0^{th} level agent model and Test

Test 0.0 (0 agent attributes; 2 hours of simulated trading time)

Initial Settings: Stock AA (initial order book is 81500 - 81502…1^{st} 3 seconds of the trading day, 160 standing orders…all of which have been modified to size 100 in accordance with the test parameters).

Population Model

- **The population will be uniform.**

Agent Model

- **Action Prompt:** agents will schedule the action prompt for action every 2 seconds. The likelihood of an action is fixed at 0.5.
- **Cancel Prompt:** an existing order will have a 25% chance of being cancelled.
- **Order Prompt:** an agent will choose to place a new order with likelihood 0.25
- **Bid Ask Prompt:** the likelihood of a bid will be fixed at 0.5
- **Bid Price Prompt:** bid price will be set by a stochastic throw at a Gaussian whose mean is the existing best bid, and whose standard deviation is the difference between the price and the best ask.
- **Bid Size Prompt:** all bids will have a size of 100 shares.
- **Ask Price Prompt:** ask price will be set by a stochastic throw at a Gaussian whose mean is the existing best ask, and whose standard deviation is the difference between the price and the best bid.
- **Ask Size Prompt:** all asks will have a size of 100 shares
- **No Individual Attributes or Order Book Statistics are used in this test.**

Trajectory Model

- **Hurst Exponent:** measuring
  - price over the simulation time
  - number of orders over the simulation time
- **Total number of orders**
- **Total number of trades**
- **Individual agent total orders, total bids, total asks**
- **Individual agent profit and loss**
  - **All agents are assumed to begin with $0 but an unlimited supply of credit and the ability to borrow stocks in order to sell them at current cost in unlimited amounts**
  - **The calculation of net worth is:**
    - **+ Sale of stock (number of shares x price at time of sale)**
    - **- Purchase of stock (number of share x price at time of sale)**
    - **+ Final stock holdings (number of shares held (or owed) x price at closing)**

0^{th} level agent model and Test

Expected Results: The results of this test should be a random price walk with steady volume of incoming orders and a steady number of open orders, and thus $H_{price} \approx 0.5$ and $H_{volume} \approx 0.5$. The proportion of market orders (orders that cross the opposite best price) to limit orders should be small. The number of orders per agent should be relatively uniform, as should the number of bids (resp. asks) for each agent. Individual agent profit and loss should conform to a normal distribution.

Test 0.1 (1 agent attribute, 2 hours of simulated trading time)

Initial Settings: Stock AA (initial order book is 81500 - 81502…1^{st} 3 seconds of the trading day, 160 standing orders…all of which have been modified to size 100 in accordance with the test parameters).

Population Model

- One **Attribute** is included in the model: **aggressiveness.**
- **Aggressiveness**: individual aggressiveness shall be drawn from a Gaussian distribution with mean 0 and standard deviation 0.25.

Agent Model

- **Action Prompt:** agents will schedule the action prompt for action every 2 seconds. The likelihood of an action is fixed at 0.5 + his individual ***aggressiveness*** attribute.
- **Cancel Prompt:** an existing order will have a 25% chance of being cancelled.
- **Order Prompt:** an agent will choose to place a new order with likelihood 0.25.
- **Bid Ask Prompt:** the likelihood of a bid will be fixed at 0.5
- **Bid Price Prompt:** bid price will be set by a stochastic throw at a Gaussian whose mean is the existing best bid, and whose standard deviation is the difference between the price and the best ask.
- **Bid Size Prompt:** all bids will have a size of 100 shares.
- **Ask Price Prompt:** ask price will be set by a stochastic throw at a Gaussian whose mean is the existing best ask, and whose standard deviation is the difference between the price and the best bid.
- **Ask Size Prompt:** all asks will have a size of 100 shares
- **No Order Book Statistics are used in this test.**

Trajectory Model

- **Hurst Exponent:** measuring

  - price over the simulation time
  - number of orders over the simulation time
- **Total number of orders**
- **Total number of trades**
- **Individual agent total orders, total bids, total asks**
- **Individual agent profit and loss**
  - **All agents are assumed to begin with $0 but an unlimited supply of credit and the ability to borrow stocks in order to sell them at current cost in unlimited amounts**
  - **The calculation of net worth is:**
    - **+ Sale of stock (number of shares x price at time of sale)**
    - **- Purchase of stock (number of share x price at time of sale)**
    - **+ Final stock holdings (number of shares held (or owed) x price at closing)**

Expected Results: The results of this test should be a steady volume of orders, $H_{volume} \approx 0.5$. Unlike Test 0.0, though, we should see a wide distribution in the number of orders (resp. bids, asks) per agent. Variations in the mean of the **aggressiveness** above or below 0 should produce changes in the overall volume compared to the neutral setting. Changes in standard deviation should not affect the overall volume but should change the distribution of the number of bids per agent. The price should remain a random walk, $H_{price} \approx 0.5$. Individual agent profit and loss should continue to conform to a normal distribution.

Test 0.2 (2 agent attributes, 2 hours of simulated trading time)

Initial Settings: Stock AA (initial order book is 81500 - 81502…1st 3 seconds of the trading day, 160 standing orders…all of which have been modified to size 100 in accordance with the test parameters).

Population Model

- One **Attribute** is included in the model: **aggressiveness.**
- **Aggressiveness**: individual aggressiveness shall be drawn from a Gaussian distribution with mean 0 and standard deviation 0.25.
- **Optimism** shall be drawn from a Gaussian with a mean of 1 and a standard deviation of 0.005.

Agent Model

0^{th} level agent model and Test

- **Action Prompt:** agents will schedule the action prompt for action every 2 seconds. The likelihood of an action is fixed at 0.5 + ***aggressiveness*** attribute.
- **Cancel Prompt:** an existing order will have a 25% chance of being cancelled.
- **Order Prompt:** an agent will choose to place a new order with likelihood 0.25.
- **Bid Ask Prompt:** the likelihood of a bid will be fixed at 0.5
- **Bid Price Prompt:** bid price will be set by the product of 1) a stochastic throw at a Gaussian whose mean is the existing best bid, and whose standard deviation is the difference between the current price and the best ask, and 2) the agent's individual **optimism** attribute.
- **Bid Size Prompt:** all bids will have a size of 100 shares.
- **Ask Price Prompt:** ask price will be set by the product of 1) a stochastic throw at a Gaussian whose mean is the existing best ask, and whose standard deviation is the difference between the current price and the best bid, and 2) the agent's individual **optimism** attribute.
- **Ask Size Prompt:** all asks will have a size of 100 shares
- **No Order Book Statistics are used in this test.**

Trajectory Model

- **Hurst Exponent:** measuring:
  - price over the simulation time
  - number of orders over the simulation time
- **Total number of orders**
- **Total number of trades**
- **Individual agent total orders, total bids, total asks**
- **Individual agent profit and loss**
  - **All agents are assumed to begin with $0 but an unlimited supply of credit and the ability to borrow stocks (in order to sell them) at current cost in unlimited amounts**
  - **The calculation of net worth is:**
    - **+ Sale of stock (number of shares x price at time of sale)**
    - **- Purchase of stock (number of share x price at time of sale)**
    - **+ Final stock holdings (number of shares held (or owed) x price at closing)**

Expected Results: The results of this test should be a steady volume of orders, but a wide distribution in the number of orders per agent, similar to that seen in Test 0.1 (and thus $H_{volume} \approx 0.5$). The optimism parameter should cause a higher proportion of market orders (the effect will be a flattening of the ideal price distribution, resulting in more bids toward the edges of that distribution). This will result in a greater volume of trade, though with optimism set a mean of 1, the price should still demonstrate a random walk, $H_{price} \approx 0.5$. Moving the mean of the optimism parameter to 1.01 should result in a rising price curve, while lowering it to 0.99 should result in a

falling price curve. In both of these cases we should see a change in the Hurst exponent for price, such that $H_{price} > 0.5$. The effect of the rising price curve ($\mu_{optimism} = 1.1$) and the herd following implicit in the order pricing should allow those agents with a higher optimism setting to profit from this effect. As a result, we might speculate that the agent wealth distribution would look more bi-modal than in the previous tests.

Test 0.3 (2 agent attributes, 1 order book statistic, 2 hours of simulated trading time)

Initial Settings: Stock AA (initial order book is 81500 - 81502…1^{st} 3 seconds of the trading day, 160 standing orders…all of which have been modified to size 100 in accordance with the test parameters).

Population Model

- One **Attribute** is included in the model: **aggressiveness.**
- **Aggressiveness**: individual aggressiveness shall be drawn from a Gaussian distribution with mean 0 and standard deviation 0.25.
- **Optimism** shall be drawn from a Gaussian with a mean of 1 and a standard deviation of 0.005.

Agent Model

- **Action Prompt:** agents will schedule the action prompt for action every 2 seconds. The likelihood of an action is fixed at 0.5 + **change in volume (an order book statistic)** + ***aggressiveness*** attribute.
- **Cancel Prompt:** an existing order will have a 25% chance of being cancelled.
- **Order Prompt:** an agent will choose to place a new order with likelihood 0.25.
- **Bid Ask Prompt:** the likelihood of a bid will be fixed at 0.5
- **Bid Price Prompt:** bid price will be set by the product of 1) a stochastic throw at a Gaussian whose mean is the existing best bid, and whose standard deviation is the difference between the current price and the best ask, and 2) the agent's individual **optimism** attribute.
- **Bid Size Prompt:** all bids will have a size of 100 shares.

0^{th} level agent model and Test

- **Ask Price Prompt:** ask price will be set by the product of 1) a stochastic throw at a Gaussian whose mean is the existing best ask, and whose standard deviation is the difference between the current price and the best bid, and 2) the agent's individual **optimism** attribute.
- **Ask Size Prompt:** all asks will have a size of 100 shares
- Order book statistics
    - **Change in volume:** the ratio of the difference in the logs of volume of order from the previous 2 second of trading to the second before that over the volume from the previous 2 second.
        - **Example:** the volume or orders at time 0→2 seconds is 15 orders, time 2→4 seconds is 11 orders (a falling volume), then the baseline action probability for all agents at time > 4 is:

$$p = 0.5 + \frac{log11 - log15}{log11} = 0.37 + \text{aggressiveness}$$

        - This is an extreme example because of the low numbers. Given that, in the base model (Test 0.0), the likelihood of an order is 0.5 (action) x 0.25 (order) every 2 seconds, then a simulation involving 2000 agents would be expected to produce (0.5 x 0.25 x 2000) = 125 orders per second.
        - There is a possibility that the change in volume could drop the likelihood of action below 0, resulting in "no action" at a step and thus a division by zero problem in the subsequent step. We can define the division by zero exception in this step to return an action probability equal to the agent's aggressiveness.

Trajectory Model

- **Hurst Exponent:** measuring:
    - price over the simulation time
    - number of orders over the simulation time
- **Total number of orders**
- **Total number of trades**
- **Individual agent total orders, total bids, total asks**
- **Individual agent profit and loss**
    - **All agents are assumed to begin with $0 but an unlimited supply of credit and the ability to borrow stocks (in order to sell them) at current cost in unlimited amounts**
    - **The calculation of net worth is:**
        - **+ Sale of stock (number of shares x price at time of sale)**
        - **- Purchase of stock (number of share x price at time of sale)**
        - **+ Final stock holdings (number of shares held (or owed) x price at closing)**

0^{th} level agent model and Test

Expected Results: The results of this test should be similar to Test 0.2, except that we should see an unsteady volume of orders, with bursts of orders followed by periods of low numbers of orders (and thus periods of high and low trade volume…in other words, $H_{volume} > 0.5$). Other results are difficult to project. The interaction of optimism, aggressiveness, and significant volume fluctuations represents a level of complexity that, even at neutral settings, is likely to produce regular patterns, but ones that are difficult to see ahead of time.