Solution of the equation $z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$ in a Free Group

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Contents

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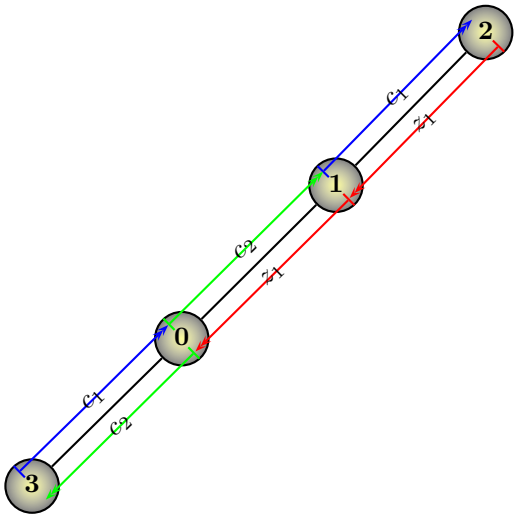
[†]This report was generated automatically by software developed with support from the National Security Agency Grant H98230-06-1-0042.

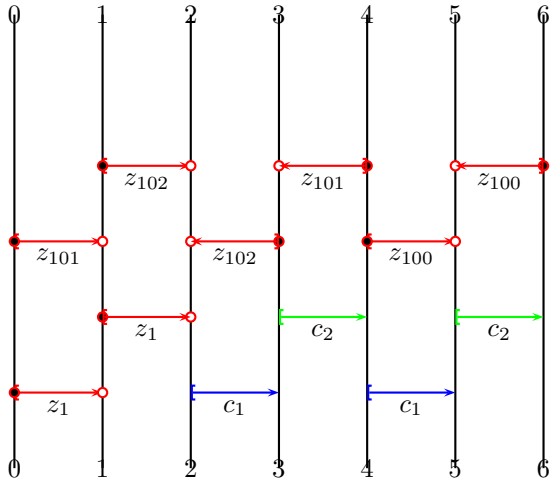
1 Cancellation scheme #1



z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$1 \leftarrow 2$
c_2	$0 \leftarrow 1$
c_1	$3 \leftarrow 0$
c_2	$0 \leftarrow 3$

Generalized Equation root-1

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-2:z1+.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=1*<1=2*
```

We proceed.

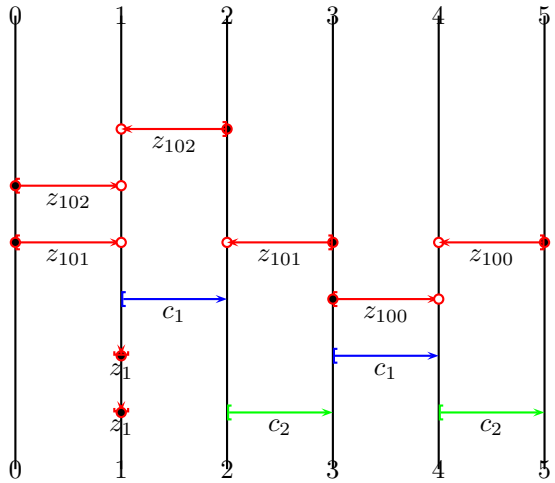
Generalized Equation root-1.1

We begin from the GE root-1 (see pp. 2). We consider its print

```
Print 1: =0=1*<1=2*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 2.
Step 2: Moved (old) base [0-1:z101+.] to (new) boundaries 1 - 2.
Step 3: Collapsed (new) base [1-2:z1+.] to the empty base (2,2).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-1.1—is illustrated below:



GE Information: Carrier: [0-1:z101+.] ; Carrier Dual: [2-3:z101-.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=3*<1=2*
```

This completes the consideration of root-1.1, as derived from the application of a print to root-1.

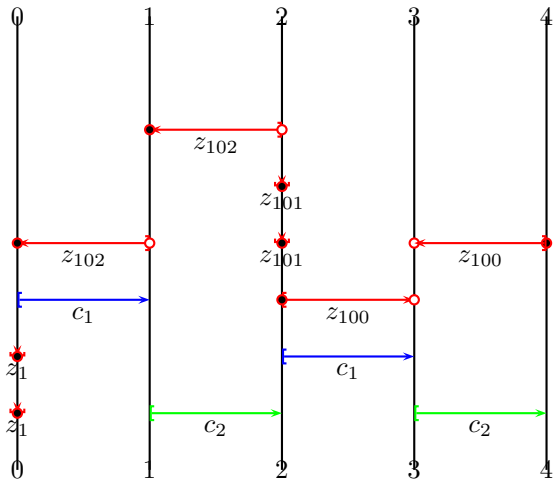
Generalized Equation root-1.1.1

We begin from the GE root-1.1 (see pp. 3). We consider its print

```
Print 1: =0=3*<1=2*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z101+.] to (new) boundaries 3 - 2.
Step 2: Moved (old) base [0-1:z102+.] to (new) boundaries 3 - 2.
Step 3: Collapsed (new) base [2-3:z101-.] to the empty base (3,3).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-1.1.1—is illustrated below:



GE Information: Carrier: [0-1:z102-.] ; Carrier Dual: [1-2:z102-.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=1*<1=2*
```

This completes the consideration of root-1.1.1, as derived from the application of a print to root-1.1.

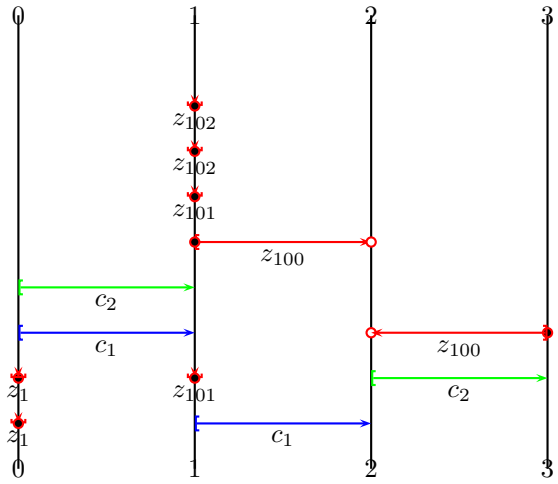
Generalized Equation root-1.1.1.1

We begin from the GE root-1.1.1 (see pp. 4). We consider its print

```
Print 1: =0=1*<1=2*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z102-.] to (new) boundaries 1 - 2.
Step 2: Moved (old) base [0-0:z1+.] to (new) boundaries 1 - 1.
Step 3: Moved (old) base [0-0:z1+.] to (new) boundaries 1 - 1.
Step 4: Moved (old) base [0-1:c1+.] to (new) boundaries 1 - 2.
Step 5: Collapsed (new) base [1-2:z102-.] to the empty base (2,2).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-1.1.1.1—is illustrated below:

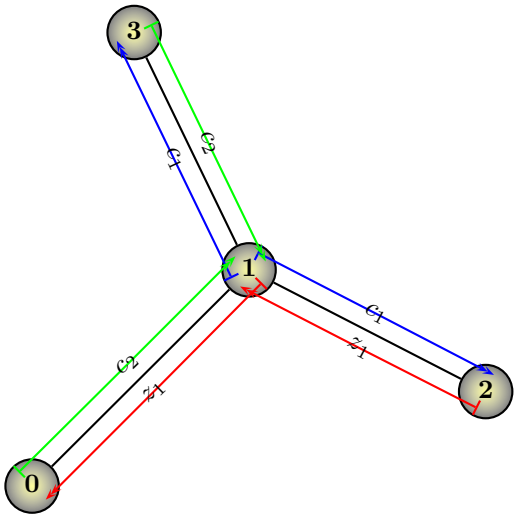


GE Information: Carrier: [1-2:z100+.] ; Carrier Dual: [2-3:z100-.] ; Critical Boundary: 2; Observe the following facts about this GE: Two distinct constants appear at boundary 0. Two distinct constants appear at boundary 0. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-1.1.1.1, as derived from the application of a print to root-1.1.1.

$z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$

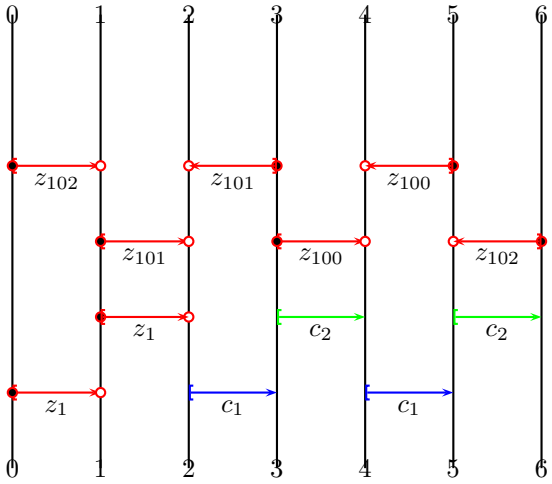
2 Cancellation scheme #2



z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$1 \leftarrow 2$
c_2	$3 \leftarrow 1$
c_1	$1 \leftarrow 3$
c_2	$0 \leftarrow 1$

Generalized Equation root-2

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-2:z1+.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=1*<1=2*
```

We proceed.

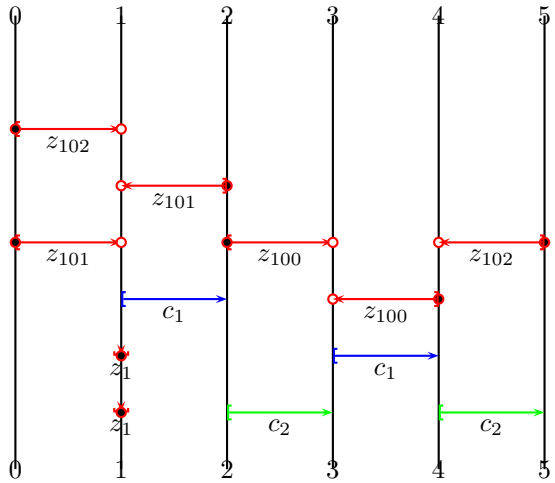
Generalized Equation root-2.1

We begin from the GE root-2 (see pp. 7). We consider its print

```
Print 1: =0=1*<1=2*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 2.
Step 2: Moved (old) base [0-1:z102+.] to (new) boundaries 1 - 2.
Step 3: Collapsed (new) base [1-2:z1+.] to the empty base (2,2).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-2.1—is illustrated below:



GE Information: Carrier: [0-1:z101+.] ; Carrier Dual: [1-2:z101-.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=2*<1=1*
```

This completes the consideration of root-2.1, as derived from the application of a print to root-2.

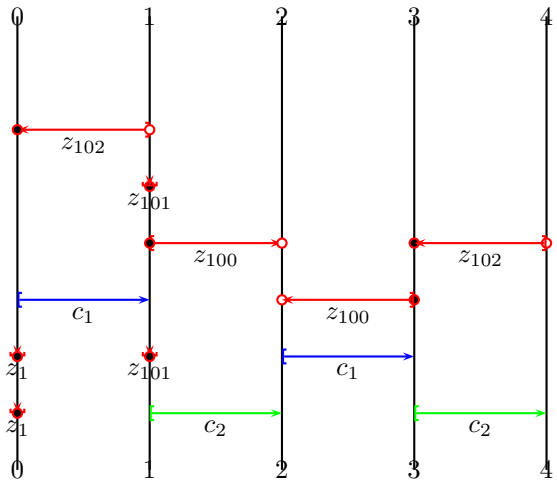
Generalized Equation root-2.1.1

We begin from the GE root-2.1 (see pp. 8). We consider its print

```
Print 1: =0=2*<1=1*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z101+.] to (new) boundaries 2 - 1.
Step 2: Moved (old) base [0-1:z102+.] to (new) boundaries 2 - 1.
Step 3: Collapsed (new) base [1-2:z101-.] to the empty base (2,2).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-2.1.1—is illustrated below:



GE Information: Carrier: [0-1:z102-.] ; Carrier Dual: [3-4:z102-.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=3*<1=4*
```

This completes the consideration of root-2.1.1, as derived from the application of a print to root-2.1.

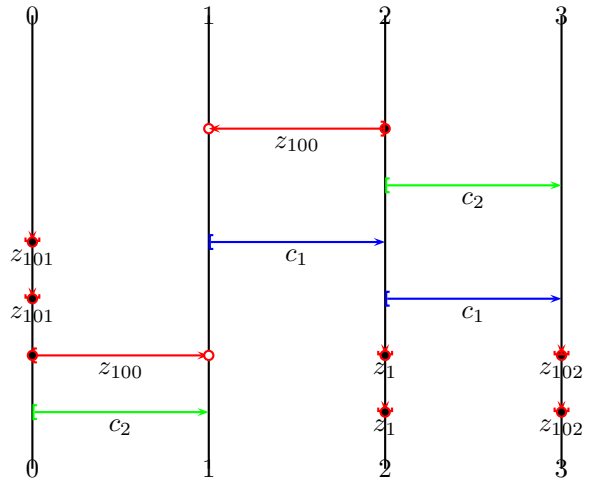
Generalized Equation root-2.1.1.1

We begin from the GE root-2.1.1 (see pp. 9). We consider its print

```
Print 1: =0=3*<1=4*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z102-.] to (new) boundaries 3 - 4.
Step 2: Moved (old) base [0-0:z1+.] to (new) boundaries 3 - 3.
Step 3: Moved (old) base [0-0:z1+.] to (new) boundaries 3 - 3.
Step 4: Moved (old) base [0-1:c1+.] to (new) boundaries 3 - 4.
Step 5: Collapsed (new) base [3-4:z102-.] to the empty base (4,4).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-2.1.1.1—is illustrated below:

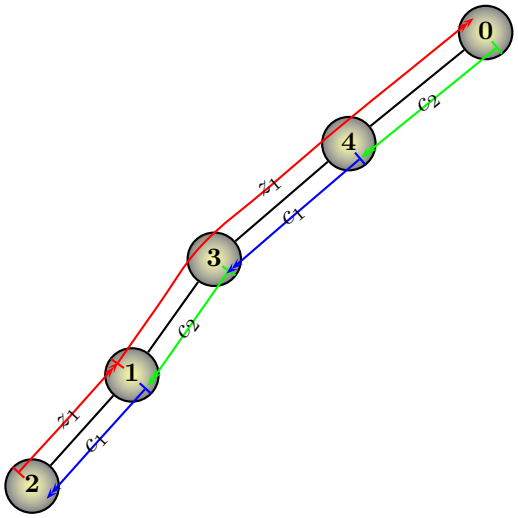


GE Information: Carrier: [0-1:z100+.] ; Carrier Dual: [1-2:z100-.] ; Critical Boundary: 1; Observe the following facts about this GE: Two distinct constants appear at boundary 2. Two distinct constants appear at boundary 2. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-2.1.1.1, as derived from the application of a print to root-2.1.1.

$$z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$$

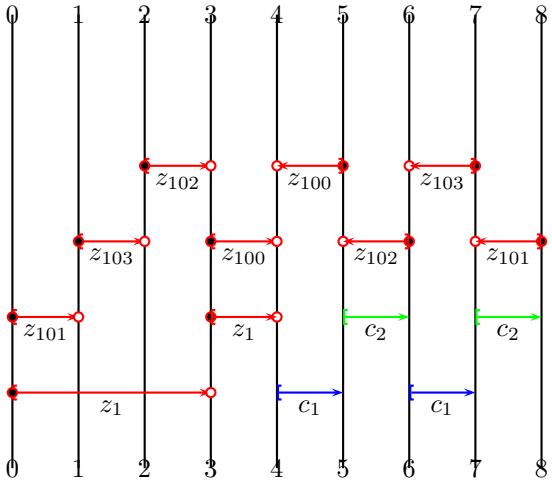
3 Cancellation scheme #3



z_1	$1 \leftarrow 3 \leftarrow 4 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$1 \leftarrow 2$
c_2	$3 \leftarrow 1$
c_1	$4 \leftarrow 3$
c_2	$0 \leftarrow 4$

Generalized Equation root-3

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-3:z1+.] ; Carrier Dual: [3-4:z1+.] ; Critical Boundary: 3; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=3*<1<2<3=4*
```

We proceed.

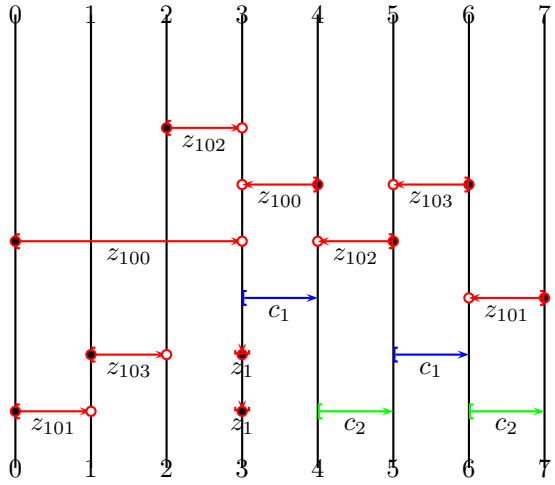
Generalized Equation root-3.1

We begin from the GE root-3 (see pp. 12). We consider its print

```
Print 1: =0=3*<1<2<3=4*
```

Sequence of actions in performing the Print 1:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 5.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 6.
Step 4: Moved (old) base [0-1:z101+.] to (new) boundaries 3 - 4.
Step 5: Moved (old) base [2-3:z102+.] to (new) boundaries 5 - 6.
Step 6: Moved (old) base [1-2:z103+.] to (new) boundaries 4 - 5.
Step 7: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

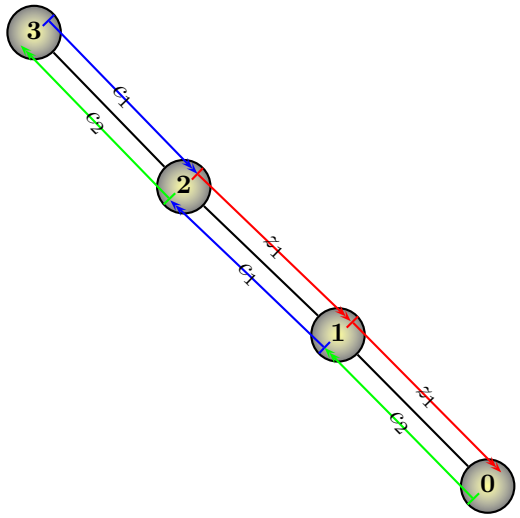
Upon applying the print, the GE we obtain—which we refer to as root-3.1—is illustrated below:



GE Information: Carrier: [0-3:z100+.] ; Carrier Dual: [3-4:z100-.] ; Critical Boundary: 3; Observe the following facts about this GE: The base [3-4:z100-.] has constraints with its dual that stretch the constant segment 3 - 4 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-3.1, as derived from the application of a print to root-3.

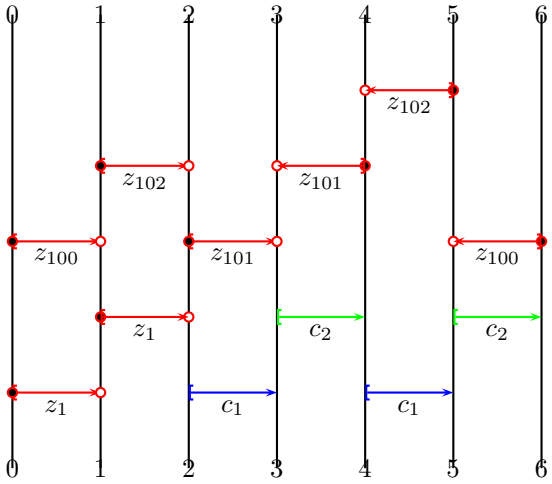
4 Cancellation scheme #4



z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$2 \leftarrow 3$
c_1	$1 \leftarrow 2$
c_2	$0 \leftarrow 1$

Generalized Equation root-4

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-2:z1+.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=1*<1=2*
```

We proceed.

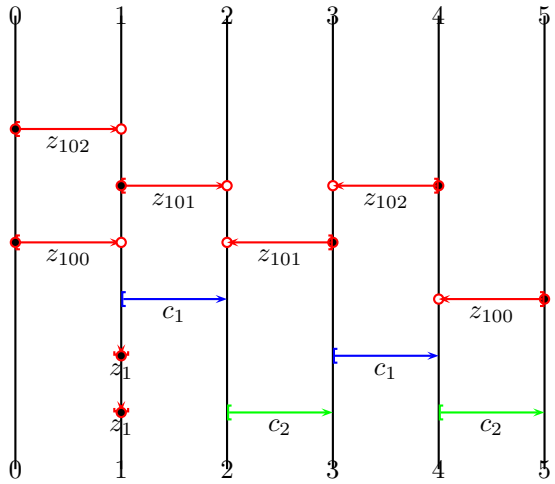
Generalized Equation root-4.1

We begin from the GE root-4 (see pp. 15). We consider its print

```
Print 1: =0=1*<1=2*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 2.
Step 2: Moved (old) base [0-1:z100+.] to (new) boundaries 1 - 2.
Step 3: Collapsed (new) base [1-2:z1+.] to the empty base (2,2).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-4.1—is illustrated below:



GE Information: Carrier: [0-1:z100+.] ; Carrier Dual: [4-5:z100-.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=5*<1=4*
```

This completes the consideration of root-4.1, as derived from the application of a print to root-4.

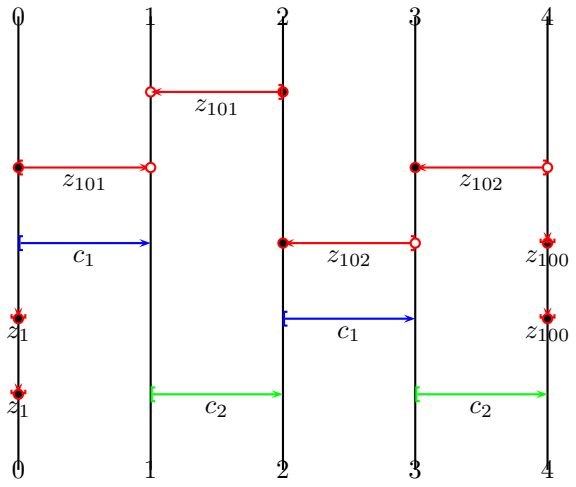
Generalized Equation root-4.1.1

We begin from the GE root-4.1 (see pp. 16). We consider its print

```
Print 1: =0=5*<1=4*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z100+.] to (new) boundaries 5 - 4.
Step 2: Moved (old) base [0-1:z102+.] to (new) boundaries 5 - 4.
Step 3: Collapsed (new) base [4-5:z100-.] to the empty base (5,5).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-4.1.1—is illustrated below:



GE Information: Carrier: [0-1:z101+.] ; Carrier Dual: [1-2:z101-.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=2*<1=1*
```

This completes the consideration of root-4.1.1, as derived from the application of a print to root-4.1.

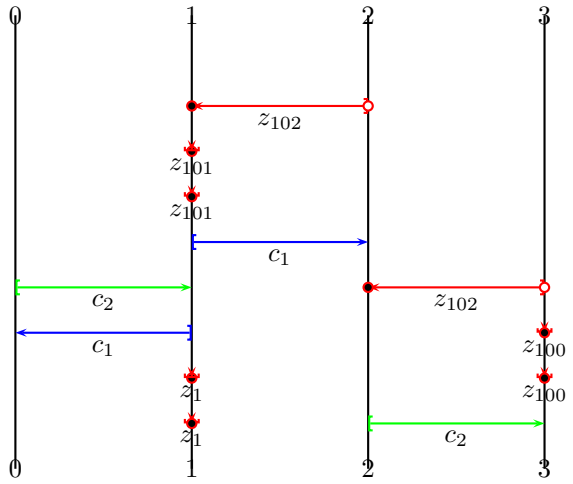
Generalized Equation root-4.1.1.1

We begin from the GE root-4.1.1 (see pp. 17). We consider its print

```
Print 1: =0=2*<1=1*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z101+.] to (new) boundaries 2 - 1.
Step 2: Moved (old) base [0-0:z1+.] to (new) boundaries 2 - 2.
Step 3: Moved (old) base [0-0:z1+.] to (new) boundaries 2 - 2.
Step 4: Moved (old) base [0-1:c1+.] to (new) boundaries 2 - 1.
Step 5: Collapsed (new) base [1-2:z101-.] to the empty base (2,2).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-4.1.1.1—is illustrated below:

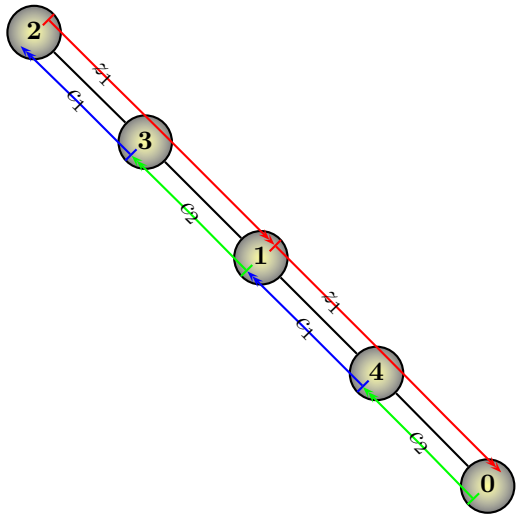


GE Information: Carrier: [1-2:z102-.] ; Carrier Dual: [2-3:z102-.] ; Critical Boundary: 2; Observe the following facts about this GE: Two distinct constants appear at boundary 0. Two distinct constants appear at boundary 0. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-4.1.1.1, as derived from the application of a print to root-4.1.1.

$$z_1 z_1 c_1 c_2 c_1 c_2 =_F 1$$

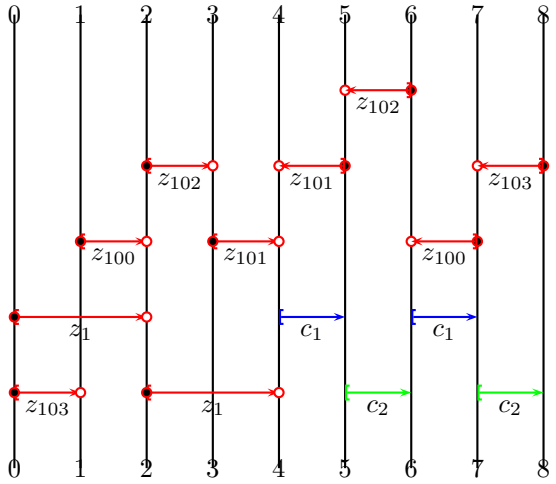
5 Cancellation scheme #5



z_1	$1 \leftarrow 4 \leftarrow 0$
z_1	$2 \leftarrow 3 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$1 \leftarrow 3$
c_1	$4 \leftarrow 1$
c_2	$0 \leftarrow 4$

Generalized Equation root-5

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-2:z1+.] ; Carrier Dual: [2-4:z1+.] ; Critical Boundary: 2; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 3 valid prints (descendents).

It has 3 legal carrier-to-dual prints, as follows:

```
Print 1: =0=2*<1=3*<2=4*
Print 2: =0=2*<1<3*<2=4*
Print 3: =0=2*<3*<1<2=4*
```

We proceed.

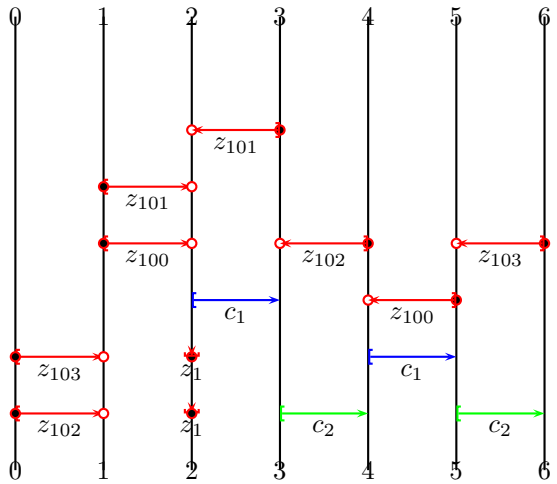
Generalized Equation root-5.1

We begin from the GE root-5 (see pp. 20). We consider its print

```
Print 1: =0=2*<1=3*<2=4*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 4.
Step 2: Moved (old) base [1-2:z100+.] to (new) boundaries 3 - 4.
Step 3: Moved (old) base [0-1:z103+.] to (new) boundaries 2 - 3.
Step 4: Collapsed (new) base [2-4:z1+.] to the empty base (4,4).
Step 5: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 6: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-5.1—is illustrated below:



GE Information: Carrier: [0-1:z102+.] ; Carrier Dual: [3-4:z102-.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=4*<1=3*
```

This completes the consideration of root-5.1, as derived from the application of a print to root-5.

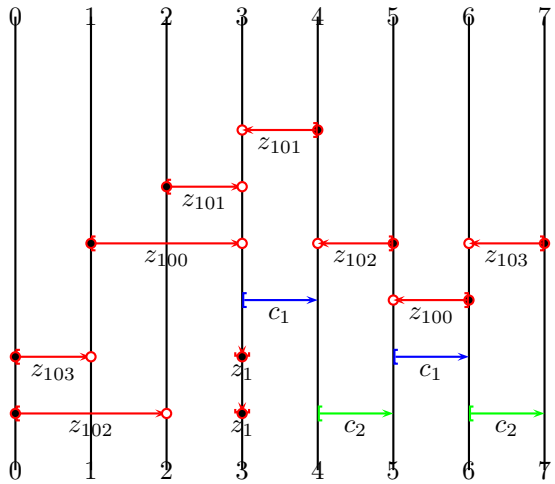
Generalized Equation root-5.2

We begin from the GE root-5 (see pp. 20). We consider its print

```
Print 2: =0=2*<1<3*<2=4*
```

Sequence of actions in performing the Print 2:
Step 1: Added (new) boundary 3.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [1-2:z100+.] to (new) boundaries 3 - 5.
Step 4: Moved (old) base [0-1:z103+.] to (new) boundaries 2 - 3.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-5.2—is illustrated below:



GE Information: Carrier: [0-2:z102+.] ; Carrier Dual: [4-5:z102-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [4-5:z102-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. The base [5-6:z100-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-5.2, as derived from the application of a print to root-5.

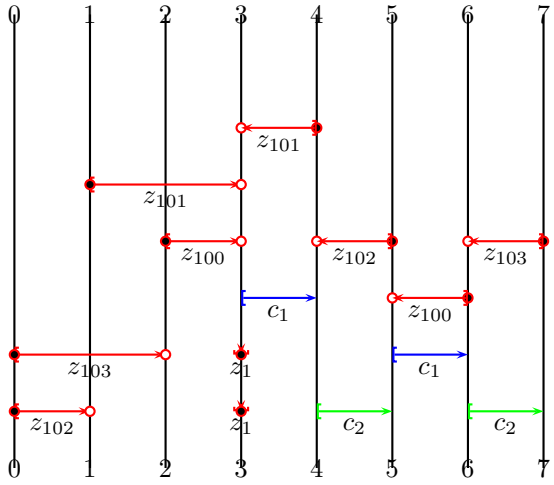
Generalized Equation root-5.3

We begin from the GE root-5 (see pp. 20). We consider its print

```
Print 3: =0=2*<3*<1<2=4*
```

Sequence of actions in performing the Print 3:
Step 1: Added (new) boundary 4.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [1-2:z100+.] to (new) boundaries 4 - 5.
Step 4: Moved (old) base [0-1:z103+.] to (new) boundaries 2 - 4.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-5.3—is illustrated below:



GE Information: Carrier: [0-2:z103+.] ; Carrier Dual: [6-7:z103-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [3-4:z101-.] has constraints with its dual that stretch the constant segment 3 - 4 to length different from 1. The base [6-7:z103-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-5.3, as derived from the application of a print to root-5.

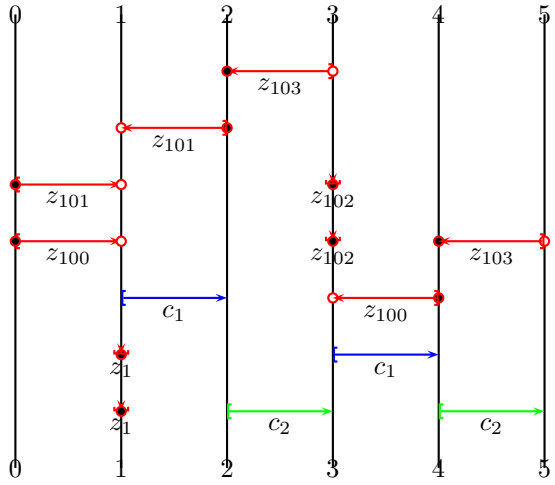
Generalized Equation root-5.1.1

We begin from the GE root-5.1 (see pp. 21). We consider its print

```
Print 1: =0=4*<1=3*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z102+.] to (new) boundaries 4 - 3.
Step 2: Moved (old) base [0-1:z103+.] to (new) boundaries 4 - 3.
Step 3: Collapsed (new) base [3-4:z102-.] to the empty base (4,4).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-5.1.1—is illustrated below:



GE Information: Carrier: [0-1:z100+.] ; Carrier Dual: [3-4:z100-.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=4*<1=3*
```

This completes the consideration of root-5.1.1, as derived from the application of a print to root-5.1.

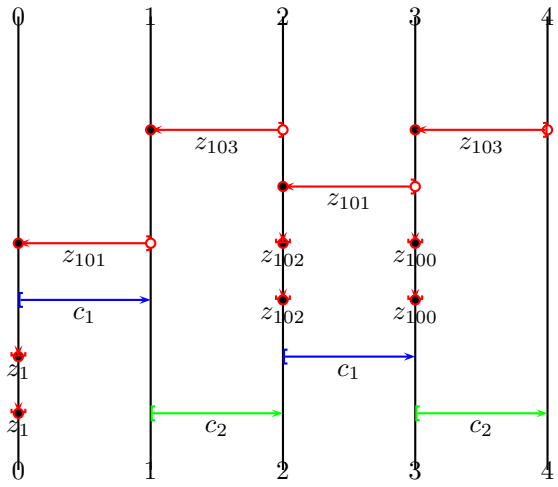
Generalized Equation root-5.1.1.1

We begin from the GE root-5.1.1 (see pp. 24). We consider its print

```
Print 1: =0=4*<1=3*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z100+.] to (new) boundaries 4 - 3.
Step 2: Moved (old) base [0-1:z101+.] to (new) boundaries 4 - 3.
Step 3: Collapsed (new) base [3-4:z100-.] to the empty base (4,4).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-5.1.1.1—is illustrated below:



GE Information: Carrier: [0-1:z101-.] ; Carrier Dual: [2-3:z101-.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=2*<1=3*
```

This completes the consideration of root-5.1.1.1, as derived from the application of a print to root-5.1.1.

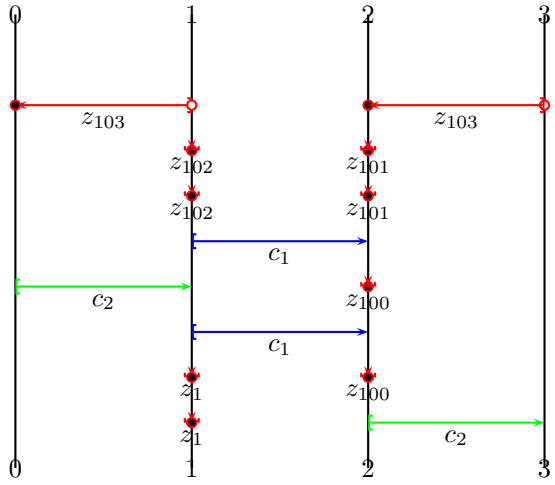
Generalized Equation root-5.1.1.1.1

We begin from the GE root-5.1.1.1 (see pp. 25). We consider its print

```
Print 1: =0=2*<1=3*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z101-.] to (new) boundaries 2 - 3.
Step 2: Moved (old) base [0-0:z1+.] to (new) boundaries 2 - 2.
Step 3: Moved (old) base [0-0:z1+.] to (new) boundaries 2 - 2.
Step 4: Moved (old) base [0-1:c1+.] to (new) boundaries 2 - 3.
Step 5: Collapsed (new) base [2-3:z101-.] to the empty base (3,3).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-5.1.1.1.1—is illustrated below:



GE Information: Carrier: [0-1:z103-.] ; Carrier Dual: [2-3:z103-.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=2*<1=3*
```

This completes the consideration of root-5.1.1.1.1, as derived from the application of a print to root-5.1.1.1.

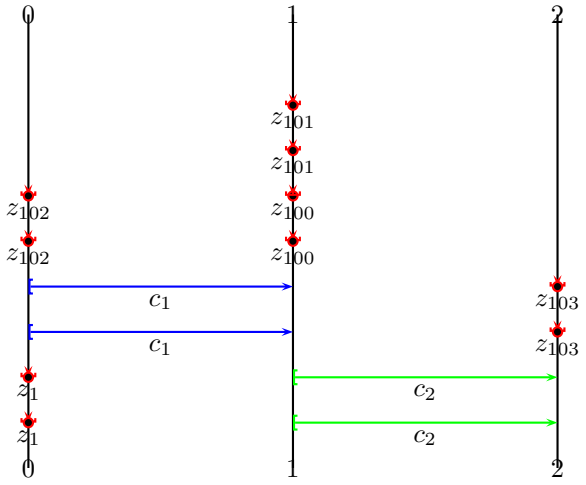
Generalized Equation root-5.1.1.1.1.1

We begin from the GE root-5.1.1.1.1 (see pp. 26). We consider its print

```
Print 1: =0=2*<1=3*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z103-.] to (new) boundaries 2 - 3.
Step 2: Moved (old) base [0-1:c2+.] to (new) boundaries 2 - 3.
Step 3: Collapsed (new) base [2-3:z103-.] to the empty base (3,3).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-5.1.1.1.1.1—is illustrated below:



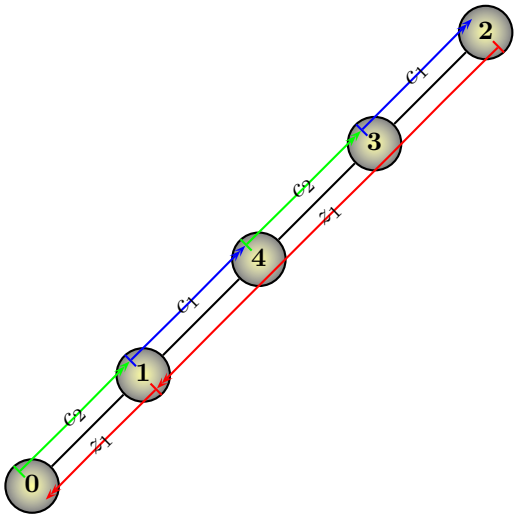
GE Information: Carrier: [0-0:z1+.] ; Carrier Dual: [0-0:z1+.] ; Critical Boundary: 0; The GE above is non-degenerate. This GE is a leaf in the GE tree. We have effectively found a solution!

variable	value
z_{101}	$c_1^{-1} =_F 1$
z_{102}	$c_2^{-1} =_F 1$
z_{103}	$c_2^{-1} =_F 1$
z_1	$c_2^{-1} c_1^{-1} =_F 1$
z_{102}^{-1}	$c_2 =_F 1$
z_{100}^{-1}	$c_1 =_F 1$
z_{100}	$c_1^{-1} =_F 1$
z_{101}^{-1}	$c_1 =_F 1$
z_{103}^{-1}	$c_2 =_F 1$
z_1^{-1}	$c_1 c_2 =_F 1$

The above table shows the values of the solution, as obtained by tracing upwards from this trivially true GE, to the root of the Makanin-Razborov tree.

This completes the consideration of root-5.1.1.1.1.1, as derived from the application of a print to root-5.1.1.1.1.

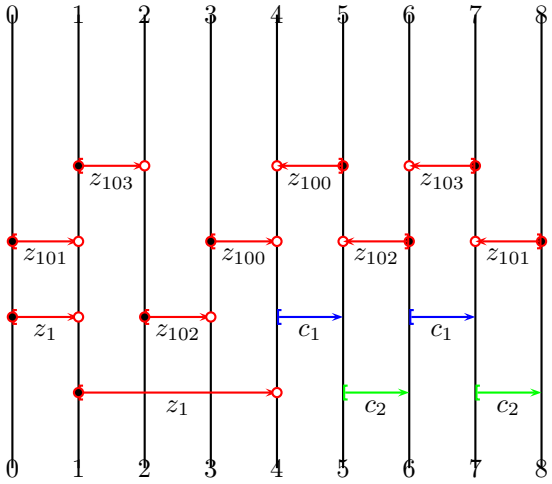
6 Cancellation scheme #6



z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 3 \leftarrow 4 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$4 \leftarrow 3$
c_1	$1 \leftarrow 4$
c_2	$0 \leftarrow 1$

Generalized Equation root-6

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-4:z1+.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=1*<2*<3*<1=4*
```

We proceed.

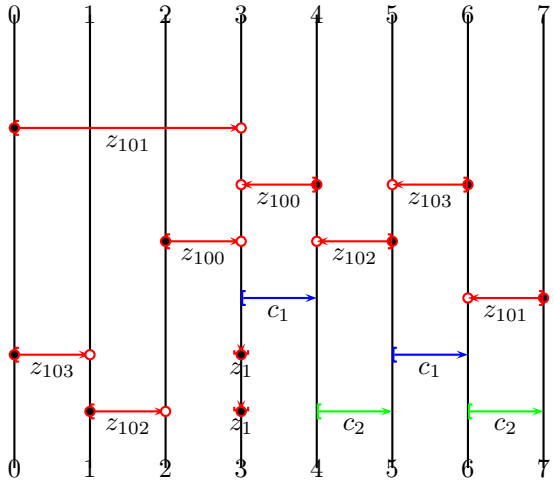
Generalized Equation root-6.1

We begin from the GE root-6 (see pp. 29). We consider its print

```
Print 1: =0=1*<2*<3*<1=4*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 4.
Step 2: Moved (old) base [0-1:z101+.] to (new) boundaries 1 - 4.
Step 3: Collapsed (new) base [1-4:z1+.] to the empty base (4,4).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-6.1—is illustrated below:

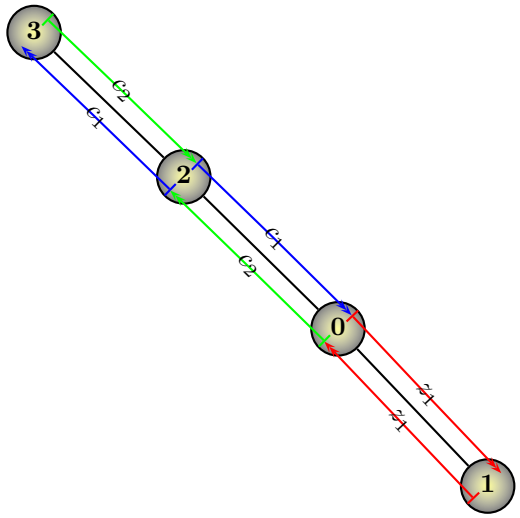


GE Information: Carrier: [0-3:z101+.] ; Carrier Dual: [6-7:z101-.] ; Critical Boundary: 3; Observe the following facts about this GE: The base [6-7:z101-.] has constraints with its dual that stretch the constant segment 6 - 7 to length

different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-6.1, as derived from the application of a print to root-6.

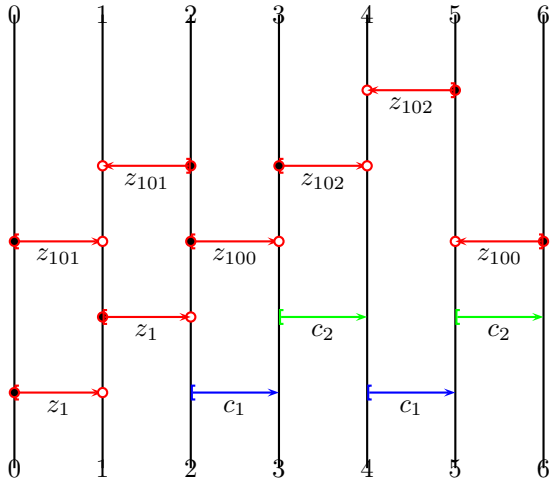
7 Cancellation scheme #7



z_1	$1 \leftarrow 0$
z_1	$0 \leftarrow 1$
c_1	$2 \leftarrow 0$
c_2	$3 \leftarrow 2$
c_1	$2 \leftarrow 3$
c_2	$0 \leftarrow 2$

Generalized Equation root-7

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-2:z1+.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=1*<1=2*
```

We proceed.

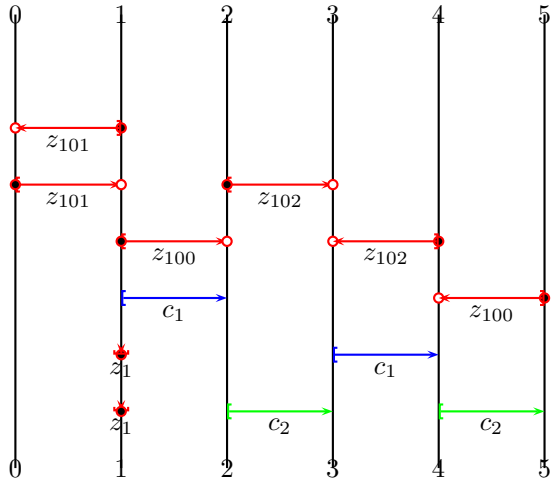
Generalized Equation root-7.1

We begin from the GE root-7 (see pp. 32). We consider its print

```
Print 1: =0=1*<1=2*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 2.
Step 2: Moved (old) base [0-1:z101+.] to (new) boundaries 1 - 2.
Step 3: Collapsed (new) base [1-2:z1+.] to the empty base (2,2).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-7.1—is illustrated below:

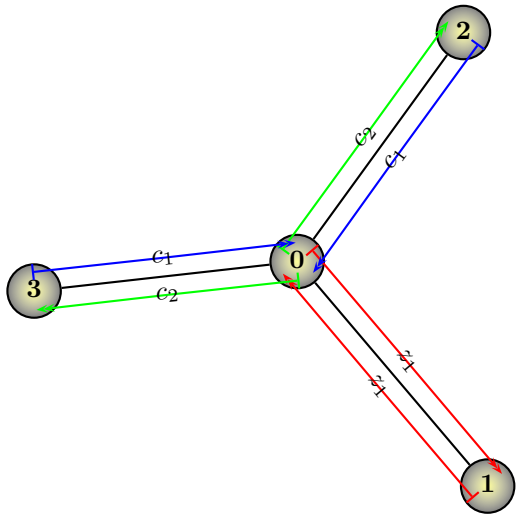


GE Information: Carrier: [0-1:z101+.] ; Carrier Dual: [0-1:z101-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [0-1:z101+.] and its dual are of opposite polarity, yet intersect. The base [0-1:z101-.] and its

dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-7.1, as derived from the application of a print to root-7.

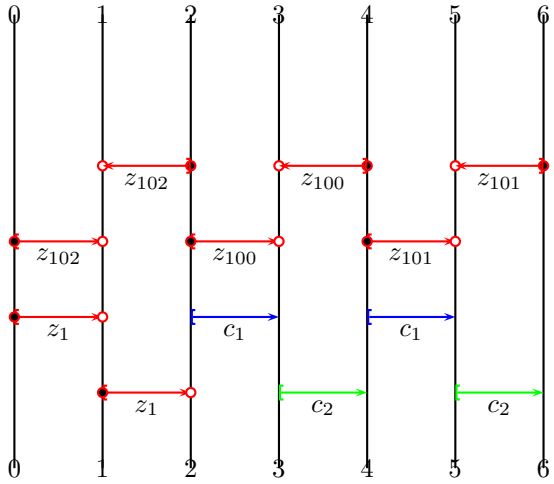
8 Cancellation scheme #8



z_1	$1 \leftarrow 0$
z_1	$0 \leftarrow 1$
c_1	$2 \leftarrow 0$
c_2	$0 \leftarrow 2$
c_1	$3 \leftarrow 0$
c_2	$0 \leftarrow 3$

Generalized Equation root-8

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-2:z1+.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=1*<1=2*
```

We proceed.

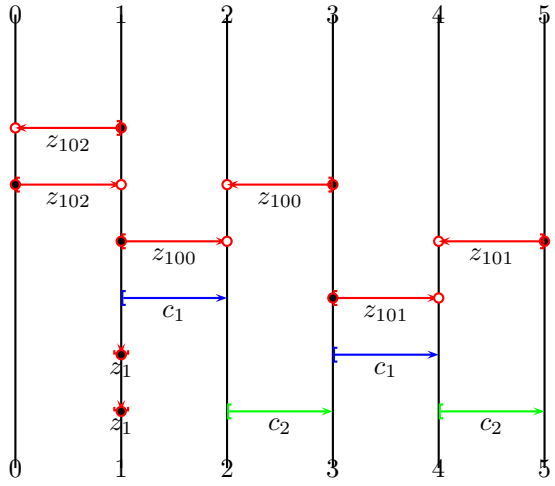
Generalized Equation root-8.1

We begin from the GE root-8 (see pp. 35). We consider its print

```
Print 1: =0=1*<1=2*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 2.
Step 2: Moved (old) base [0-1:z102+.] to (new) boundaries 1 - 2.
Step 3: Collapsed (new) base [1-2:z1+.] to the empty base (2,2).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-8.1—is illustrated below:

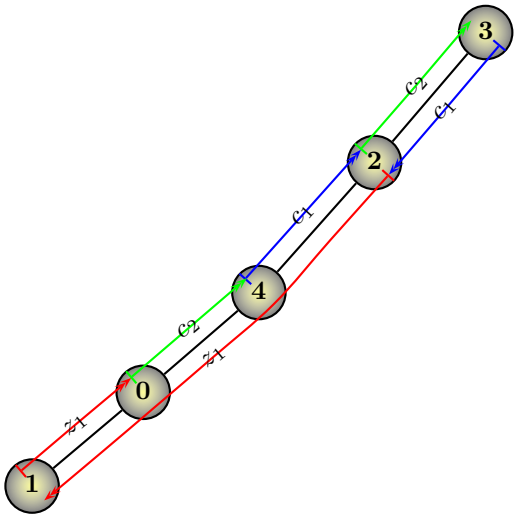


GE Information: Carrier: [0-1:z102+.] ; Carrier Dual: [0-1:z102-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [0-1:z102+.] and its dual are of opposite polarity, yet intersect. The base [0-1:z102-.] and its

dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-8.1, as derived from the application of a print to root-8.

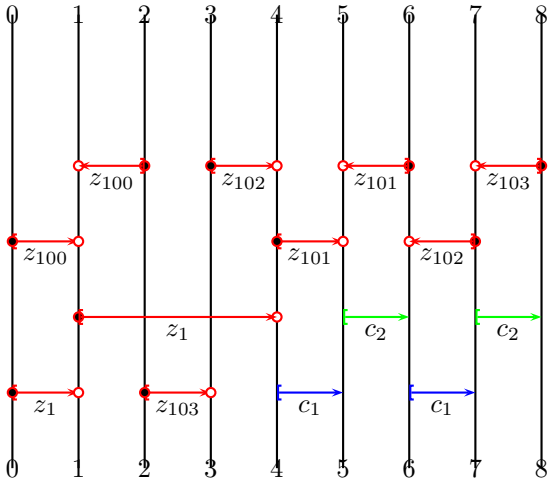
9 Cancellation scheme #9



z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 4 \leftarrow 0 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$2 \leftarrow 3$
c_1	$4 \leftarrow 2$
c_2	$0 \leftarrow 4$

Generalized Equation root-9

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-4:z1+.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=1*<2*<3*<1=4*
```

We proceed.

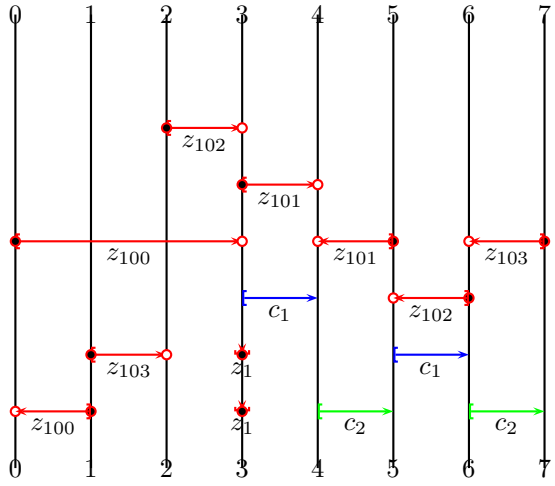
Generalized Equation root-9.1

We begin from the GE root-9 (see pp. 38). We consider its print

```
Print 1: =0=1*<2*<3*<1=4*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 4.
Step 2: Moved (old) base [0-1:z100+.] to (new) boundaries 1 - 4.
Step 3: Collapsed (new) base [1-4:z1+.] to the empty base (4,4).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-9.1—is illustrated below:

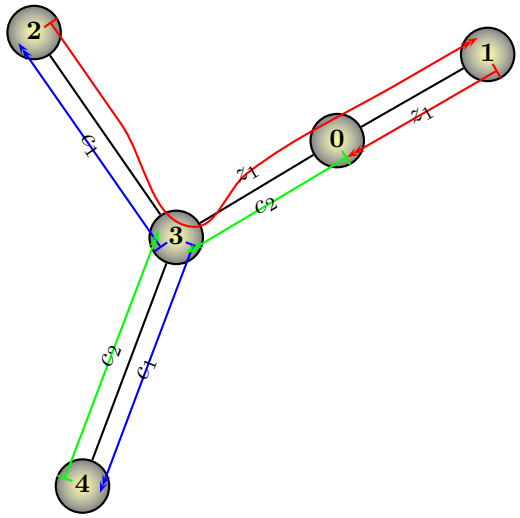


GE Information: Carrier: [0-3:z100+.] ; Carrier Dual: [0-1:z100-.] ; Critical Boundary: 3; Observe the following facts about this GE: The base [0-3:z100+.] and its dual are of opposite polarity, yet intersect. The base [0-1:z100-.] and its

dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-9.1, as derived from the application of a print to root-9.

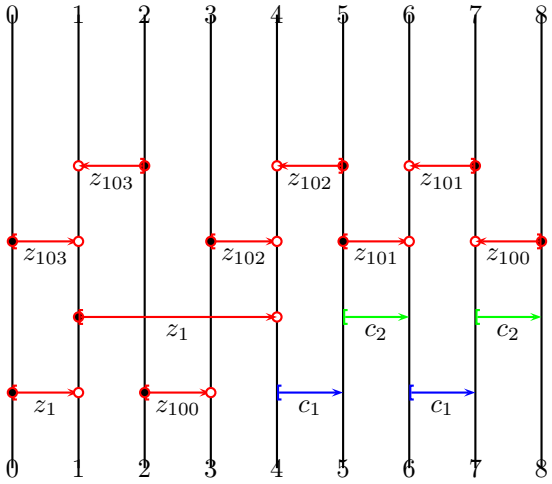
10 Cancellation scheme #10



z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 3 \leftarrow 0 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$4 \leftarrow 3$
c_1	$3 \leftarrow 4$
c_2	$0 \leftarrow 3$

Generalized Equation root-10

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-4:z1+.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=1*<2*<3*<1=4*
```

We proceed.

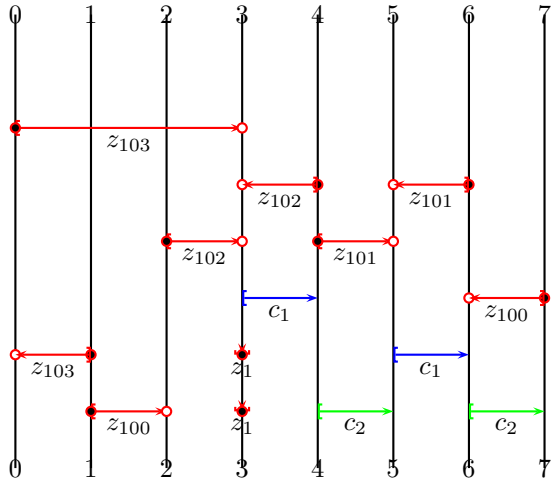
Generalized Equation root-10.1

We begin from the GE root-10 (see pp. 41). We consider its print

```
Print 1: =0=1*<2*<3*<1=4*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 4.
Step 2: Moved (old) base [0-1:z103+.] to (new) boundaries 1 - 4.
Step 3: Collapsed (new) base [1-4:z1+.] to the empty base (4,4).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-10.1—is illustrated below:

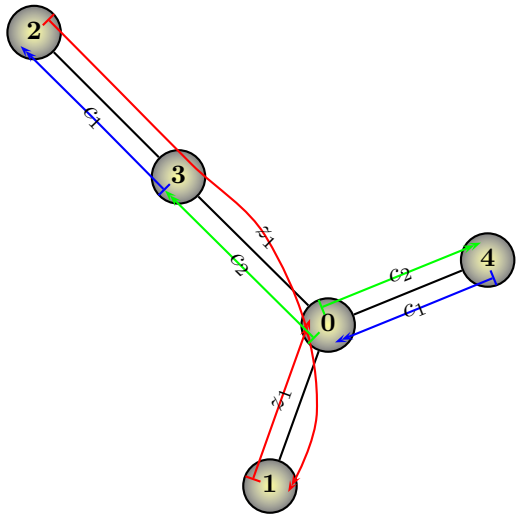


GE Information: Carrier: [0-3:z103+.] ; Carrier Dual: [0-1:z103-.] ; Critical Boundary: 3; Observe the following facts about this GE: The base [0-3:z103+.] and its dual are of opposite polarity, yet intersect. The base [0-1:z103-.] and its

dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-10.1, as derived from the application of a print to root-10.

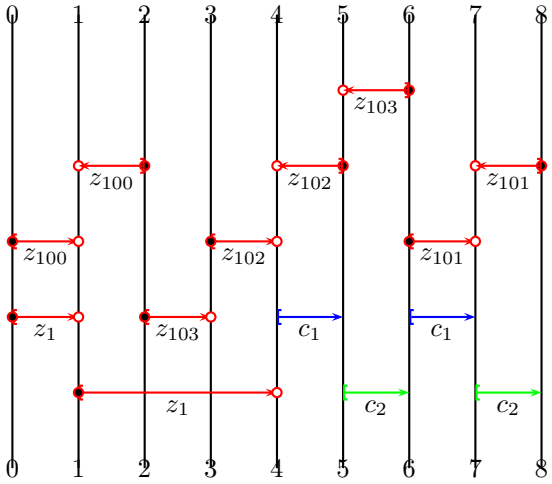
11 Cancellation scheme #11



z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 3 \leftarrow 0 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$0 \leftarrow 3$
c_1	$4 \leftarrow 0$
c_2	$0 \leftarrow 4$

Generalized Equation root-11

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-4:z1+.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=1*<2*<3*<1=4*
```

We proceed.

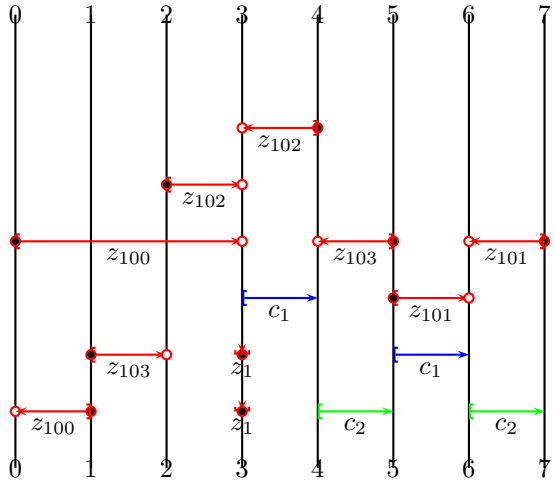
Generalized Equation root-11.1

We begin from the GE root-11 (see pp. 44). We consider its print

```
Print 1: =0=1*<2*<3*<1=4*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 4.
Step 2: Moved (old) base [0-1:z100+.] to (new) boundaries 1 - 4.
Step 3: Collapsed (new) base [1-4:z1+.] to the empty base (4,4).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-11.1—is illustrated below:

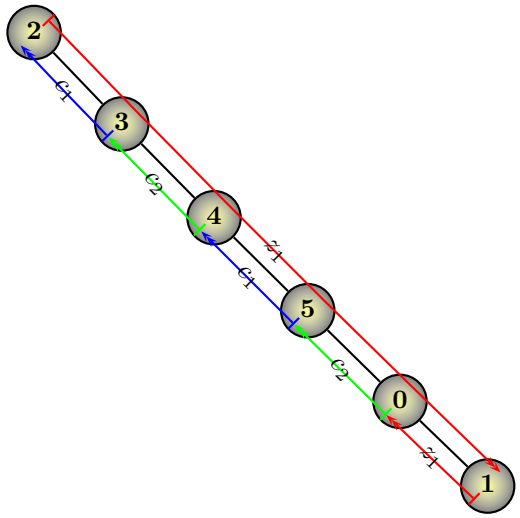


GE Information: Carrier: [0-3:z100+.] ; Carrier Dual: [0-1:z100-.] ; Critical Boundary: 3; Observe the following facts about this GE: The base [0-3:z100+.] and its dual are of opposite polarity, yet intersect. The base [0-1:z100-.] and its

dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-11.1, as derived from the application of a print to root-11.

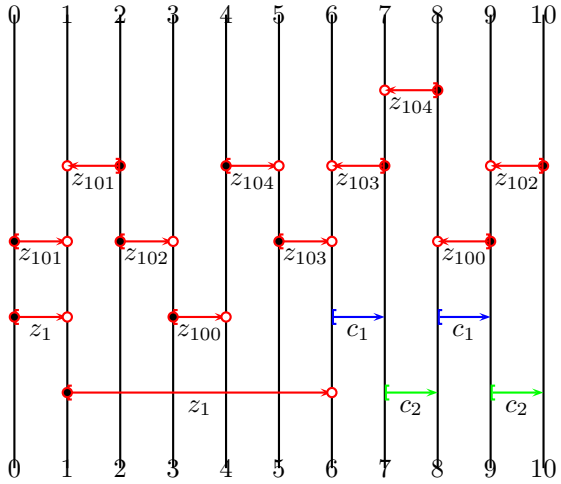
12 Cancellation scheme #12



z_1	$1 \leftarrow 0$
z_1	$2 \leftarrow 3 \leftarrow 4 \leftarrow 5 \leftarrow 0 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$4 \leftarrow 3$
c_1	$5 \leftarrow 4$
c_2	$0 \leftarrow 5$

Generalized Equation root-12

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-1:z1+.] ; Carrier Dual: [1-6:z1+.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=1*<2*<3*<4*<5*<1=6*
```

We proceed.

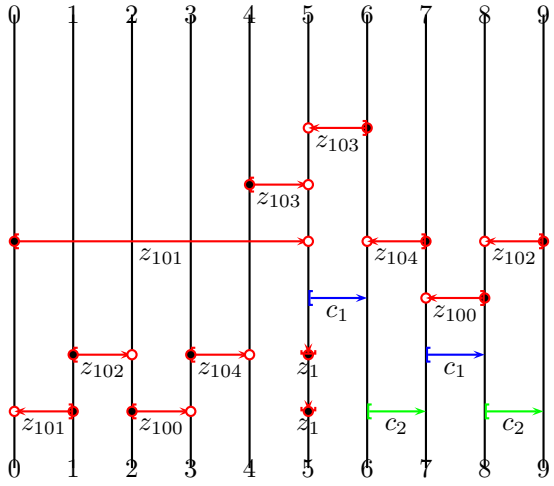
Generalized Equation root-12.1

We begin from the GE root-12 (see pp. 47). We consider its print

```
Print 1: =0=1*<2*<3*<4*<5*<1=6*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z1+.] to (new) boundaries 1 - 6.
Step 2: Moved (old) base [0-1:z101+.] to (new) boundaries 1 - 6.
Step 3: Collapsed (new) base [1-6:z1+.] to the empty base (6,6).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-12.1—is illustrated below:

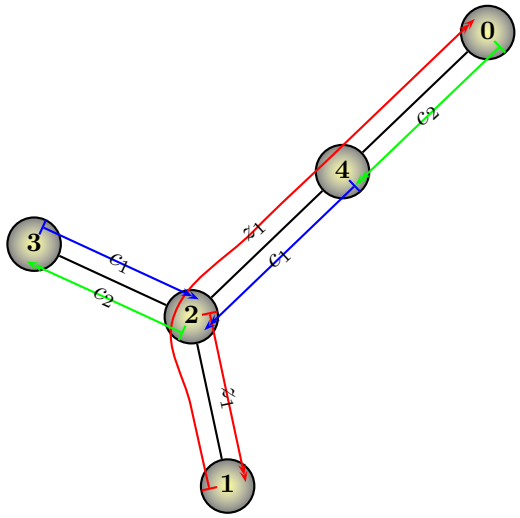


GE Information: Carrier: [0-5:z101+.] ; Carrier Dual: [0-1:z101-.] ; Critical Boundary: 5; Observe the following facts about this GE: The base [0-5:z101+.] and its dual are of opposite polarity, yet intersect. The base [0-1:z101-.] and its

dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-12.1, as derived from the application of a print to root-12.

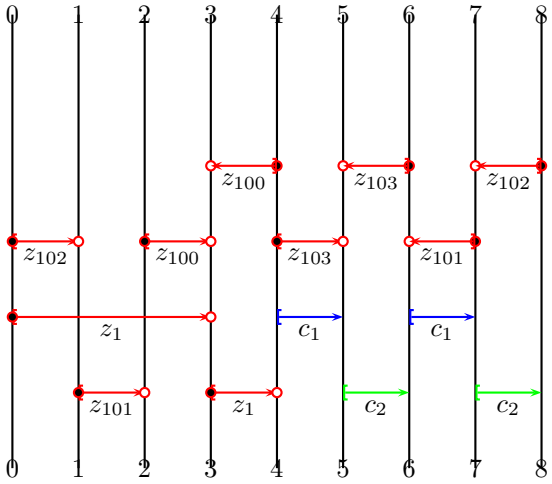
13 Cancellation scheme #13



z_1	$1 \leftarrow 2 \leftarrow 4 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$2 \leftarrow 3$
c_1	$4 \leftarrow 2$
c_2	$0 \leftarrow 4$

Generalized Equation root-13

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-3:z1+.] ; Carrier Dual: [3-4:z1+.] ; Critical Boundary: 3; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=3*<1<2<3=4*
```

We proceed.

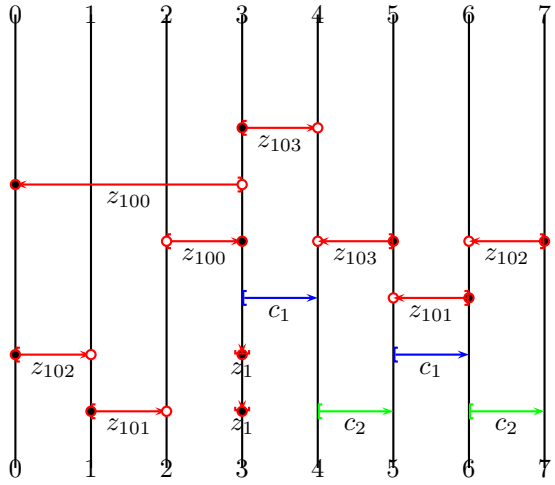
Generalized Equation root-13.1

We begin from the GE root-13 (see pp. 50). We consider its print

```
Print 1: =0=3*<1<2<3=4*
```

Sequence of actions in performing the Print 1:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 5.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 6.
Step 4: Moved (old) base [2-3:z100+.] to (new) boundaries 5 - 6.
Step 5: Moved (old) base [1-2:z101+.] to (new) boundaries 4 - 5.
Step 6: Moved (old) base [0-1:z102+.] to (new) boundaries 3 - 4.
Step 7: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

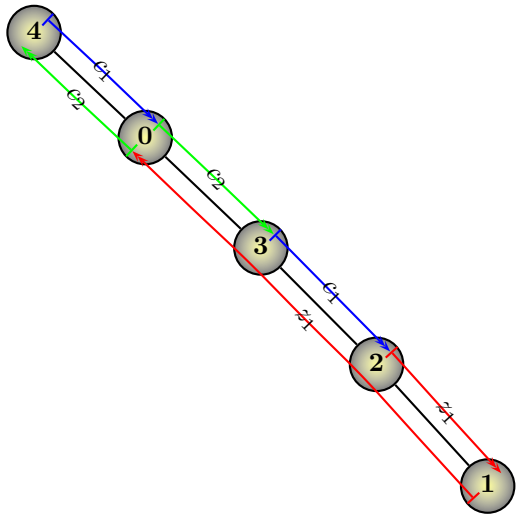
Upon applying the print, the GE we obtain—which we refer to as root-13.1—is illustrated below:



GE Information: Carrier: [0-3:z100-.] ; Carrier Dual: [2-3:z100+.] ; Critical Boundary: 3; Observe the following facts about this GE: The base [2-3:z100+.] and its dual are of opposite polarity, yet intersect. The base [0-3:z100-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-13.1, as derived from the application of a print to root-13.

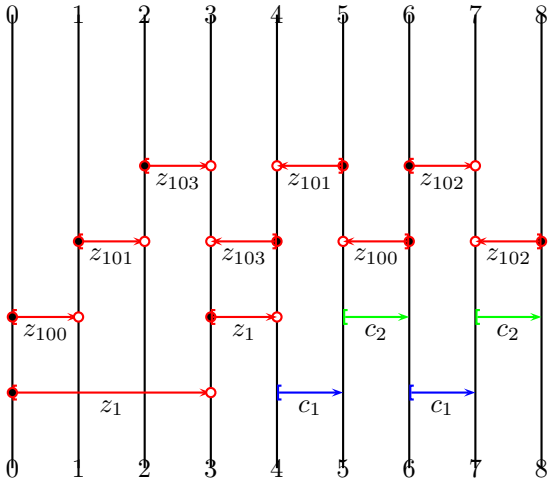
14 Cancellation scheme #14



z_1	$1 \leftarrow 2 \leftarrow 3 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$0 \leftarrow 3$
c_1	$4 \leftarrow 0$
c_2	$0 \leftarrow 4$

Generalized Equation root-14

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-3:z1+.] ; Carrier Dual: [3-4:z1+.] ; Critical Boundary: 3; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=3*<1<2<3=4*
```

We proceed.

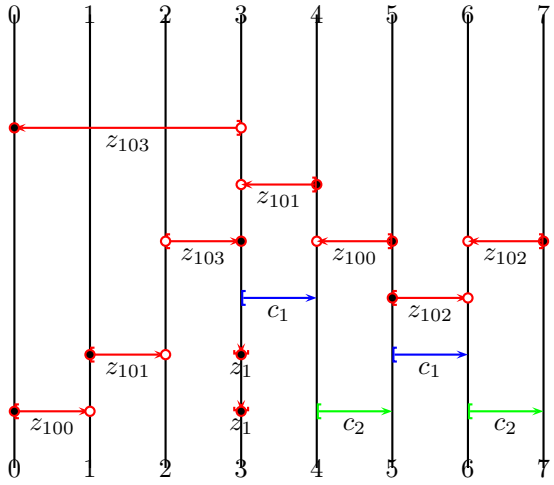
Generalized Equation root-14.1

We begin from the GE root-14 (see pp. 53). We consider its print

```
Print 1: =0=3*<1<2<3=4*
```

Sequence of actions in performing the Print 1:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 5.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 6.
Step 4: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 4.
Step 5: Moved (old) base [1-2:z101+.] to (new) boundaries 4 - 5.
Step 6: Moved (old) base [2-3:z103+.] to (new) boundaries 5 - 6.
Step 7: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

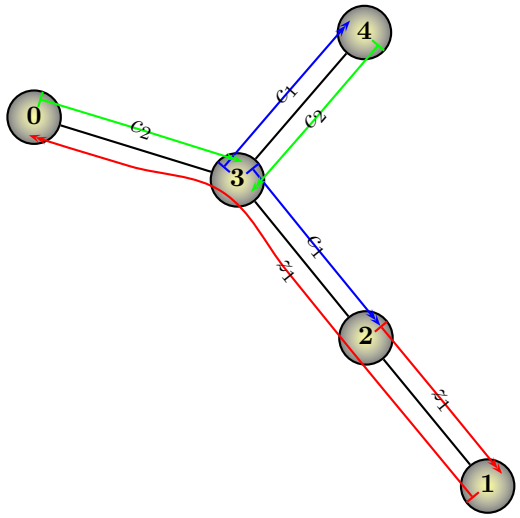
Upon applying the print, the GE we obtain—which we refer to as root-14.1—is illustrated below:



GE Information: Carrier: [0-3:z103-.] ; Carrier Dual: [2-3:z103+.] ; Critical Boundary: 3; Observe the following facts about this GE: The base [2-3:z103+.] and its dual are of opposite polarity, yet intersect. The base [0-3:z103-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-14.1, as derived from the application of a print to root-14.

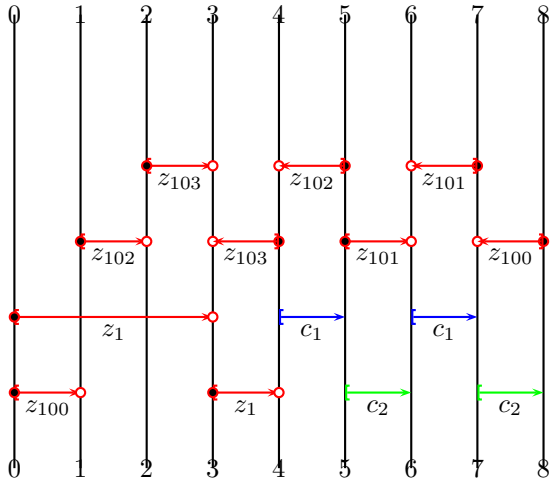
15 Cancellation scheme #15



z_1	$1 \leftarrow 2 \leftarrow 3 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$4 \leftarrow 3$
c_1	$3 \leftarrow 4$
c_2	$0 \leftarrow 3$

Generalized Equation root-15

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-3:z1+.] ; Carrier Dual: [3-4:z1+.] ; Critical Boundary: 3; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=3*<1<2<3=4*
```

We proceed.

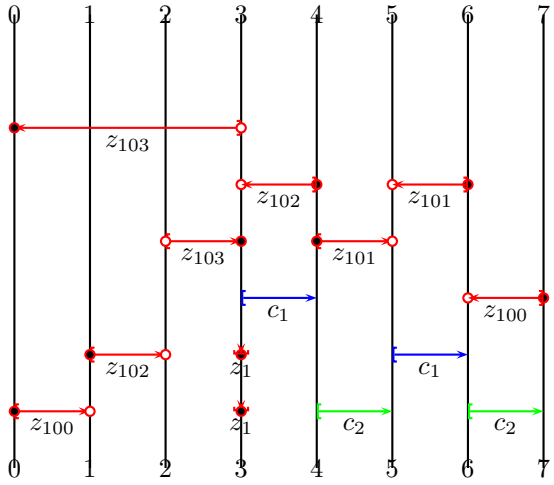
Generalized Equation root-15.1

We begin from the GE root-15 (see pp. 56). We consider its print

```
Print 1: =0=3*<1<2<3=4*
```

Sequence of actions in performing the Print 1:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 5.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 6.
Step 4: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 4.
Step 5: Moved (old) base [1-2:z102+.] to (new) boundaries 4 - 5.
Step 6: Moved (old) base [2-3:z103+.] to (new) boundaries 5 - 6.
Step 7: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

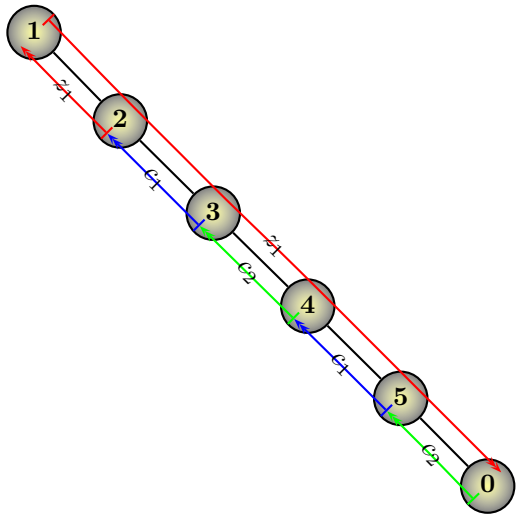
Upon applying the print, the GE we obtain—which we refer to as root-15.1—is illustrated below:



GE Information: Carrier: [0-3:z103-.] ; Carrier Dual: [2-3:z103+.] ; Critical Boundary: 3; Observe the following facts about this GE: The base [2-3:z103+.] and its dual are of opposite polarity, yet intersect. The base [0-3:z103-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-15.1, as derived from the application of a print to root-15.

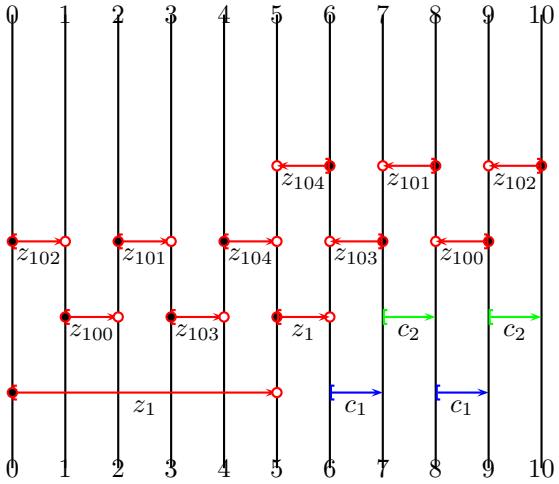
16 Cancellation scheme #16



z_1	$1 \leftarrow 2 \leftarrow 3 \leftarrow 4 \leftarrow 5 \leftarrow 0$
z_1	$2 \leftarrow 1$
c_1	$3 \leftarrow 2$
c_2	$4 \leftarrow 3$
c_1	$5 \leftarrow 4$
c_2	$0 \leftarrow 5$

Generalized Equation root-16

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-5:z1+.] ; Carrier Dual: [5-6:z1+.] ; Critical Boundary: 5; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=5*<1<2<3<4<5=6*
```

We proceed.

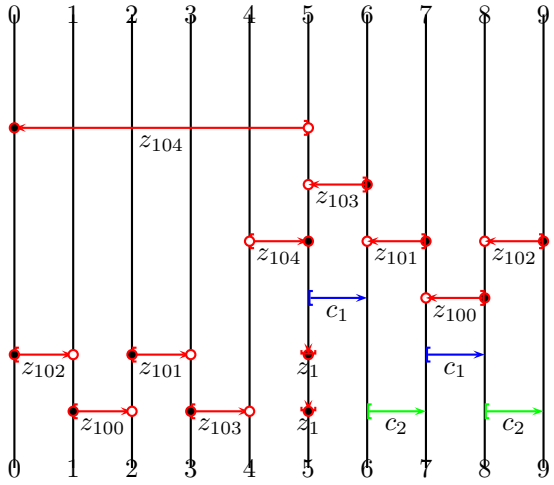
Generalized Equation root-16.1

We begin from the GE root-16 (see pp. 59). We consider its print

```
Print 1: =0=5*<1<2<3<4<5=6*
```

Sequence of actions in performing the Print 1:
Step 1: Added (new) boundary 6.
Step 2: Added (new) boundary 7.
Step 3: Added (new) boundary 8.
Step 4: Added (new) boundary 9.
Step 5: Moved (old) base [0-5:z1+.] to (new) boundaries 5 - 10.
Step 6: Moved (old) base [1-2:z100+.] to (new) boundaries 6 - 7.
Step 7: Moved (old) base [2-3:z101+.] to (new) boundaries 7 - 8.
Step 8: Moved (old) base [0-1:z102+.] to (new) boundaries 5 - 6.
Step 9: Moved (old) base [3-4:z103+.] to (new) boundaries 8 - 9.
Step 10: Moved (old) base [4-5:z104+.] to (new) boundaries 9 - 10.
Step 11: Collapsed (new) base [5-10:z1+.] to the empty base (10,10).
Step 12: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 13: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 14: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 15: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 16: Deleted (new) boundary 4 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

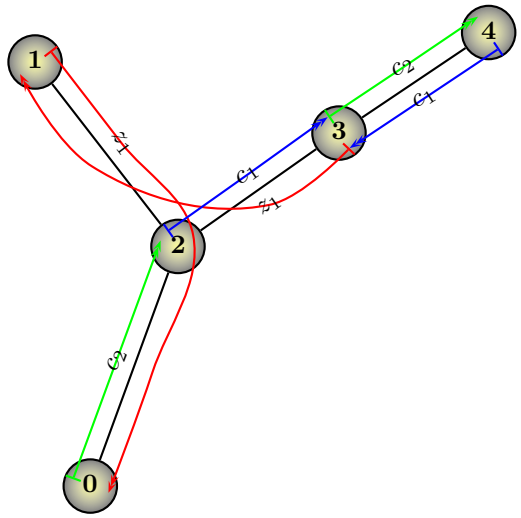
Upon applying the print, the GE we obtain—which we refer to as root-16.1—is illustrated below:



GE Information: Carrier: [0-5:z104-.] ; Carrier Dual: [4-5:z104+.] ; Critical Boundary: 5; Observe the following facts about this GE: The base [4-5:z104+.] and its dual are of opposite polarity, yet intersect. The base [0-5:z104-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-16.1, as derived from the application of a print to root-16.

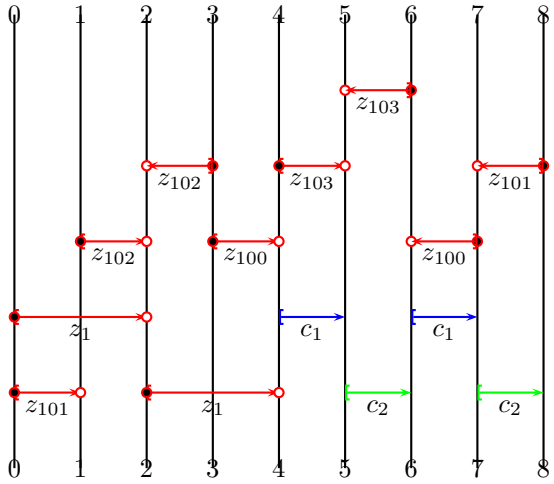
17 Cancellation scheme #17



z_1	$1 \leftarrow 2 \leftarrow 0$
z_1	$3 \leftarrow 2 \leftarrow 1$
c_1	$4 \leftarrow 3$
c_2	$3 \leftarrow 4$
c_1	$2 \leftarrow 3$
c_2	$0 \leftarrow 2$

Generalized Equation root-17

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-2:z1+.] ; Carrier Dual: [2-4:z1+.] ; Critical Boundary: 2; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 2 valid prints (descendents).

It has 2 legal carrier-to-dual prints, as follows:

```
Print 1: =0=2*<1<3*<2=4*
Print 2: =0=2*<3*<1<2=4*
```

We proceed.

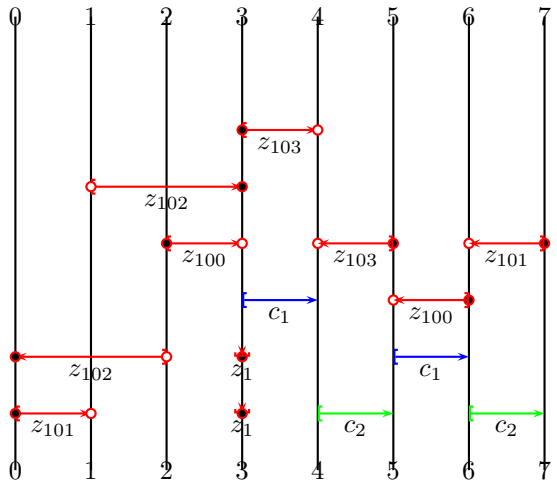
Generalized Equation root-17.1

We begin from the GE root-17 (see pp. 62). We consider its print

```
Print 1: =0=2*<1<3*<2=4*
```

Sequence of actions in performing the Print 1:
Step 1: Added (new) boundary 3.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [0-1:z101+.] to (new) boundaries 2 - 3.
Step 4: Moved (old) base [1-2:z102+.] to (new) boundaries 3 - 5.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-17.1—is illustrated below:



GE Information: Carrier: [0-2:z102-.] ; Carrier Dual: [1-3:z102+.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [1-3:z102+.] and its dual are of opposite polarity, yet intersect. The base [0-2:z102-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-17.1, as derived from the application of a print to root-17.

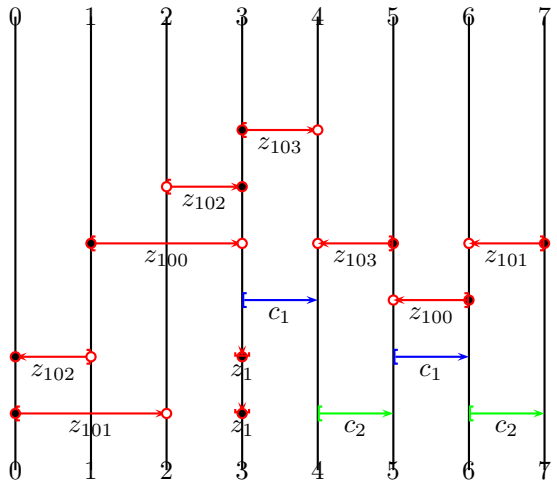
Generalized Equation root-17.2

We begin from the GE root-17 (see pp. 62). We consider its print

```
Print 2: =0=2*<3*<1<2=4*
```

Sequence of actions in performing the Print 2:
Step 1: Added (new) boundary 4.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [0-1:z101+.] to (new) boundaries 2 - 4.
Step 4: Moved (old) base [1-2:z102+.] to (new) boundaries 4 - 5.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

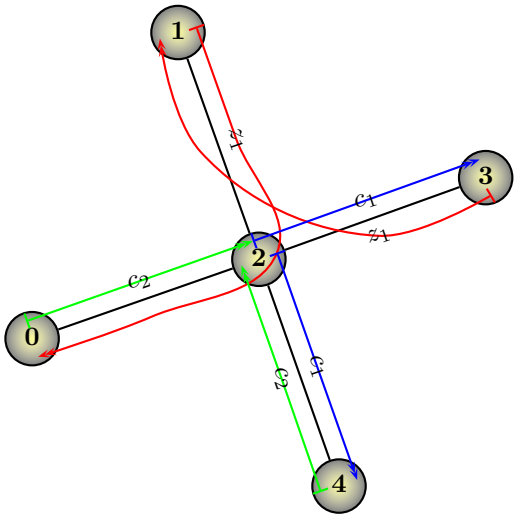
Upon applying the print, the GE we obtain—which we refer to as root-17.2—is illustrated below:



GE Information: Carrier: [0-2:z101+.] ; Carrier Dual: [6-7:z101-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [5-6:z100-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [6-7:z101-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-17.2, as derived from the application of a print to root-17.

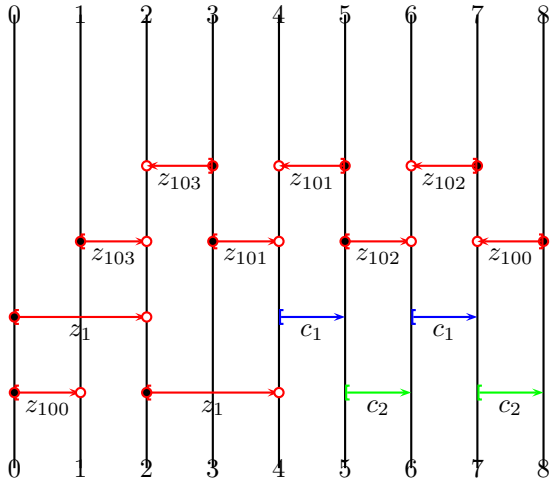
18 Cancellation scheme #18



z_1	$1 \leftarrow 2 \leftarrow 0$
z_1	$3 \leftarrow 2 \leftarrow 1$
c_1	$2 \leftarrow 3$
c_2	$4 \leftarrow 2$
c_1	$2 \leftarrow 4$
c_2	$0 \leftarrow 2$

Generalized Equation root-18

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-2:z1+.] ; Carrier Dual: [2-4:z1+.] ; Critical Boundary: 2; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 2 valid prints (descendents).

It has 2 legal carrier-to-dual prints, as follows:

```
Print 1: =0=2*<1<3*<2=4*
Print 2: =0=2*<3*<1<2=4*
```

We proceed.

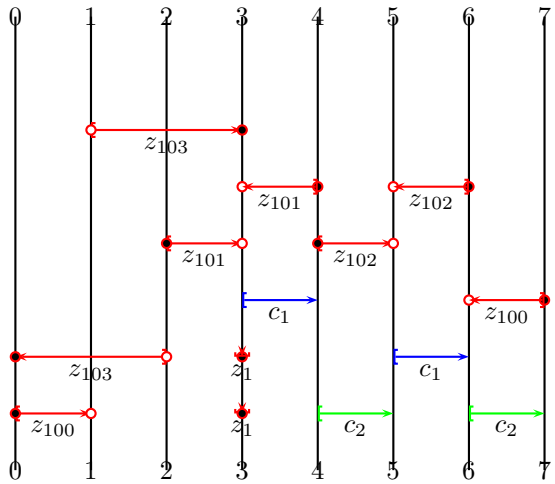
Generalized Equation root-18.1

We begin from the GE root-18 (see pp. 66). We consider its print

```
Print 1: =0=2*<1<3*<2=4*
```

Sequence of actions in performing the Print 1:
Step 1: Added (new) boundary 3.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [0-1:z100+.] to (new) boundaries 2 - 3.
Step 4: Moved (old) base [1-2:z103+.] to (new) boundaries 3 - 5.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-18.1—is illustrated below:



GE Information: Carrier: [0-2:z103-.] ; Carrier Dual: [1-3:z103+.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [1-3:z103+.] and its dual are of opposite polarity, yet intersect. The base [0-2:z103-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-18.1, as derived from the application of a print to root-18.

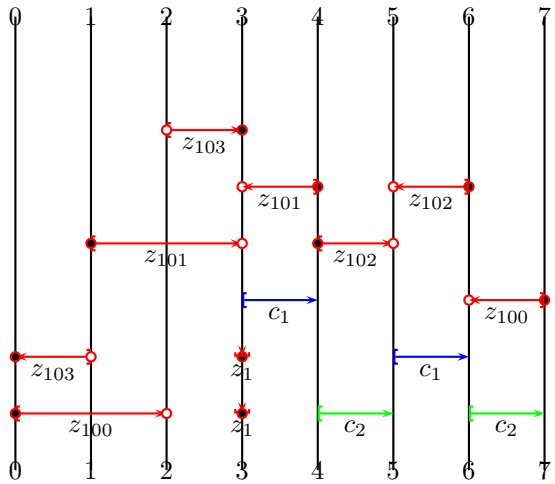
Generalized Equation root-18.2

We begin from the GE root-18 (see pp. 66). We consider its print

```
Print 2: =0=2*<3*<1<2=4*
```

Sequence of actions in performing the Print 2:
Step 1: Added (new) boundary 4.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [0-1:z100+.] to (new) boundaries 2 - 4.
Step 4: Moved (old) base [1-2:z103+.] to (new) boundaries 4 - 5.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

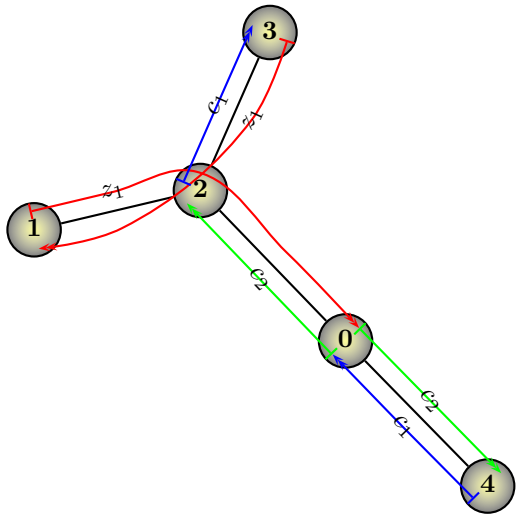
Upon applying the print, the GE we obtain—which we refer to as root-18.2—is illustrated below:



GE Information: Carrier: [0-2:z100+.] ; Carrier Dual: [6-7:z100-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [3-4:z101-.] has constraints with its dual that stretch the constant segment 3 - 4 to length different from 1. The base [6-7:z100-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-18.2, as derived from the application of a print to root-18.

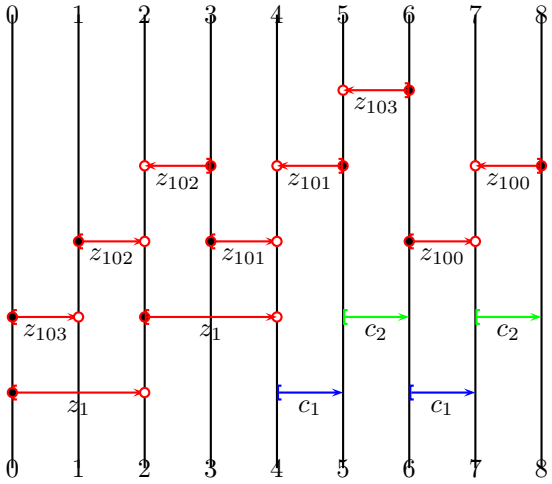
19 Cancellation scheme #19



z_1	$1 \leftarrow 2 \leftarrow 0$
z_1	$3 \leftarrow 2 \leftarrow 1$
c_1	$2 \leftarrow 3$
c_2	$0 \leftarrow 2$
c_1	$4 \leftarrow 0$
c_2	$0 \leftarrow 4$

Generalized Equation root-19

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-2:z1+.] ; Carrier Dual: [2-4:z1+.] ; Critical Boundary: 2; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 2 valid prints (descendents).

It has 2 legal carrier-to-dual prints, as follows:

```
Print 1: =0=2*<1<3*<2=4*
Print 2: =0=2*<3*<1<2=4*
```

We proceed.

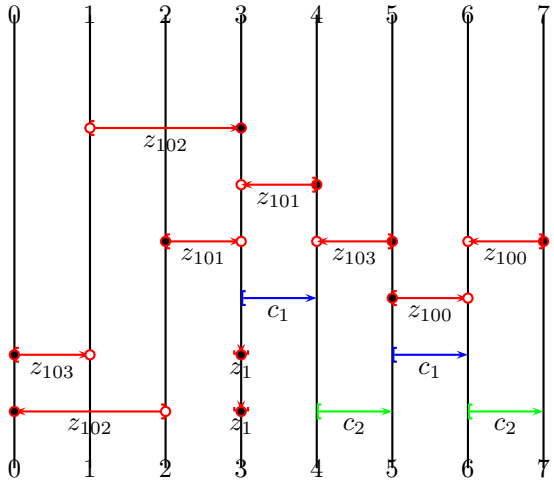
Generalized Equation root-19.1

We begin from the GE root-19 (see pp. 70). We consider its print

```
Print 1: =0=2*<1<3*<2=4*
```

Sequence of actions in performing the Print 1:
Step 1: Added (new) boundary 3.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [1-2:z102+.] to (new) boundaries 3 - 5.
Step 4: Moved (old) base [0-1:z103+.] to (new) boundaries 2 - 3.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-19.1—is illustrated below:



GE Information: Carrier: [0-2:z102-.] ; Carrier Dual: [1-3:z102+.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [1-3:z102+.] and its dual are of opposite polarity, yet intersect. The base [0-2:z102-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-19.1, as derived from the application of a print to root-19.

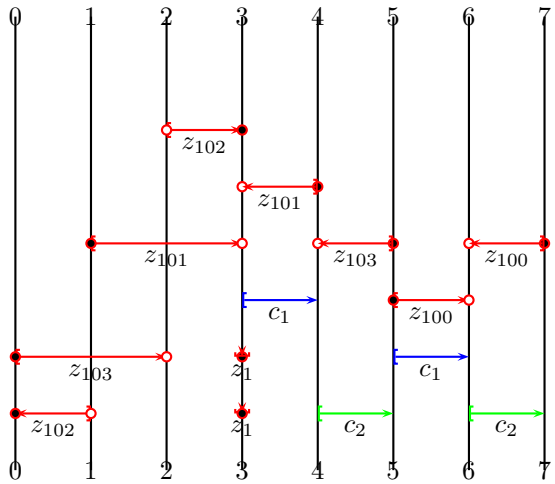
Generalized Equation root-19.2

We begin from the GE root-19 (see pp. 70). We consider its print

```
Print 2: =0=2*<3*<1<2=4*
```

Sequence of actions in performing the Print 2:
Step 1: Added (new) boundary 4.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 5.
Step 3: Moved (old) base [1-2:z102+.] to (new) boundaries 4 - 5.
Step 4: Moved (old) base [0-1:z103+.] to (new) boundaries 2 - 4.
Step 5: Collapsed (new) base [2-5:z1+.] to the empty base (5,5).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

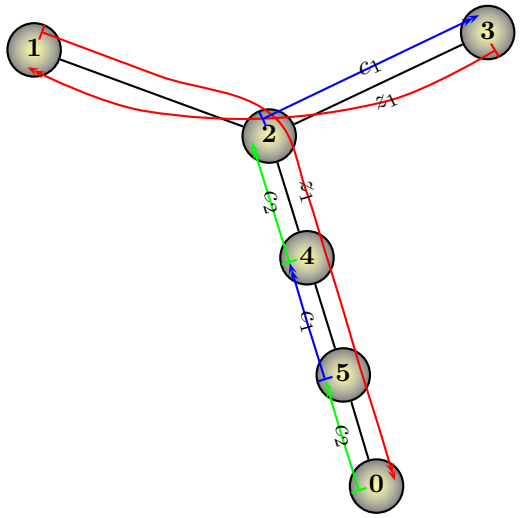
Upon applying the print, the GE we obtain—which we refer to as root-19.2—is illustrated below:



GE Information: Carrier: [0-2:z103+.] ; Carrier Dual: [4-5:z103-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [3-4:z101-.] has constraints with its dual that stretch the constant segment 3 - 4 to length different from 1. The base [4-5:z103-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-19.2, as derived from the application of a print to root-19.

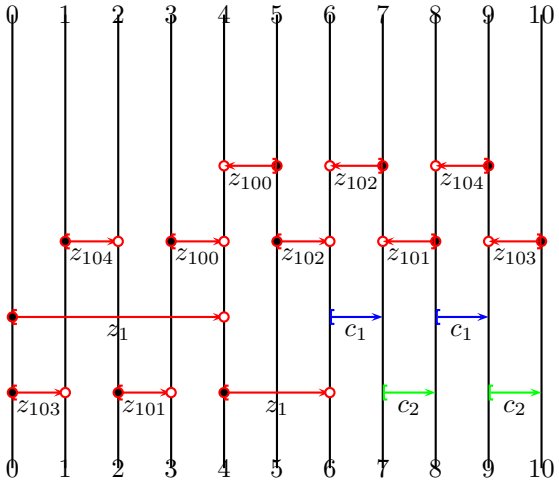
20 Cancellation scheme #20



z_1	$1 \leftarrow 2 \leftarrow 4 \leftarrow 5 \leftarrow 0$
z_1	$3 \leftarrow 2 \leftarrow 1$
c_1	$2 \leftarrow 3$
c_2	$4 \leftarrow 2$
c_1	$5 \leftarrow 4$
c_2	$0 \leftarrow 5$

Generalized Equation root-20

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-4:z1+.] ; Carrier Dual: [4-6:z1+.] ; Critical Boundary: 4; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 6 valid prints (descendents).

It has 6 legal carrier-to-dual prints, as follows:

```
Print 1: =0=4*<1<2<3<5*<4=6*
Print 2: =0=4*<1<2=5*<3<4=6*
Print 3: =0=4*<1<2<5*<3<4=6*
Print 4: =0=4*<1=5*<2<3<4=6*
Print 5: =0=4*<1<5*<2<3<4=6*
Print 6: =0=4*<5*<1<2<3<4=6*
```

We proceed.

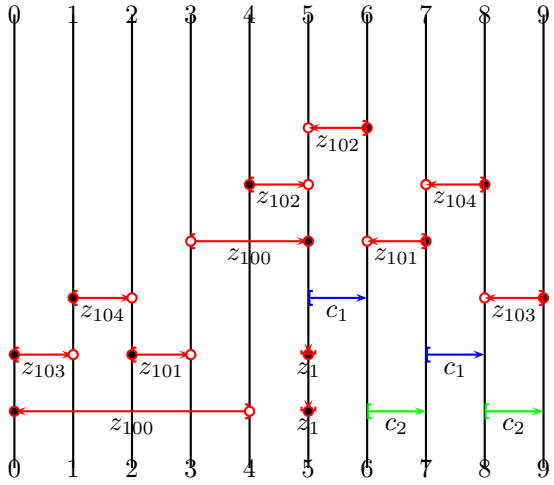
Generalized Equation root-20.1

We begin from the GE root-20 (see pp. 74). We consider its print

```
Print 1: =0=4*<1<2<3<5*<4=6*
```

Sequence of actions in performing the Print 1:
Step 1: Added (new) boundary 5.
Step 2: Added (new) boundary 6.
Step 3: Added (new) boundary 7.
Step 4: Moved (old) base [0-4:z1+.] to (new) boundaries 4 - 9.
Step 5: Moved (old) base [3-4:z100+.] to (new) boundaries 7 - 9.
Step 6: Moved (old) base [2-3:z101+.] to (new) boundaries 6 - 7.
Step 7: Moved (old) base [0-1:z103+.] to (new) boundaries 4 - 5.
Step 8: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 6.
Step 9: Collapsed (new) base [4-9:z1+.] to the empty base (9,9).
Step 10: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 11: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 12: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 13: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-20.1—is illustrated below:



GE Information: Carrier: [0-4:z100-.] ; Carrier Dual: [3-5:z100+.] ; Critical Boundary: 3; Observe the following facts about this GE: The base [3-5:z100+.] and its dual are of opposite polarity, yet intersect. The base [0-4:z100-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-20.1, as derived from the application of a print to root-20.

Generalized Equation root-20.2

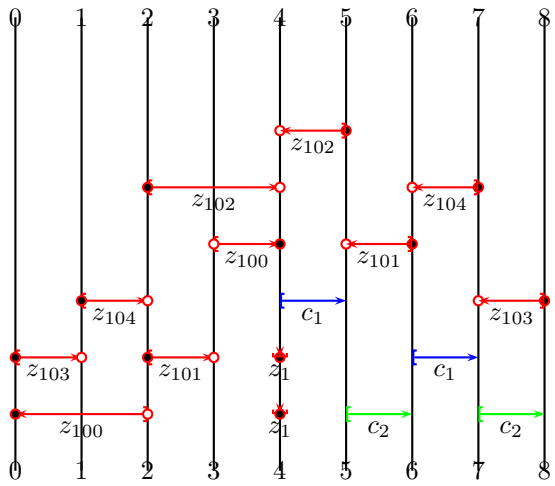
We begin from the GE root-20 (see pp. 74). We consider its print

```
Print 2: =0=4*<1<2=5*<3<4=6*
```

Sequence of actions in performing the Print 2:
Step 1: Added (new) boundary 5.
Step 2: Added (new) boundary 7.
Step 3: Moved (old) base [0-4:z1+.] to (new) boundaries 4 - 8.
Step 4: Moved (old) base [3-4:z100+.] to (new) boundaries 7 - 8.
Step 5: Moved (old) base [2-3:z101+.] to (new) boundaries 6 - 7.
Step 6: Moved (old) base [0-1:z103+.] to (new) boundaries 4 - 5.
Step 7: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 6.
Step 8: Collapsed (new) base [4-8:z1+.] to the empty base (8,8).
Step 9: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 1 because it is not used inside any base. This

will cause renumbering of higher numbered boundaries.
Step 11: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 12: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-20.2—is illustrated below:



GE Information: Carrier: [0-2:z100-.] ; Carrier Dual: [3-4:z100+.] ; Critical Boundary: 2; Observe the following facts about this GE: The base [4-5:z102-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-20.2, as derived from the application of a print to root-20.

Generalized Equation root-20.3

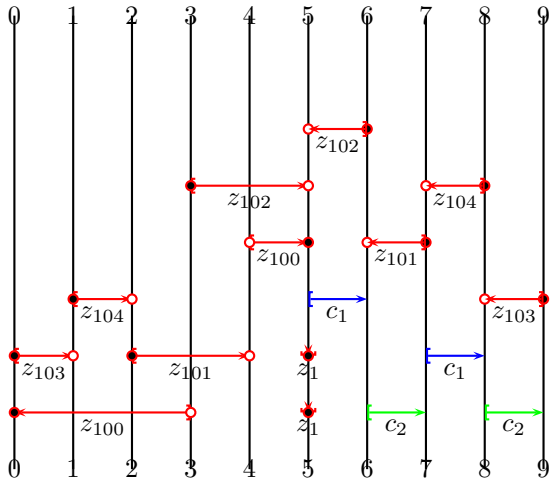
We begin from the GE root-20 (see pp. 74). We consider its print

```
Print 3: =0=4*<1<2<5*<3<4=6*
```

Sequence of actions in performing the Print 3:
Step 1: Added (new) boundary 5.
Step 2: Added (new) boundary 6.
Step 3: Added (new) boundary 8.

Step 4: Moved (old) base [0-4:z1+.] to (new) boundaries 4 - 9.
Step 5: Moved (old) base [3-4:z100+.] to (new) boundaries 8 - 9.
Step 6: Moved (old) base [2-3:z101+.] to (new) boundaries 6 - 8.
Step 7: Moved (old) base [0-1:z103+.] to (new) boundaries 4 - 5.
Step 8: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 6.
Step 9: Collapsed (new) base [4-9:z1+.] to the empty base (9,9).
Step 10: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 11: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 12: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 13: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-20.3—is illustrated below:



GE Information: Carrier: [0-3:z100-.] ; Carrier Dual: [4-5:z100+.] ; Critical Boundary: 2; Observe the following facts about this GE: The base [5-6:z102-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [6-7:z101-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-20.3, as derived from the application of a print to root-20.

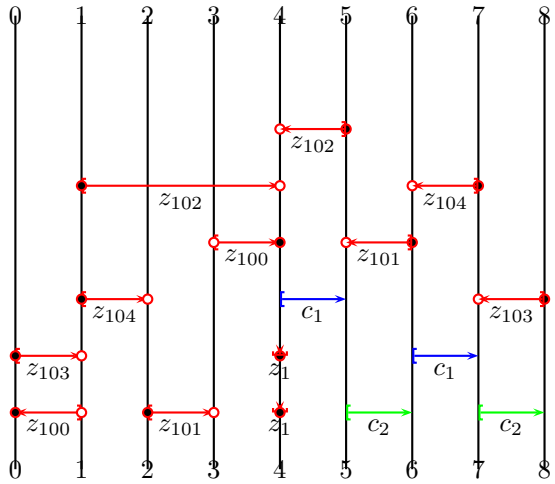
Generalized Equation root-20.4

We begin from the GE root-20 (see pp. 74). We consider its print

```
Print 4: =0=4*<1=5*<2<3<4=6*
```

Sequence of actions in performing the Print 4:
Step 1: Added (new) boundary 6.
Step 2: Added (new) boundary 7.
Step 3: Moved (old) base [0-4:z1+.] to (new) boundaries 4 - 8.
Step 4: Moved (old) base [3-4:z100+.] to (new) boundaries 7 - 8.
Step 5: Moved (old) base [2-3:z101+.] to (new) boundaries 6 - 7.
Step 6: Moved (old) base [0-1:z103+.] to (new) boundaries 4 - 5.
Step 7: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 6.
Step 8: Collapsed (new) base [4-8:z1+.] to the empty base (8,8).
Step 9: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 11: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 12: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-20.4—is illustrated below:



GE Information: Carrier: [0-1:z100-.] ; Carrier Dual: [3-4:z100+.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [4-5:z102-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. These observations show that the GE above is degenerate.

This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-20.4, as derived from the application of a print to root-20.

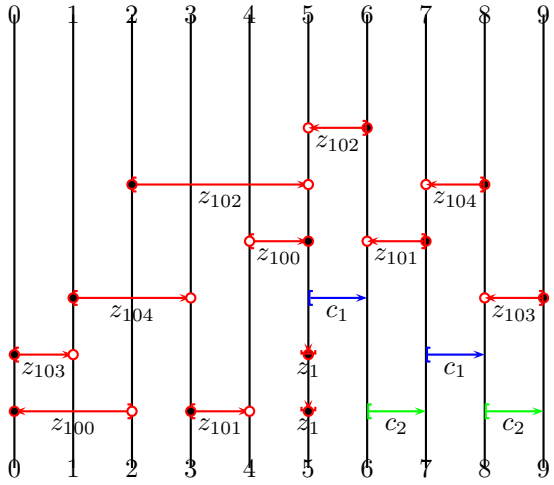
Generalized Equation root-20.5

We begin from the GE root-20 (see pp. 74). We consider its print

```
Print 5: =0=4*<1<5*<2<3<4=6*
```

Sequence of actions in performing the Print 5:
Step 1: Added (new) boundary 5.
Step 2: Added (new) boundary 7.
Step 3: Added (new) boundary 8.
Step 4: Moved (old) base [0-4:z1+.] to (new) boundaries 4 - 9.
Step 5: Moved (old) base [3-4:z100+.] to (new) boundaries 8 - 9.
Step 6: Moved (old) base [2-3:z101+.] to (new) boundaries 7 - 8.
Step 7: Moved (old) base [0-1:z103+.] to (new) boundaries 4 - 5.
Step 8: Moved (old) base [1-2:z104+.] to (new) boundaries 5 - 7.
Step 9: Collapsed (new) base [4-9:z1+.] to the empty base (9,9).
Step 10: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 11: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 12: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 13: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-20.5—is illustrated below:



GE Information: Carrier: [0-2:z100-.] ; Carrier Dual: [4-5:z100+.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [5-6:z102-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [7-8:z104-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-20.5, as derived from the application of a print to root-20.

Generalized Equation root-20.6

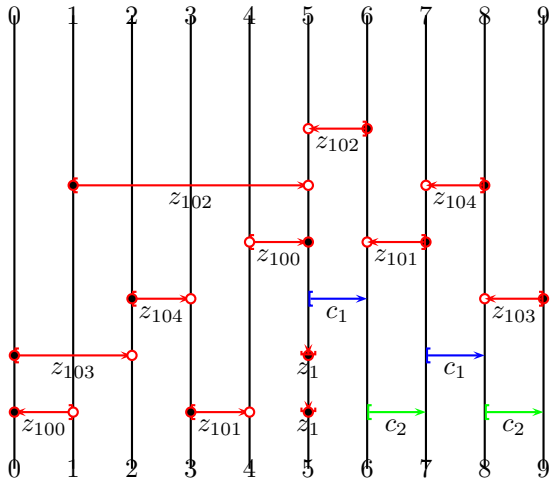
We begin from the GE root-20 (see pp. 74). We consider its print

```
Print 6: =0=4*<5*<1<2<3<4=6*
```

Sequence of actions in performing the Print 6:
Step 1: Added (new) boundary 6.
Step 2: Added (new) boundary 7.
Step 3: Added (new) boundary 8.
Step 4: Moved (old) base [0-4:z1+.] to (new) boundaries 4 - 9.
Step 5: Moved (old) base [3-4:z100+.] to (new) boundaries 8 - 9.
Step 6: Moved (old) base [2-3:z101+.] to (new) boundaries 7 - 8.
Step 7: Moved (old) base [0-1:z103+.] to (new) boundaries 4 - 6.
Step 8: Moved (old) base [1-2:z104+.] to (new) boundaries 6 - 7.
Step 9: Collapsed (new) base [4-9:z1+.] to the empty base (9,9).
Step 10: Deleted (new) boundary 0 because it is not used inside any base. This

will cause renumbering of higher numbered boundaries.
Step 11: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 12: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 13: Deleted (new) boundary 3 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

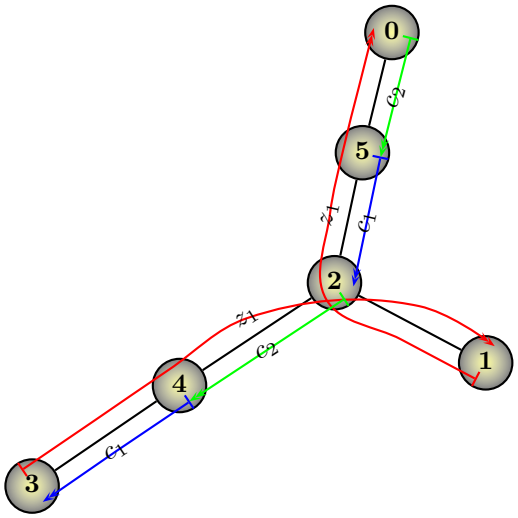
Upon applying the print, the GE we obtain—which we refer to as root-20.6—is illustrated below:



GE Information: Carrier: [0-2:z103+.] ; Carrier Dual: [8-9:z103-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [5-6:z102-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [8-9:z103-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-20.6, as derived from the application of a print to root-20.

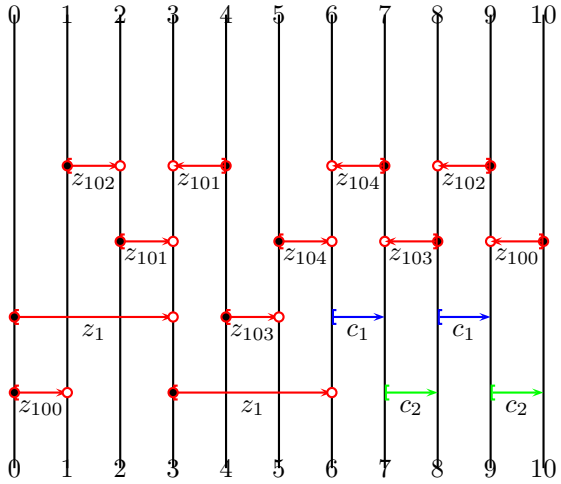
21 Cancellation scheme #21



z_1	$1 \leftarrow 2 \leftarrow 5 \leftarrow 0$
z_1	$3 \leftarrow 4 \leftarrow 2 \leftarrow 1$
c_1	$4 \leftarrow 3$
c_2	$2 \leftarrow 4$
c_1	$5 \leftarrow 2$
c_2	$0 \leftarrow 5$

Generalized Equation root-21

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-3:z1+.] ; Carrier Dual: [3-6:z1+.] ; Critical Boundary: 3; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 12 valid prints (descendents).

It has 12 legal carrier-to-dual prints, as follows:

```
Print 1: =0=3*<1<2<4*<5*<3=6*
Print 2: =0=3*<1=4*<2=5*<3=6*
Print 3: =0=3*<1=4*<2<5*<3=6*
Print 4: =0=3*<1=4*<5*<2<3=6*
Print 5: =0=3*<1<4*<2=5*<3=6*
Print 6: =0=3*<1<4*<2<5*<3=6*
Print 7: =0=3*<1<4*<5*<2<3=6*
Print 8: =0=3*<4*<1<2=5*<3=6*
Print 9: =0=3*<4*<1<2<5*<3=6*
Print 10: =0=3*<4*<1=5*<2<3=6*
Print 11: =0=3*<4*<1<5*<2<3=6*
Print 12: =0=3*<4*<5*<1<2<3=6*
```

We proceed.

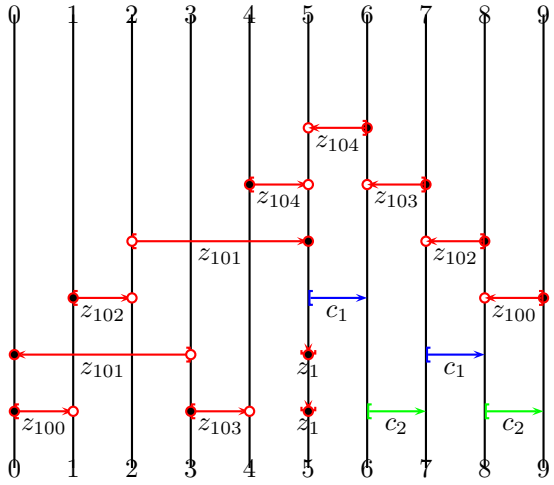
Generalized Equation root-21.1

We begin from the GE root-21 (see pp. 83). We consider its print

```
Print 1: =0=3*<1<2<4*<5*<3=6*
```

Sequence of actions in performing the Print 1:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 5.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 8.
Step 4: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 4.
Step 5: Moved (old) base [2-3:z101+.] to (new) boundaries 5 - 8.
Step 6: Moved (old) base [1-2:z102+.] to (new) boundaries 4 - 5.
Step 7: Collapsed (new) base [3-8:z1+.] to the empty base (8,8).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.1—is illustrated below:



GE Information: Carrier: [0-3:z101-.] ; Carrier Dual: [2-5:z101+.] ; Critical Boundary: 2; Observe the following facts about this GE: The base [2-5:z101+.] and its dual are of opposite polarity, yet intersect. The base [0-3:z101-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-21.1, as derived from the application of a print to root-21.

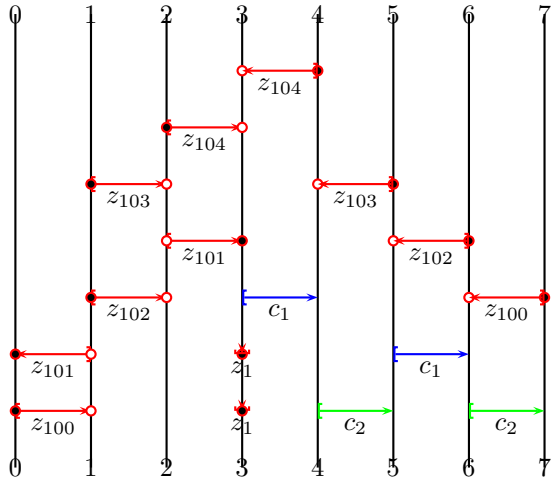
Generalized Equation root-21.2

We begin from the GE root-21 (see pp. 83). We consider its print

```
Print 2: =0=3*<1=4*<2=5*<3=6*
```

Sequence of actions in performing the Print 2:
Step 1: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 6.
Step 2: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 4.
Step 3: Moved (old) base [2-3:z101+.] to (new) boundaries 5 - 6.
Step 4: Moved (old) base [1-2:z102+.] to (new) boundaries 4 - 5.
Step 5: Collapsed (new) base [3-6:z1+.] to the empty base (6,6).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 8: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.2—is illustrated below:



GE Information: Carrier: [0-1:z100+.] ; Carrier Dual: [6-7:z100-.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=7*<1=6*
```

This completes the consideration of root-21.2, as derived from the application of a print to root-21.

Generalized Equation root-21.3

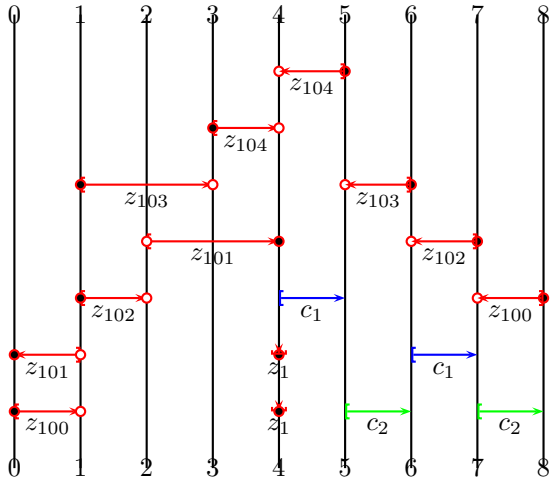
We begin from the GE root-21 (see pp. 83). We consider its print

```
Print 3: =0=3*<1=4*<2<5*<3=6*
```

Sequence of actions in performing the Print 3:
Step 1: Added (new) boundary 5.
Step 2: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 7.
Step 3: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 4.
Step 4: Moved (old) base [2-3:z101+.] to (new) boundaries 5 - 7.
Step 5: Moved (old) base [1-2:z102+.] to (new) boundaries 4 - 5.
Step 6: Collapsed (new) base [3-7:z1+.] to the empty base (7,7).
Step 7: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Step 8: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.3—is illustrated below:



GE Information: Carrier: [0-1:z100+.] ; Carrier Dual: [7-8:z100-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [5-6:z103-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-21.3, as derived from the application of a print to root-21.

Generalized Equation root-21.4

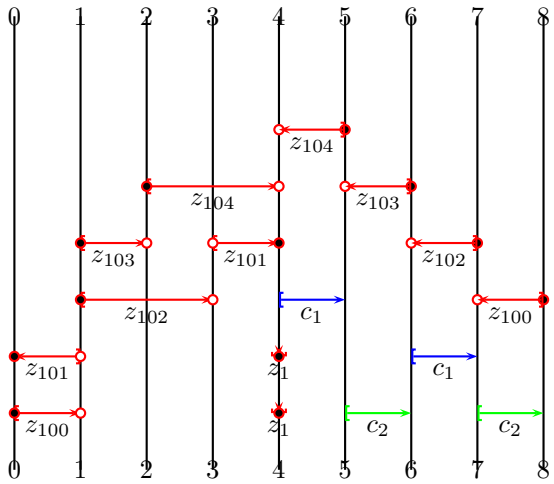
We begin from the GE root-21 (see pp. 83). We consider its print

```
Print 4: =0=3*<1=4*<5*<2<3=6*
```

Sequence of actions in performing the Print 4:
Step 1: Added (new) boundary 6.
Step 2: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 7.
Step 3: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 4.
Step 4: Moved (old) base [2-3:z101+.] to (new) boundaries 6 - 7.

Step 5: Moved (old) base [1-2:z102+.] to (new) boundaries 4 - 6.
Step 6: Collapsed (new) base [3-7:z1+.] to the empty base (7,7).
Step 7: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 8: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.4—is illustrated below:



GE Information: Carrier: [0-1:z100+.] ; Carrier Dual: [7-8:z100-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [4-5:z104-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. The base [6-7:z102-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-21.4, as derived from the application of a print to root-21.

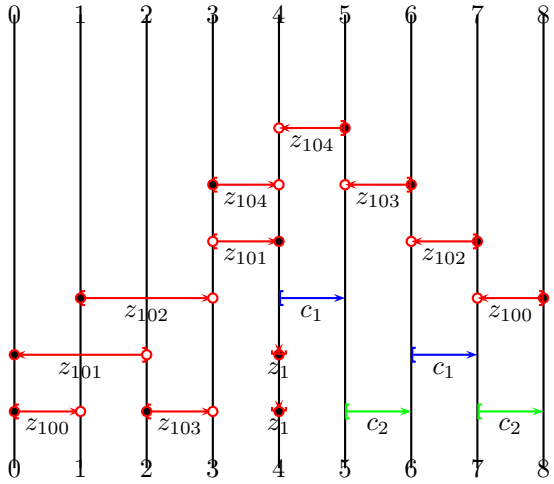
Generalized Equation root-21.5

We begin from the GE root-21 (see pp. 83). We consider its print

```
Print 5: =0=3*<1<4*<2=5*<3=6*
```

Sequence of actions in performing the Print 5:
Step 1: Added (new) boundary 4.
Step 2: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 7.
Step 3: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 4.
Step 4: Moved (old) base [2-3:z101+.] to (new) boundaries 6 - 7.
Step 5: Moved (old) base [1-2:z102+.] to (new) boundaries 4 - 6.
Step 6: Collapsed (new) base [3-7:z1+.] to the empty base (7,7).
Step 7: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 8: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.5—is illustrated below:



GE Information: Carrier: [0-2:z101-.] ; Carrier Dual: [3-4:z101+.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [6-7:z102-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-21.5, as derived from the application of a print to root-21.

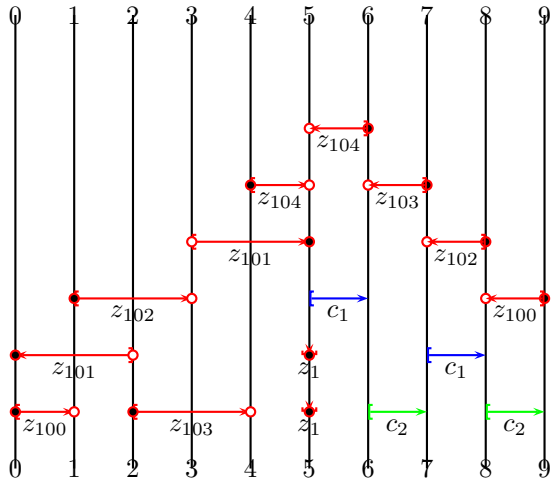
Generalized Equation root-21.6

We begin from the GE root-21 (see pp. 83). We consider its print

```
Print 6: =0=3*<1<4*<2<5*<3=6*
```

Sequence of actions in performing the Print 6:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 6.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 8.
Step 4: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 4.
Step 5: Moved (old) base [2-3:z101+.] to (new) boundaries 6 - 8.
Step 6: Moved (old) base [1-2:z102+.] to (new) boundaries 4 - 6.
Step 7: Collapsed (new) base [3-8:z1+.] to the empty base (8,8).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.6—is illustrated below:



GE Information: Carrier: [0-2:z101-.] ; Carrier Dual: [3-5:z101+.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [6-7:z103-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. The base [7-8:z102-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-21.6, as derived from the application of a print to root-21.

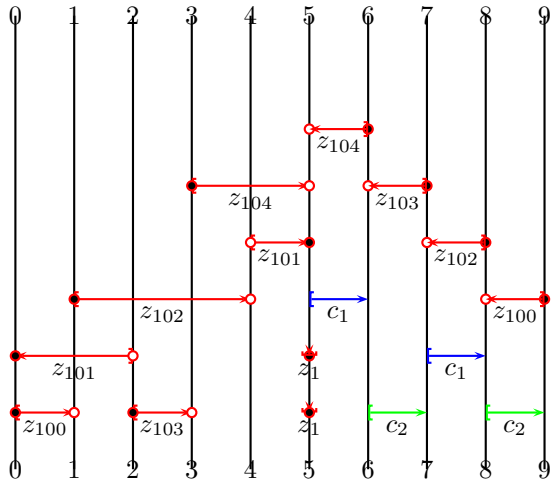
Generalized Equation root-21.7

We begin from the GE root-21 (see pp. 83). We consider its print

```
Print 7: =0=3*<1<4*<5*<2<3=6*
```

Sequence of actions in performing the Print 7:
Step 1: Added (new) boundary 4.
Step 2: Added (new) boundary 7.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 8.
Step 4: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 4.
Step 5: Moved (old) base [2-3:z101+.] to (new) boundaries 7 - 8.
Step 6: Moved (old) base [1-2:z102+.] to (new) boundaries 4 - 7.
Step 7: Collapsed (new) base [3-8:z1+.] to the empty base (8,8).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.7—is illustrated below:



GE Information: Carrier: [0-2:z101-.] ; Carrier Dual: [4-5:z101+.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [5-6:z104-.]

has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [7-8:z102-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-21.7, as derived from the application of a print to root-21.

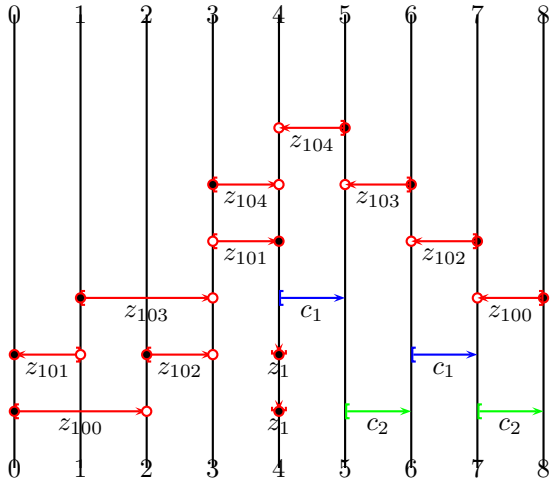
Generalized Equation root-21.8

We begin from the GE root-21 (see pp. 83). We consider its print

```
Print 8: =0=3*<4*<1<2=5*<3=6*
```

Sequence of actions in performing the Print 8:
Step 1: Added (new) boundary 5.
Step 2: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 7.
Step 3: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 5.
Step 4: Moved (old) base [2-3:z101+.] to (new) boundaries 6 - 7.
Step 5: Moved (old) base [1-2:z102+.] to (new) boundaries 5 - 6.
Step 6: Collapsed (new) base [3-7:z1+.] to the empty base (7,7).
Step 7: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 8: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.8—is illustrated below:



GE Information: Carrier: [0-2:z100+.] ; Carrier Dual: [7-8:z100-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [5-6:z103-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [7-8:z100-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-21.8, as derived from the application of a print to root-21.

Generalized Equation root-21.9

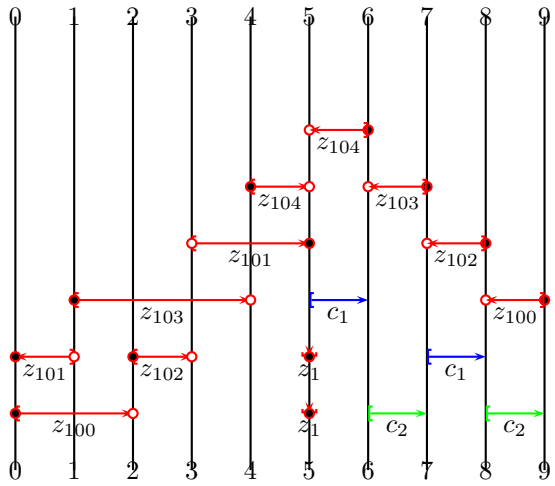
We begin from the GE root-21 (see pp. 83). We consider its print

```
Print 9: =0=3*<4*<1<2<5*<3=6*
```

Sequence of actions in performing the Print 9:
Step 1: Added (new) boundary 5.
Step 2: Added (new) boundary 6.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 8.
Step 4: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 5.
Step 5: Moved (old) base [2-3:z101+.] to (new) boundaries 6 - 8.
Step 6: Moved (old) base [1-2:z102+.] to (new) boundaries 5 - 6.
Step 7: Collapsed (new) base [3-8:z1+.] to the empty base (8,8).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This

will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.9—is illustrated below:



GE Information: Carrier: [0-2:z100+.] ; Carrier Dual: [8-9:z100-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [6-7:z103-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. The base [8-9:z100-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-21.9, as derived from the application of a print to root-21.

Generalized Equation root-21.10

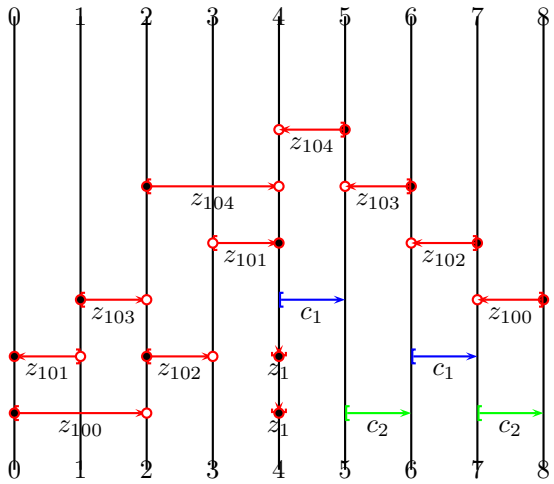
We begin from the GE root-21 (see pp. 83). We consider its print

```
Print 10: =0=3*<4*<1=5*<2<3=6*
```

Sequence of actions in performing the Print 10:
Step 1: Added (new) boundary 6.
Step 2: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 7.
Step 3: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 5.
Step 4: Moved (old) base [2-3:z101+.] to (new) boundaries 6 - 7.

Step 5: Moved (old) base [1-2:z102+.] to (new) boundaries 5 - 6.
Step 6: Collapsed (new) base [3-7:z1+.] to the empty base (7,7).
Step 7: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 8: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.10—is illustrated below:



GE Information: Carrier: [0-2:z100+.] ; Carrier Dual: [7-8:z100-.] ; Critical Boundary: 2; Observe the following facts about this GE: The base [4-5:z104-.] has constraints with its dual that stretch the constant segment 4 - 5 to length different from 1. The base [7-8:z100-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-21.10, as derived from the application of a print to root-21.

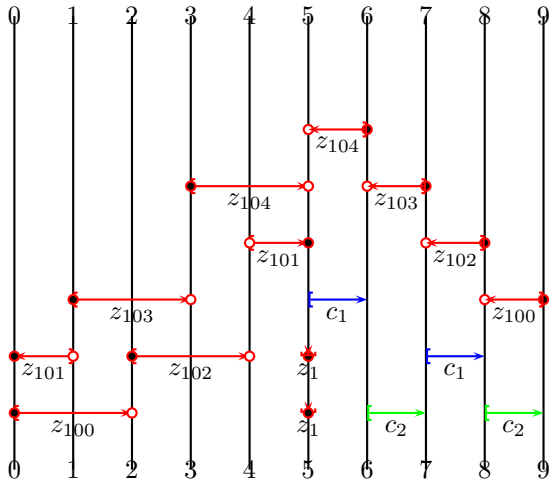
Generalized Equation root-21.11

We begin from the GE root-21 (see pp. 83). We consider its print

```
Print 11: =0=3*<4*<1<5*<2<3=6*
```

Sequence of actions in performing the Print 11:
Step 1: Added (new) boundary 5.
Step 2: Added (new) boundary 7.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 8.
Step 4: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 5.
Step 5: Moved (old) base [2-3:z101+.] to (new) boundaries 7 - 8.
Step 6: Moved (old) base [1-2:z102+.] to (new) boundaries 5 - 7.
Step 7: Collapsed (new) base [3-8:z1+.] to the empty base (8,8).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.11—is illustrated below:



GE Information: Carrier: [0-2:z100+.] ; Carrier Dual: [8-9:z100-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [5-6:z104-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [6-7:z103-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. The base [7-8:z102-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. The base [8-9:z100-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-21.11, as derived from the application

of a print to root-21.

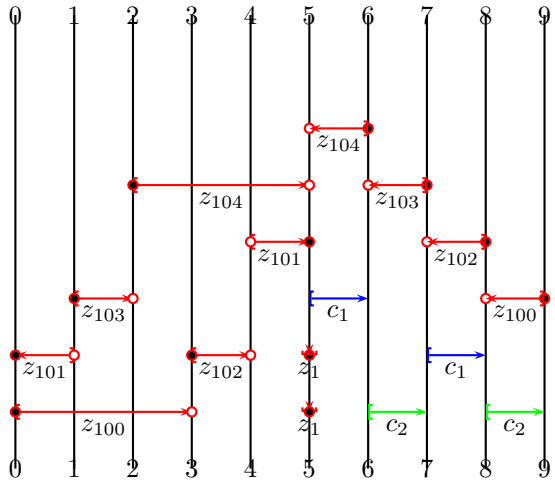
Generalized Equation root-21.12

We begin from the GE root-21 (see pp. 83). We consider its print

```
Print 12: =0=3*<4*<5*<1<2<3=6*
```

Sequence of actions in performing the Print 12:
Step 1: Added (new) boundary 6.
Step 2: Added (new) boundary 7.
Step 3: Moved (old) base [0-3:z1+.] to (new) boundaries 3 - 8.
Step 4: Moved (old) base [0-1:z100+.] to (new) boundaries 3 - 6.
Step 5: Moved (old) base [2-3:z101+.] to (new) boundaries 7 - 8.
Step 6: Moved (old) base [1-2:z102+.] to (new) boundaries 6 - 7.
Step 7: Collapsed (new) base [3-8:z1+.] to the empty base (8,8).
Step 8: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 9: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 10: Deleted (new) boundary 2 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.12—is illustrated below:



GE Information: Carrier: [0-3:z100+.] ; Carrier Dual: [8-9:z100-.] ; Critical Boundary: 2; Observe the following facts about this GE: The base [5-6:z104-.] has constraints with its dual that stretch the constant segment 5 - 6 to length

different from 1. The base [8-9:z100-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-21.12, as derived from the application of a print to root-21.

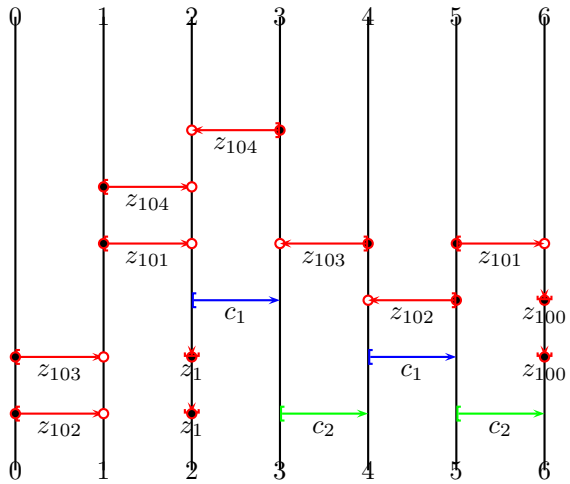
Generalized Equation root-21.2.1

We begin from the GE root-21.2 (see pp. 85). We consider its print

```
Print 1: =0=7*<1=6*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z100+.] to (new) boundaries 7 - 6.
Step 2: Moved (old) base [0-1:z101-.] to (new) boundaries 7 - 6.
Step 3: Collapsed (new) base [6-7:z100-.] to the empty base (7,7).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.2.1—is illustrated below:



GE Information: Carrier: [0-1:z102+.] ; Carrier Dual: [4-5:z102-.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=5*<1=4*
```

This completes the consideration of root-21.2.1, as derived from the application of a print to root-21.2.

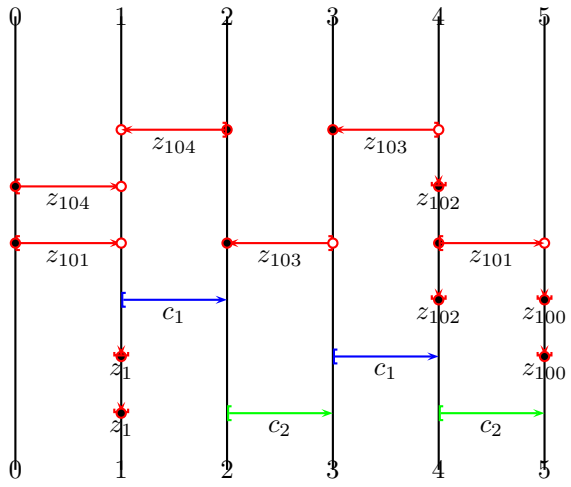
Generalized Equation root-21.2.1.1

We begin from the GE root-21.2.1 (see pp. 98). We consider its print

```
Print 1: =0=5*<1=4*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z102+.] to (new) boundaries 5 - 4.
Step 2: Moved (old) base [0-1:z103+.] to (new) boundaries 5 - 4.
Step 3: Collapsed (new) base [4-5:z102-.] to the empty base (5,5).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.2.1.1—is illustrated below:



GE Information: Carrier: [0-1:z101+.] ; Carrier Dual: [4-5:z101+.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=4*<1=5*
```

This completes the consideration of root-21.2.1.1, as derived from the application of a print to root-21.2.1.

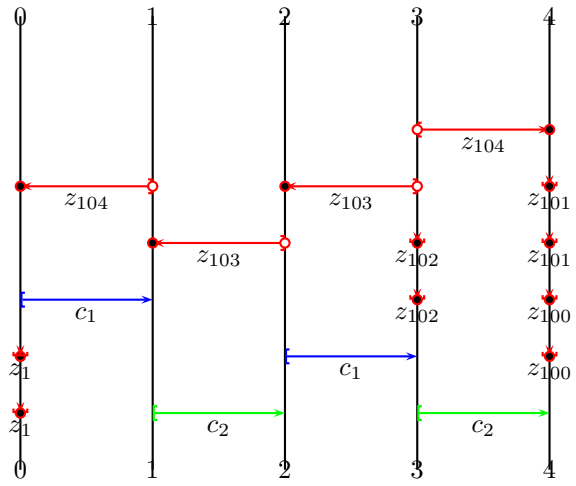
Generalized Equation root-21.2.1.1.1

We begin from the GE root-21.2.1.1 (see pp. 99). We consider its print

```
Print 1: =0=4*<1=5*
```

Sequence of actions in performing the Print 1:
Step 1: Moved (old) base [0-1:z101+.] to (new) boundaries 4 - 5.
Step 2: Moved (old) base [0-1:z104+.] to (new) boundaries 4 - 5.
Step 3: Collapsed (new) base [4-5:z101+.] to the empty base (5,5).
Step 4: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-21.2.1.1.1—is illustrated below:



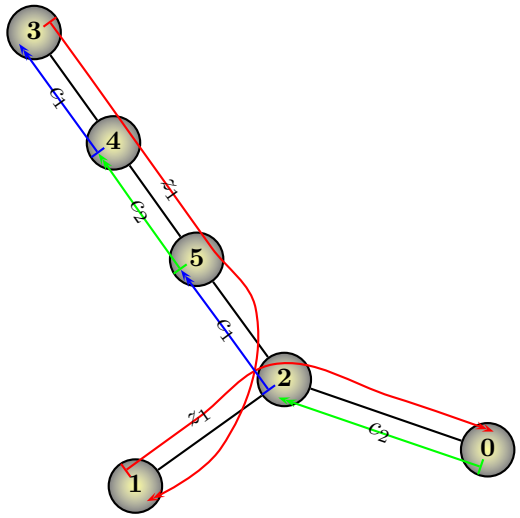
GE Information: Carrier: [0-1:z104-.] ; Carrier Dual: [3-4:z104+.] ; Critical Boundary: 1; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 1 valid prints (descendents).

It has 1 legal carrier-to-dual prints, as follows:

```
Print 1: =0=4*<1=3*
```

This completes the consideration of root-21.2.1.1.1, as derived from the application of a print to root-21.2.1.1.

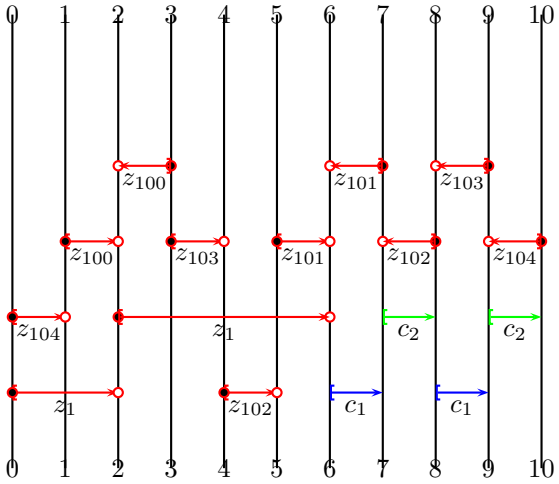
22 Cancellation scheme #22



z_1	$1 \leftarrow 2 \leftarrow 0$
z_1	$3 \leftarrow 4 \leftarrow 5 \leftarrow 2 \leftarrow 1$
c_1	$4 \leftarrow 3$
c_2	$5 \leftarrow 4$
c_1	$2 \leftarrow 5$
c_2	$0 \leftarrow 2$

Generalized Equation root-22

Below is the root GE obtained from the cancellation diagram above.



GE Information: Carrier: [0-2:z1+.] ; Carrier Dual: [2-6:z1+.] ; Critical Boundary: 2; The GE above is non-degenerate. This GE is *not* a leaf in the GE tree. It has 6 valid prints (descendents).

It has 6 legal carrier-to-dual prints, as follows:

```
Print 1: =0=2*<1<3*<4*<5*<2=6*
Print 2: =0=2*<3*<1=4*<5*<2=6*
Print 3: =0=2*<3*<1<4*<5*<2=6*
Print 4: =0=2*<3*<4*<1=5*<2=6*
Print 5: =0=2*<3*<4*<1<5*<2=6*
Print 6: =0=2*<3*<4*<5*<1<2=6*
```

We proceed.

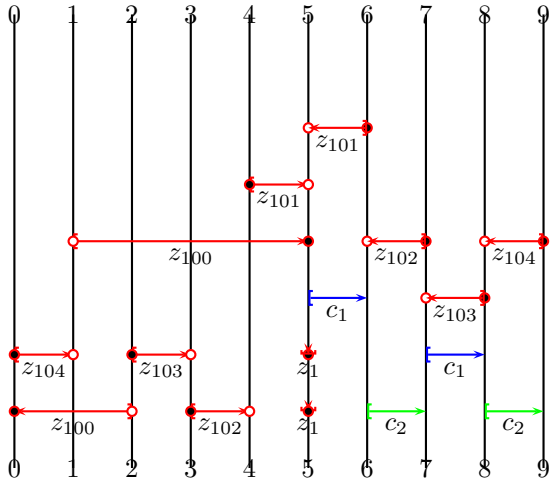
Generalized Equation root-22.1

We begin from the GE root-22 (see pp. 101). We consider its print

```
Print 1: =0=2*<1<3*<4*<5*<2=6*
```

Sequence of actions in performing the Print 1:
Step 1: Added (new) boundary 3.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 7.
Step 3: Moved (old) base [1-2:z100+.] to (new) boundaries 3 - 7.
Step 4: Moved (old) base [0-1:z104+.] to (new) boundaries 2 - 3.
Step 5: Collapsed (new) base [2-7:z1+.] to the empty base (7,7).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-22.1—is illustrated below:



GE Information: Carrier: [0-2:z100-.] ; Carrier Dual: [1-5:z100+.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [1-5:z100+.] and its dual are of opposite polarity, yet intersect. The base [0-2:z100-.] and its dual are of opposite polarity, yet intersect. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-22.1, as derived from the application of a print to root-22.

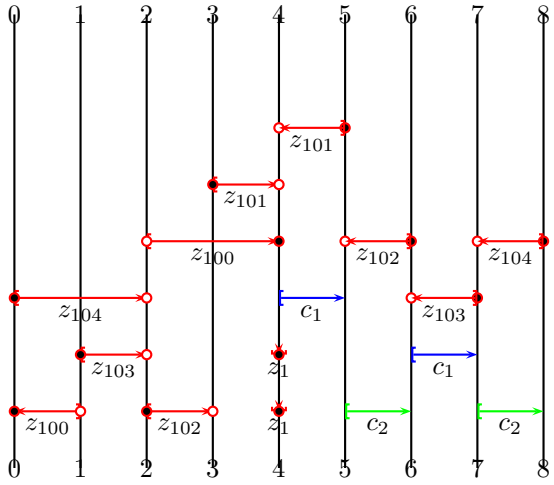
Generalized Equation root-22.2

We begin from the GE root-22 (see pp. 101). We consider its print

```
Print 2: =0=2*<3*<1=4*<5*<2=6*
```

Sequence of actions in performing the Print 2:
Step 1: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 6.
Step 2: Moved (old) base [1-2:z100+.] to (new) boundaries 4 - 6.
Step 3: Moved (old) base [0-1:z104+.] to (new) boundaries 2 - 4.
Step 4: Collapsed (new) base [2-6:z1+.] to the empty base (6,6).
Step 5: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 6: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-22.2—is illustrated below:



GE Information: Carrier: [0-2:z104+.] ; Carrier Dual: [7-8:z104-.] ; Critical Boundary: 2; Observe the following facts about this GE: The base [7-8:z104-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-22.2, as derived from the application of a print to root-22.

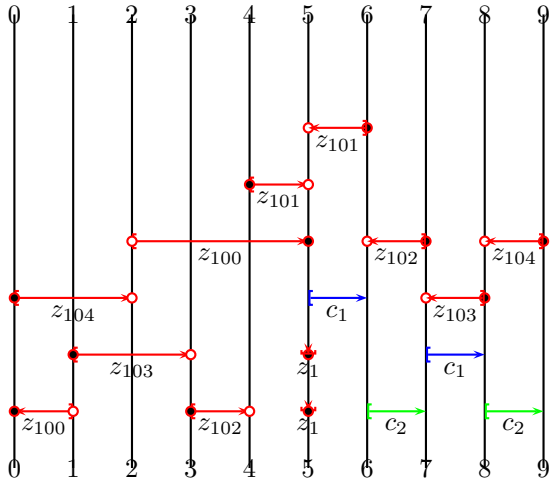
Generalized Equation root-22.3

We begin from the GE root-22 (see pp. 101). We consider its print

```
Print 3: =0=2*<3*<1<4*<5*<2=6*
```

Sequence of actions in performing the Print 3:
Step 1: Added (new) boundary 4.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 7.
Step 3: Moved (old) base [1-2:z100+.] to (new) boundaries 4 - 7.
Step 4: Moved (old) base [0-1:z104+.] to (new) boundaries 2 - 4.
Step 5: Collapsed (new) base [2-7:z1+.] to the empty base (7,7).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-22.3—is illustrated below:



GE Information: Carrier: [0-2:z104+.] ; Carrier Dual: [8-9:z104-.] ; Critical Boundary: 1; Observe the following facts about this GE: The base [7-8:z103-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. The base [8-9:z104-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-22.3, as derived from the application of a print to root-22.

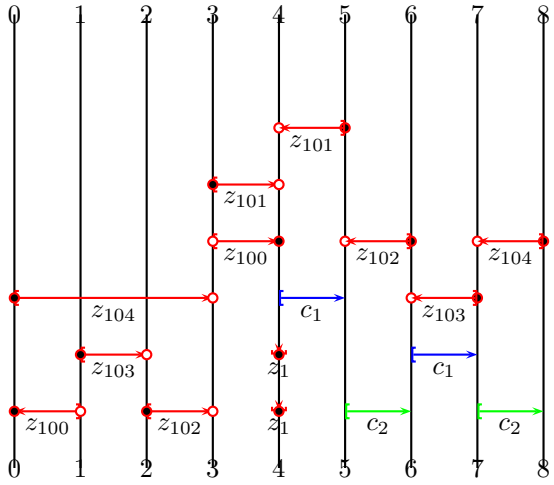
Generalized Equation root-22.4

We begin from the GE root-22 (see pp. 101). We consider its print

```
Print 4: =0=2*<3*<4*<1=5*<2=6*
```

Sequence of actions in performing the Print 4:
Step 1: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 6.
Step 2: Moved (old) base [1-2:z100+.] to (new) boundaries 5 - 6.
Step 3: Moved (old) base [0-1:z104+.] to (new) boundaries 2 - 5.
Step 4: Collapsed (new) base [2-6:z1+.] to the empty base (6,6).
Step 5: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 6: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-22.4—is illustrated below:



GE Information: Carrier: [0-3:z104+.] ; Carrier Dual: [7-8:z104-.] ; Critical Boundary: 3; Observe the following facts about this GE: The base [7-8:z104-.] has constraints with its dual that stretch the constant segment 7 - 8 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-22.4, as derived from the application of a print to root-22.

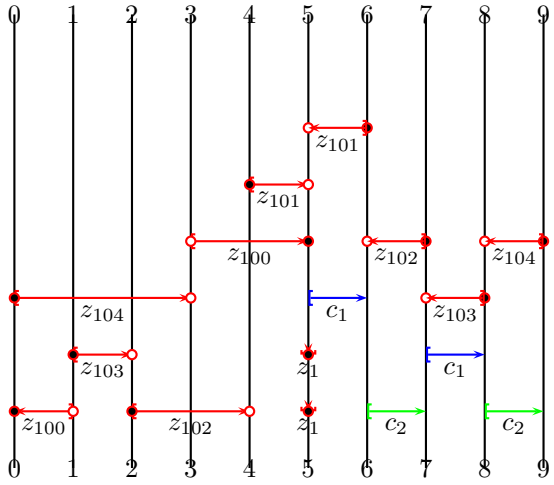
Generalized Equation root-22.5

We begin from the GE root-22 (see pp. 101). We consider its print

```
Print 5: =0=2*<3*<4*<1<5*<2=6*
```

Sequence of actions in performing the Print 5:
Step 1: Added (new) boundary 5.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 7.
Step 3: Moved (old) base [1-2:z100+.] to (new) boundaries 5 - 7.
Step 4: Moved (old) base [0-1:z104+.] to (new) boundaries 2 - 5.
Step 5: Collapsed (new) base [2-7:z1+.] to the empty base (7,7).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-22.5—is illustrated below:



GE Information: Carrier: [0-3:z104+.] ; Carrier Dual: [8-9:z104-.] ; Critical Boundary: 2; Observe the following facts about this GE: The base [6-7:z102-.] has constraints with its dual that stretch the constant segment 6 - 7 to length different from 1. The base [8-9:z104-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-22.5, as derived from the application of a print to root-22.

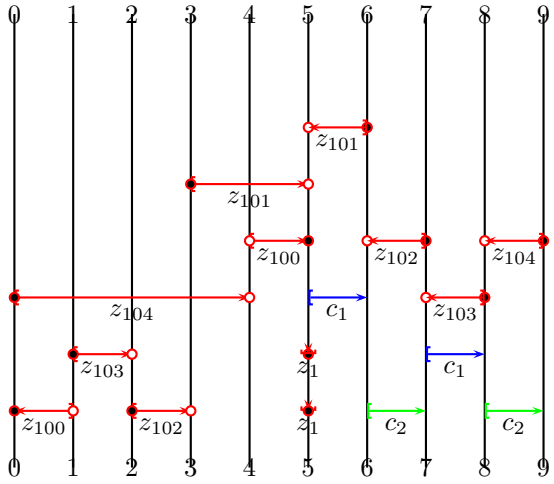
Generalized Equation root-22.6

We begin from the GE root-22 (see pp. 101). We consider its print

```
Print 6: =0=2*<3*<4*<5*<1<2=6*
```

Sequence of actions in performing the Print 6:
Step 1: Added (new) boundary 6.
Step 2: Moved (old) base [0-2:z1+.] to (new) boundaries 2 - 7.
Step 3: Moved (old) base [1-2:z100+.] to (new) boundaries 6 - 7.
Step 4: Moved (old) base [0-1:z104+.] to (new) boundaries 2 - 6.
Step 5: Collapsed (new) base [2-7:z1+.] to the empty base (7,7).
Step 6: Deleted (new) boundary 0 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.
Step 7: Deleted (new) boundary 1 because it is not used inside any base. This will cause renumbering of higher numbered boundaries.

Upon applying the print, the GE we obtain—which we refer to as root-22.6—is illustrated below:



GE Information: Carrier: [0-4:z104+.] ; Carrier Dual: [8-9:z104-.] ; Critical Boundary: 3; Observe the following facts about this GE: The base [5-6:z101-.] has constraints with its dual that stretch the constant segment 5 - 6 to length different from 1. The base [8-9:z104-.] has constraints with its dual that stretch the constant segment 8 - 9 to length different from 1. These observations show that the GE above is degenerate. This GE is a leaf in the GE tree. This branch of the tree has led us to a dead end.

This completes the consideration of root-22.6, as derived from the application of a print to root-22.

23 Acknowledgements

The authors acknowledge that this report was generated by software developed as part of a funded project supported by a research grant (H98230-06-1-0042) from the National Security Agency. We also give special thanks to Alexei Miasnikov and Olga Kharlampovich for many helpful discussions along the way.