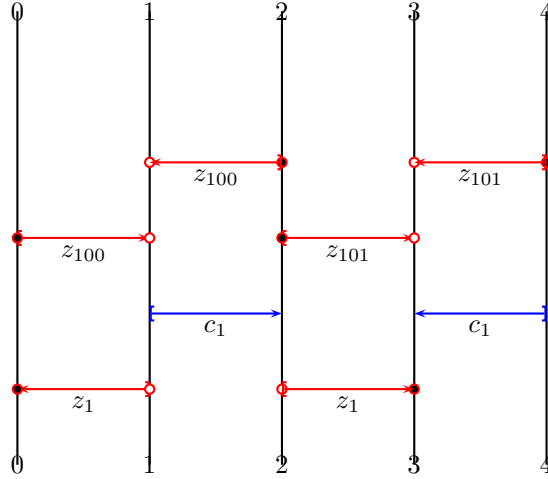# The Prints of the Generalized Equations of
$z_1^{-1}c_1z_1c_1^{-1} =_F 1$
in a Free Group

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1   Generalized Equation #1

Quadratic System: $z_1^{-1}c_1z_1c_1^{-1} =_F 1$.

GE Information: Carrier: [0-1:z1-.] ; Carrier Dual: [2-3:z1+.] ; Critical Boundary: 1; **Prints**

```
Print 0: =0=3*<1=2*
```

Total number of prints: 1
Next, we consider

```
Print 1: =0=3*<1=2*
```

Sequence of Actions in performing the Print 1:

Step 1: Moved (old) base [0-1:z1-.] to (new) boundaries 3 - 2.

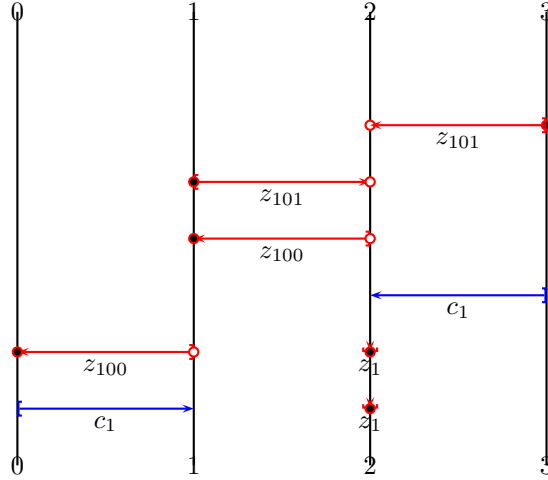Step 2: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 2.

Step 3: Collapsed (new) base [2-3:z1+.] to the empty base (3,3).

Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying
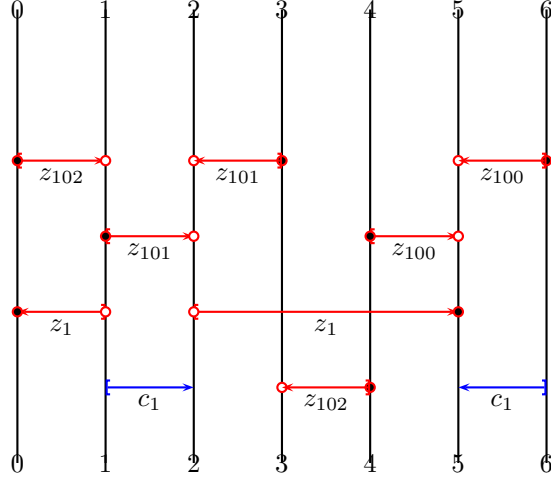
```
Print 1: =0=3*<1=2*
```

is shown below:

The GE above is non-degenerate.

This completes the consideration of Print 1.

2 Generalized Equation #2

Quadratic System: $z_1^{-1}c_1z_1c_1^{-1} =_F 1$.

GE Information: Carrier: [0-1:z1-.] ; Carrier Dual: [2-5:z1+.] ; Critical Boundary: 1; **Prints**

```
Print 0: =0=5*<4*<3*<1=2*
```

Total number of prints: 1
Next, we consider

```
Print 1: =0=5*<4*<3*<1=2*
```

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1-.] to (new) boundaries 5 - 2.
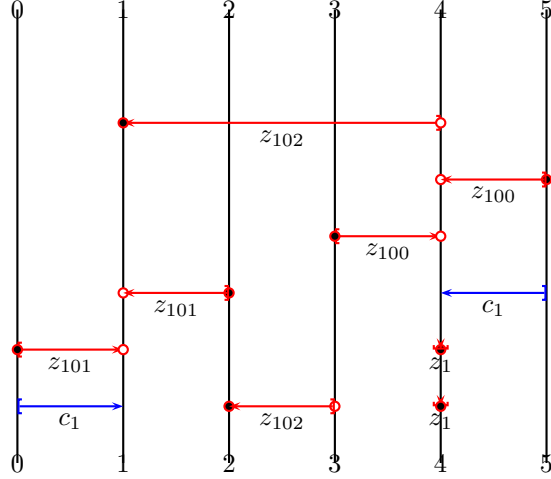Step 2: Moved (old) base [0-1:z102+.] to (new) boundaries 5 - 2.
Step 3: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying
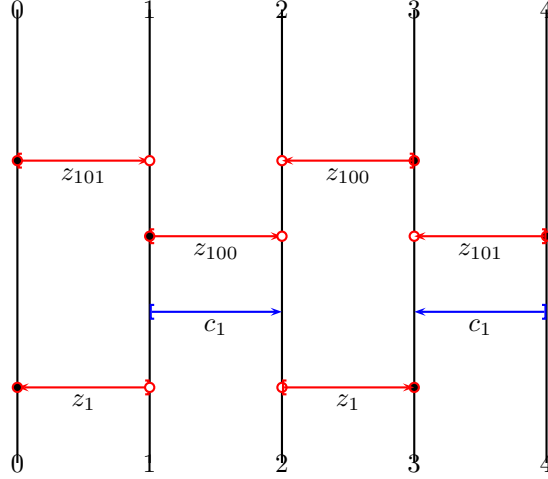
```
Print 1: =0=5*<4*<3*<1=2*
```

is shown below:

Observe the following facts about this GE: The base [1-4:z102-.] and its dual are of the same polarity, yet one properly contains the other. The base [2-3:z102-.] and its dual are of the same polarity, yet one properly contains the other. These observations show that the GE above is degenerate.

This completes the consideration of Print 1.

3 Generalized Equation #3

Quadratic System: $z_1^{-1} c_1 z_1 c_1^{-1} =_F 1$.

0 1 2 3 4

z_{101} z_{100}

z_{100} z_{101}

c_1 c_1

z_1 z_1

0 1 2 3 4

GE Information: Carrier: [0-1:z1-.] ; Carrier Dual: [2-3:z1+.] ; Critical Boundary: 1; **Prints**

    Print 0: =0=3*<1=2*

Total number of prints: 1
Next, we consider

    Print 1: =0=3*<1=2*

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1-.] to (new) boundaries 3 - 2.
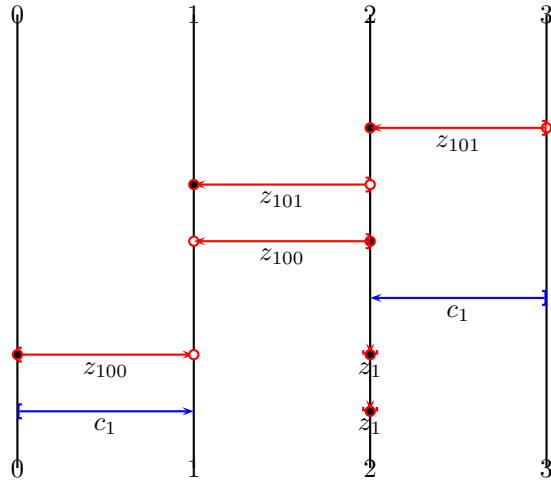Step 2: Moved (old) base [0-1:z101+.] to (new) boundaries 3 - 2.
Step 3: Collapsed (new) base [2-3:z1+.] to the empty base (3,3).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

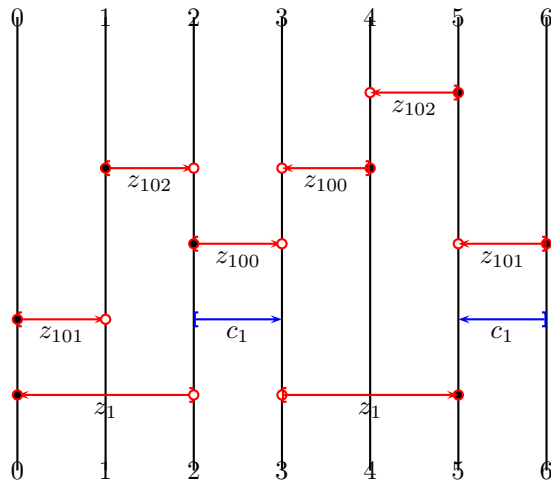   Print 1: =0=3*<1=2*

is shown below:

The GE above is non-degenerate.

This completes the consideration of Print 1.

4 Generalized Equation #4

Quadratic System: $z_1^{-1} c_1 z_1 c_1^{-1} =_F 1$.

GE Information: Carrier: [0-2:z1-.] ; Carrier Dual: [3-5:z1+.] ; Critical Boundary: 2; **Prints**

```
Print 0: =0=5*<1=4*<2=3*
Print 1: =0=5*<1<4*<2=3*
Print 2: =0=5*<4*<1<2=3*
```

Total number of prints: 3

Next, we consider

```
Print 1: =0=5*<1=4*<2=3*
```

Sequence of Actions in performing the Print 1:

Step 1: Moved (old) base [0-2:z1-.] to (new) boundaries 5 - 3.

Step 2: Moved (old) base [0-1:z101+.] to (new) boundaries 5 - 4.

Step 3: Moved (old) base [1-2:z102+.] to (new) boundaries 4 - 3.

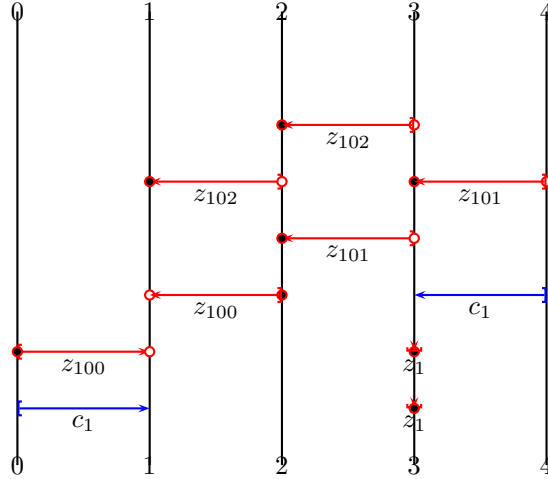Step 4: Collapsed (new) base [3-5:z1+.] to the empty base (5,5).

Step 5: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Step 6: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=5*<1=4*<2=3*
```

is shown below:

The GE above is non-degenerate.

This completes the consideration of Print 1.

Next, we consider

```
Print 2: =0=5*<1<4*<2=3*
```

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Sequence of Actions in performing the Print 2:
Step 1: Added (new) boundary 5.
Step 2: Moved (old) base [0-2:z1-.] to (new) boundaries 6 - 3.
Step 3: Moved (old) base [0-1:z101+.] to (new) boundaries 6 - 5.
Step 4: Moved (old) base [1-2:z102+.] to (new) boundaries 5 - 3.
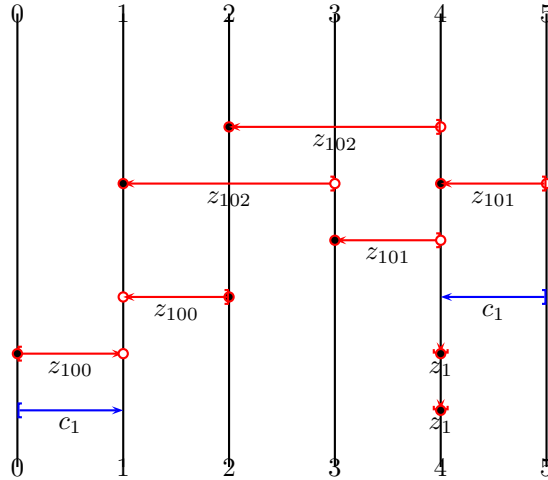Step 5: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 2: =0=5*<1<4*<2=3*
```

is shown below:

The GE above is non-degenerate.

This completes the consideration of Print 2.

Next, we consider

```
Print 3: =0=5*<4*<1<2=3*
```

Sequence of Actions in performing the Print 3:
Step 1: Added (new) boundary 4.
Step 2: Moved (old) base [0-2:z1-.] to (new) boundaries 6 - 3.
Step 3: Moved (old) base [0-1:z101+.] to (new) boundaries 6 - 4.
Step 4: Moved (old) base [1-2:z102+.] to (new) boundaries 4 - 3.
Step 5: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This
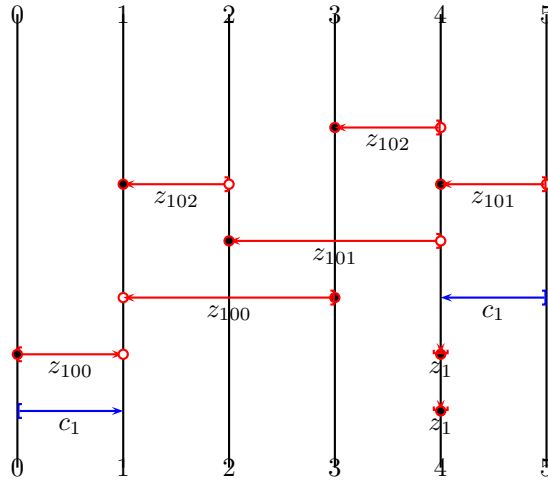
will cause renumbering of higher numbered boundaries.

Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 3: =0=5*<4*<1<2=3*
```

is shown below:

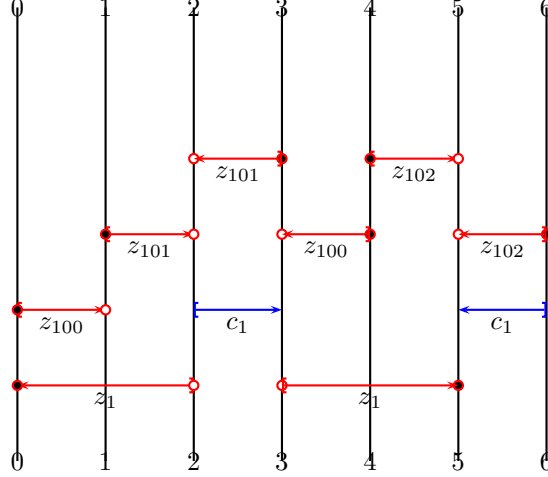Observe the following facts about this GE: The base [0-1:z100+.] has constraints with its dual that stretch the constant segment 0 - 1 to length different from 1. The base [4-5:z101-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 3.

5   Generalized Equation #5

Quadratic System: $z_1^{-1} c_1 z_1 c_1^{-1} =_F 1$.

GE Information: Carrier: [0-2:z1-.] ; Carrier Dual: [3-5:z1+.] ; Critical Boundary: 2; **Prints**

```
Print 0: =0=5*<1=4*<2=3*
Print 1: =0=5*<1<4*<2=3*
Print 2: =0=5*<4*<1<2=3*
```

Total number of prints: 3
Next, we consider

```
Print 1: =0=5*<1=4*<2=3*
```

Sequence of Actions in performing the Print 1:
Step 1: Moved (old) base [0-2:z1-.] to (new) boundaries 5 - 3.
Step 2: Moved (old) base [0-1:z100+.] to (new) boundaries 5 - 4.
Step 3: Moved (old) base [1-2:z101+.] to (new) boundaries 4 - 3.
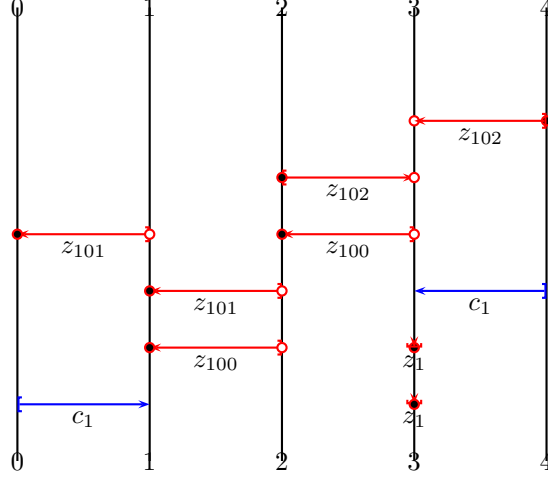Step 4: Collapsed (new) base [3-5:z1+.] to the empty base (5,5).
Step 5: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 6: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=5*<1=4*<2=3*
```

is shown below:

The GE above is non-degenerate.

This completes the consideration of Print 1.

Next, we consider

```
Print 2: =0=5*<1<4*<2=3*
```

Sequence of Actions in performing the Print 2:
Step 1: Added (new) boundary 5.
Step 2: Moved (old) base [0-2:z1-.] to (new) boundaries 6 - 3.
Step 3: Moved (old) base [0-1:z100+.] to (new) boundaries 6 - 5.
Step 4: Moved (old) base [1-2:z101+.] to (new) boundaries 5 - 3.
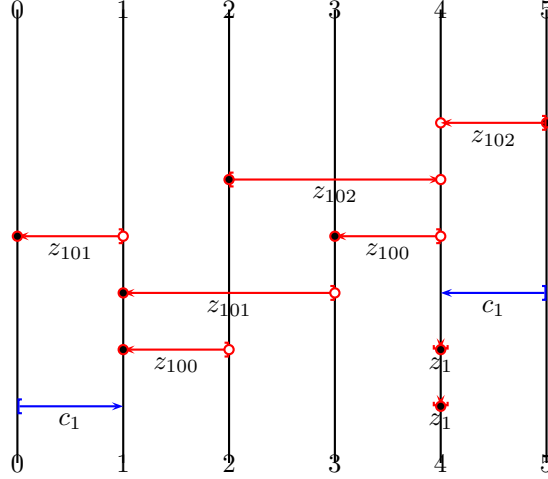Step 5: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 2: =0=5*<1<4*<2=3*
```

is shown below:

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Observe the following facts about this GE: The base [0-1:z101-.] has constraints with its dual that stretch the constant segment 0 - 1 to length different from 1. The base [4-5:z102-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. These observations show that the GE above is degenerate.

This completes the consideration of Print 2.

Next, we consider

    Print 3: =0=5*<4*<1<2=3*

Sequence of Actions in performing the Print 3:
Step 1: Added (new) boundary 4.
Step 2: Moved (old) base [0-2:z1-.] to (new) boundaries 6 - 3.
Step 3: Moved (old) base [0-1:z100+.] to (new) boundaries 6 - 4.
Step 4: Moved (old) base [1-2:z101+.] to (new) boundaries 4 - 3.
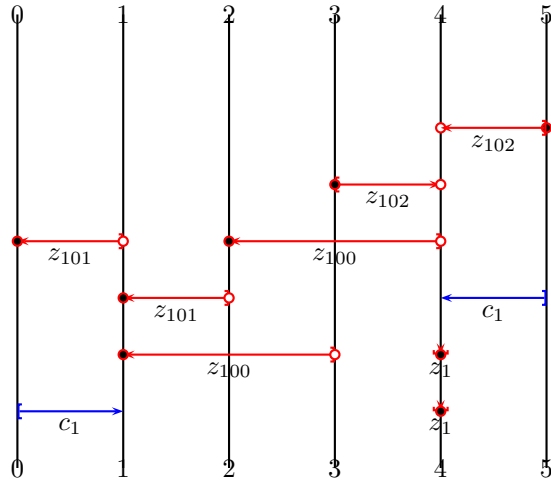Step 5: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

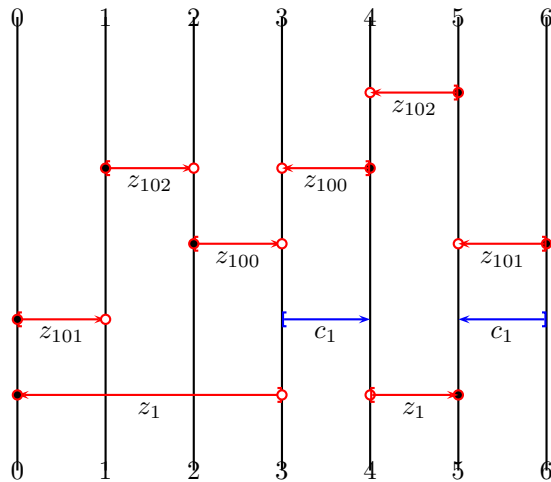   Print 3: =0=5*<4*<1<2=3*

is shown below:

12

The GE above is non-degenerate.

This completes the consideration of Print 3.

6   Generalized Equation #6

Quadratic System: $z_1^{-1}c_1z_1c_1^{-1} =_F 1$.

GE Information: Carrier: [0-3:z1-.] ; Carrier Dual: [4-5:z1+.] ; Critical Boundary: 3; **Prints**

```
Print 0: =0=5*<1<2<3=4*
```

Total number of prints: 1
Next, we consider

```
Print 1: =0=5*<1<2<3=4*
```

Sequence of Actions in performing the Print 1:
Step 1: Added (new) boundary 5.
Step 2: Added (new) boundary 6.
Step 3: Moved (old) base [0-3:z1-.] to (new) boundaries 7 - 4.
Step 4: Moved (old) base [2-3:z100+.] to (new) boundaries 5 - 4.
Step 5: Moved (old) base [0-1:z101+.] to (new) boundaries 7 - 6.
Step 6: Moved (old) base [1-2:z102+.] to (new) boundaries 6 - 5.
Step 7: Collapsed (new) base [4-7:z1+.] to the empty base (7,7).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
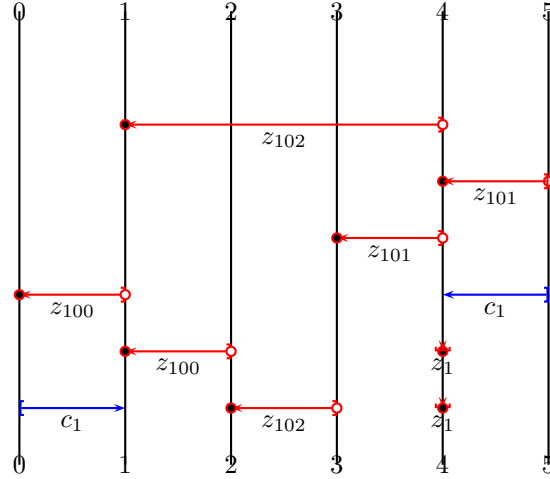Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Summarizing, the GE we obtain after applying

```
Print 1: =0=5*<1<2<3=4*
```

is shown below:

Observe the following facts about this GE: The base [2-3:z102-.] and its dual are of the same polarity, yet one properly contains the other. The base [1-4:z102-.] and its dual are of the same polarity, yet one properly contains the other. These

observations show that the GE above is degenerate.

This completes the consideration of Print 1.