Problem Section 7

Elements of Significance Testing

Learning Outcomes

The problems are designed to build conceptual understanding and problem-solving skills. The emphasis is on learning to find, evaluate and build confidence. The specific tasks include:

- Calculate one and two-sided P-values
- Find the Type I and Type II errors for a given decision rule
- Find an empirical P-value
- Back up and support work with relevant explanations

Instructions

- Please work in groups of three to answer the problems below.
- Each member of the group must write up their work on a separate sheet of paper. Be sure to clearly write your full name on the top as it appears in the gradebook.

Exercises

1. A children's game uses a six sided die with a picture of a ghost named Hugo on one side and numbers on the other sides. If the die is fair, the ghost should be rolled 1/6 of the time. You test the die by rolling it $n = 10$ times and and the ghost is rolled $x = 3$ times. Calculate the P-value for an exact binomial test of the hypothesis

$$H_0 : \pi = \frac{1}{6} \quad H_1 : \pi \neq \frac{1}{6}$$

2. As input for a new inflation model, economists predicted that the average cost of a hypothetical "food basket" in western WA in July would be \$145.75. The standard deviation (σ_0) of basket prices was assumed to be \$9.50, a figure that has held fairly constant over the years. To check their prediction, a sample of twenty-five baskets representing different parts of the region were checked in late July, and the average cost was \$149.75.

a. Let μ_0 denote the true mean price of the food basket in July in Western WA. Write the null and alternative hypothesis.

b. Suppose the test will be based on $\bar{X}$ the sample mean. What is its sampling distribution? (You may assume the CLT applies)

c. Calculate the P-value associated with $\bar{x} = \$149.75$.

3. An experimenter takes a sample of size 4 - X_1, X_2, X_3, X_4 - from the Poisson probability model,

$$f(x) = e^{-\lambda_0} \frac{\lambda_0^x}{x!} \quad x = 0, 1, 2, \ldots$$

and wishes to test $H0 : \lambda_0 = 6$ versus $H_1 : \lambda_0 < 6$. The test will be based on the statistic $S = X_1 + X_2 + X_3 + X_4$.

a. Find the P-value associated with observing $s_{obs} = 15$.

b. Suppose we decide to conduct the test at level $\alpha = 0.1$. What values of s_{obs} will you reject H_0 for?

c. Find the Type I error rate for your test in part b.

d. Calculate the Type II error rate for your test in part b. when $\lambda_0 = 4$.

4. An urn contains ten marbles: an unknown number of them are white, the rest red. We wish to test:

$$H_0 : \text{exactly half are white}$$

versus

$$H_1 : \text{more than half are white}$$

We will draw randomly, without replacement, three marbles and reject H_0 if two or more are white.

a. Find the Type I error rate of this test. (Hint: the number of white marbles among the three drawn is called a hypergeometric random variable)

b. Find the Type II error rate in two situations.

- 60% of the marbles in the urn are white
- 70% of the marbles in the urn are white.