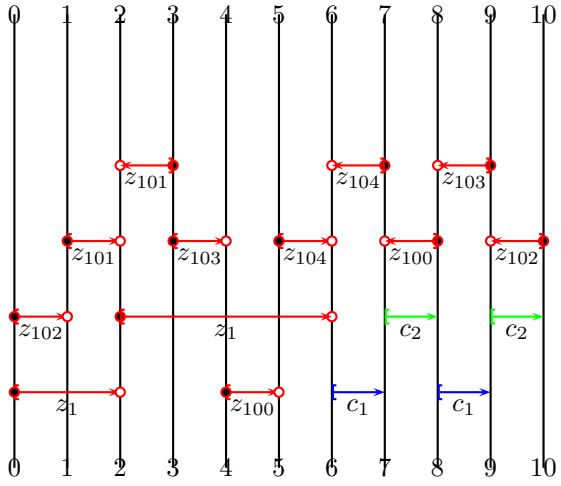
z_1	$1 \leftarrow 2 \leftarrow 5 \leftarrow 0$
z_1	$3 \leftarrow 4 \leftarrow 2 \leftarrow 1$
c_1	$4 \leftarrow 3$
c_2	$2 \leftarrow 4$
c_1	$5 \leftarrow 2$
c_2	$0 \leftarrow 5$



Cancellation diagram root.1.5.3.3.1:



z_1	$1 \leftarrow 2 \leftarrow 0$
z_1	$3 \leftarrow 4 \leftarrow 5 \leftarrow 2 \leftarrow 1$
c_1	$4 \leftarrow 3$
c_2	$5 \leftarrow 4$
c_1	$2 \leftarrow 5$
c_2	$0 \leftarrow 2$



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