Problem Section 2

Mon Oct 9 2023

Learning Outcomes

The problems are designed to build conceptual understanding and problem-solving skills. The emphasis is on learning to find, evaluate and build confidence. The specific tasks include:

- Enumerate and count

- Apply Bayes' rule

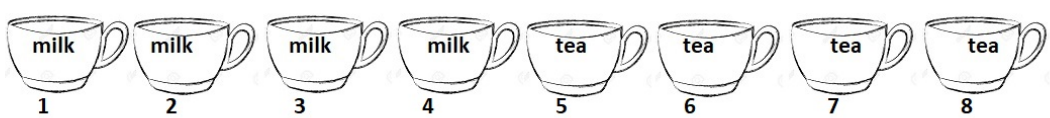
- Calculate probabilities for independent events

- Back up and support work with relevant explanations

Exercises

1. (*Lady Tasting Tea*) In a famous experiment conducted by Sir. R. A. Fisher, 8 cups of tea - 4 prepared by pouring the milk first and then the tea and 4 prepared by pouring the tea first and then the milk – were presented in random order to Muriel Bristol who claimed she could tell the difference.

The methods employed in the experiment were fully disclosed to the subject and she had to select the 4 cups prepared with the "milk poured first".



a. The sample space S for this experiment consists of all groupings of 4 cups selected from eight. How many such groups are there? Write your answer symbolically first and also calculate it in the code chunk below.

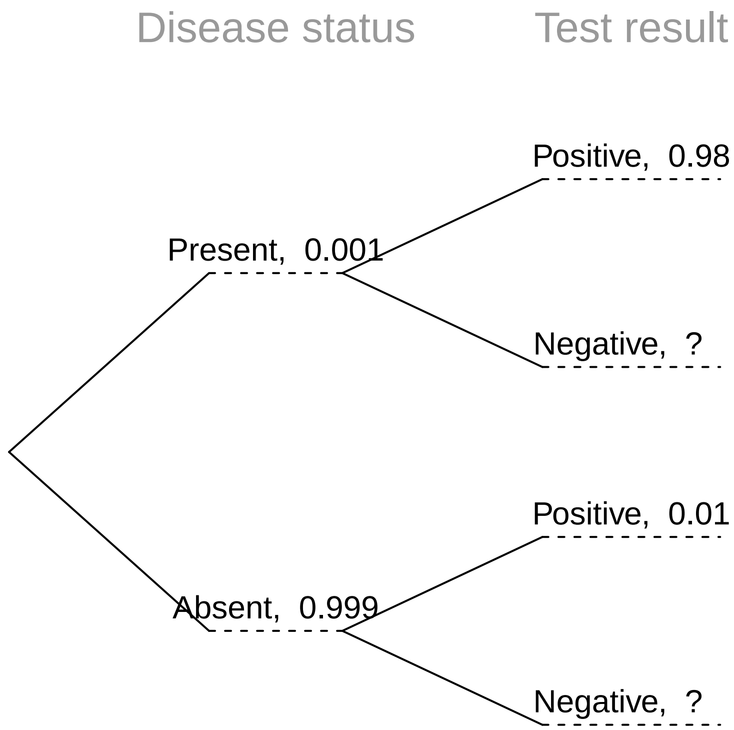
```
#calculate number of elements in S
## in the code chunk below
```

b. Let E denote the event that there are 3 cups with "milk poured first" and 1 cup with "tea poured first". How many elements are in E? Write your answer symbolically first and also calculate it in a code chunk below.

```
#calculate number of elements in E
```

c. If the lady has no ESP and is just guessing randomly, it is reasonable to assume that every outcome in S is equally likely. Calculate $P(E)$ under this assumption. You may do this in a code chunk.

2. One common application of conditional probabilities is in calculating inverse probabilities. For example, consider the following tree diagram of probabilities relating a screening test for a disease with the actual presence of disease.



Let D denote that a randomly selected individual has the disease, and T the event that the screening test is positive.

The first (primary) branch of the tree gives *unconditional* probabilities of disease being present or absent:

$$P(D) = 0.001, P(D^c) = 0.999$$

while the secondary branches state conditional probabilities of the test coming out positive (or negative) given the disease status. For example, you are given that

$$P(T|D) = 0.98, P(T|D^c) = 0.01.$$

a. Fill in the values in the tree diagram for the conditional probabilities $P(T^c|D)$ and $P(T^c|D^c)$ indicated by a "?" (Hint: conditional probability follows Law of Complements.)

b. Suppose the test is positive. What is the probability that the person actually has disease? In other words, find the inverse conditional probability: $P(D|T)$

Hint: see example 4.2 from "Chapter 4: Conditional Probability"

3. Suppose that string of tree lights you just bought has twenty-four bulbs wired in series. If each bulb has a 99.9% chance of "working" when the current is applied, what is the probability that the string itself will not work? You may assume that bulb failures are independent.