Problem Section 2

Continuous Distributions & Independence

Learning Outcomes

The problems are designed to build conceptual understanding and problem-solving skills. The emphasis is on learning to find, evaluate and build confidence. The specific tasks include:

- Eyeballing if two random variables are independent of each other
- Performing probability calculations with joint PDFs
- Calculating the mean and standard deviation of linear combinations of independent random variables.
- Back up and support work with relevant explanations

Instructions

- Please work in groups of three to answer the problems below.
- Each member of the group must write up their work on a separate sheet of paper. Be sure to clearly write your full name on the top as it appears in the gradebook.

Exercises

1. Suppose the probabilistic behavior of two random variables X and Y is defined by the joint PDF:

$$f(x,y) = 2x + y - 2xy \quad 0 \leq x < 1, \quad 0 \leq y < 1$$

a. Just by eyeballing, are X and Y independent? Why or why not?

b. Calculate $P(X < Y)$.

2. A mason is contracted to build a patio retaining wall. Plans call for the base of the wall to be a row of fifty 10-inch bricks, each separated by $\frac{1}{2}$ inch thick mortar. Suppose that the bricks used are randomly chosen from a population of bricks whose mean length is 10 inches and whose standard deviation is $\frac{1}{32}$ inch. Also, suppose that the mason, on the average, will make the mortar $\frac{1}{2}$ inch thick, but that the actual dimension will vary from brick to brick, the standard deviation of the thicknesses being $\frac{1}{16}$ inch. What is the standard deviation of L, the length of the first row of the wall? What assumption are you making?

3. Suppose X and Y are independent random variables, with $Var\left[X\right] = Var\left[Y\right] = 1$. Consider the new random variable formed by the linear transformation

$$W = c\,X + (1-c)\,Y.$$

Find the value of c that minimizes the variance of W.