Problem Section 1

Discrete Distributions

Learning Outcomes

The problems are designed to build conceptual understanding and problem-solving skills. The emphasis is on learning to find, evaluate and build confidence. The specific tasks include:

- Construct a table of joint probabilities and calculate marginal and conditional distributions.
- Calculate probabilities for the trinomial experiment
- Work with conditional probabilities
- Back up and support work with relevant explanations

Instructions

- Please work in groups of three to answer the problems below.
- Each member of the group must write up their work on a separate sheet of paper. Be sure to clearly write your full name on the top as it appears in the gradebook.

Exercises

1. A fair coin is tossed four times. X is the number of heads that come up on the first three tosses and Y is the number of heads that come up on tosses 2, 3, 4.

   a. Fill in the cells of the table showing the joint PMF of $\langle X, Y \rangle$

Table 1: Joint PMF of $\langle X, Y \rangle$

y \ x	0	1	2	3
0				
1				
2				
3				

   b. Find the marginal distribution of X and also of Y.

   c. Find the conditional probabilities $P(X = 2|Y = 2)$ and $P(Y = 3|X = 0)$.

2. Suppose that five independent observations are drawn from the continuous PDF

$$f(t) = 2t \quad 0 \leq t < 1$$

   Let X denote the number of t's that fall in the interval $0 \leq t < \frac{1}{3}$ and let Y denote the number of t's in the interval $\frac{1}{3} \leq t < \frac{2}{3}$.

a. Find the probability that $X = 2$ and $Y = 1$. How will you calculate this in R? Go ahead and use R to find the probability.

b. Find the probability that $X = 1$. How will you calculate this in R Go ahead and use R to find the probability.

c. How many observations should you expect will lie in the interval $0 \le t < \frac{1}{3}$. That is, find $E[X]$. What is the standard deviation of X?

3. (For HW 1 problem 3) The binomial theorem states that for real numbers a and b and an integer n:

$$(a+b)^n = \binom{n}{0} a^0 b^n + \binom{n}{1} a^1 b^{n-1} + \binom{n}{2} a^2 b^{n-2} + \cdots + \binom{n}{n} a^n b^0$$

Show, using the binomial theorem, that

$$\sum_{x=0}^{n} \binom{n}{x} = 2^n.$$